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BENJAMIN GREENLEAF, A. M.

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INTRODUCTION.

CULTURE is progressive in its nature. Higher, still higher, is the true educational spirit.

Advance in methods of instruction makes new, improved text-books a necessity; and, to subserve wants apparently not heretofore fully provided for, this work has been carefully prepared.

Without being in any respect redundant, it is intended to be complete in details and comprehensive in scope;

Combining with processes the most scientific the greatest simplicity;

Developing principles by inductive methods, deducing rules from rational solutions, and encouraging self-reliance and originality by numerous exercises in analysis;

Making written arithmetic in all its steps intellectual; and

Keeping prominently in view the practical uses of numbers, by various applications of a business character.

While it avoids obsolete or useless material, it properly treats new topics requiring attention, such as the Metric System of Weights and Measures, Annual Interest, Internal Revenue, etc.; and

Enforces thorough educational results, by orderly arrangement of subjects, and by systematic review questions and exercises.

The prominence given in this book to the enunciation of Principles, will, it is believed, commend itself to the enlightened educator, since, without a knowledge of these principles, the art of using numbers becomes mere mechanical ciphering.

Multiplication and Division of Decimal Fractions have been much simplified by assimilating their processes to those of like cases in Common Fractions, and by making the corresponding rules substantially the same.

By treating of Fractions before Compound Denominate Numbers, the Reduction of the latter is made more thorough, and a number of special rules is avoided.

Many rules of limited application are also dispensed with, by analyzing single examples, of some anomalous kind, as a guide to the solution of all others of its class.

The examples have been selected with special reference to their adaptation to the present wants of active life.

Grateful expression of indebtedness is due to Hon. John A. Kasson of Iowa, to Prof. H. A. Newton of Yale College, and to many others, for favors received while preparing this volume.

PERMANENT EDITION.

The generous favor extended to this book, and its wide introduction into the best schools, have led to the sale of several large editions, in a few months. Encouraged by this marked appreciation, the work has been critically re-examined, and put in a permanent form.

H. B. MAGLATHLIN.

KINGSTON, MASS., March, 1867.

CONTENTS.

PRACTICAL ARITHMETIC.

DEFINITIONS.

1. A **Unit** is a single thing, or one. Thus,

A dollar is a unit, an apple is a unit, etc.

2. **Quantity** is any thing that can be measured, or computed. Thus,

Distance is quantity, since it can be measured, so as to be named miles, rods, etc.

3. A **Number** is a unit, or a collection of units of the same kind.

One, two, three, four, etc., which show how many units there are of any quantity, express numbers.

4. The **Unit of a Number** is one of that number. Thus,

One dollar is the unit of three dollars, and one the unit of five.

5. **Like Numbers** are such as have the same unit. Thus,

Five *dollars* and seven *dollars* are like numbers.

6. A **Concrete Number** is a number in which some kind of unit is named. Thus,

Two *books*, five *days*, seven *dollars*, are concrete numbers.

What is a Unit? A Quantity? A Number? The Unit of a Number? Like Numbers? A Concrete Number?

7. An **Abstract Number** is a number in which no particular kind of unit is named. Thus,

Two, five, in which no particular unit is named, are abstract numbers.

8. An **Operation** is a process performed with numbers.

9. An **Answer** is the result of a correct operation.

10. A **Solution** is an explanation of the operation.

11. A **Rule** is a direction for performing an operation.

12. An **Example** is an application of a rule.

13. An **Exercise** is a lesson for practice.

14. **Arithmetic** is the science of numbers and the art of using them.

15. **Practical Arithmetic** treats of the methods of applying numbers to practical or business purposes.

NOTATION AND NUMERATION.

16. **Notation** is the method of writing numbers.

17. **Numeration** is the method of reading numbers.

18. **Figures** are certain marks or characters used to express numbers.

The method of expressing numbers by figures is called the *Arabic,* because it was used by the Arabs.

Ten different figures are used in expressing numbers.

What is an Abstract Number? What is an Operation? An Answer? A Solution? A Rule? An Example? An Exercise? Arithmetic? Practical Arithmetic? Notation? Numeration? What are Figures? What is the method of expressing numbers by figures called? Why so called?

Names, or value denoted.	Figures as printed.	Figures as written.	Names, or value denoted.	Figures as printed.	Figures as written.
Cipher,	0	*0*	Five,	5	*5*
One,	1	*1*	Six,	6	*6*
Two,	2	*2*	Seven	7	*7*
Three,	3	*3*	Eight,	8	*8*
Four,	4	*4*	Nine,	9	*9*

19. The figures 1, 2, 3, 4, 5, 6, 7, 8, 9, are called *significant* figures, because each *signifies*, or stands for, the number, which its name denotes.

The figure 0, or cipher, is sometimes called *zero*, or *naught*, because, when used alone, it stands for *no number*. Thus,

0 dollars means *no* dollars.

No number higher than nine can be expressed by a single figure, but by combining figures all other numbers may be denoted.

UNITS, TENS, HUNDREDS.

20. In naming numbers, nine units and one more are regarded as forming a single group, or collection, called TEN.

One ten and one are called ELEVEN; one ten and two, TWELVE; one ten and three, THIRTEEN; one ten and four, FOURTEEN; etc., "*teen*" meaning "*and ten*."

Two tens are called TWENTY; three tens, THIRTY; four tens, FORTY; etc., "*ty*" meaning "*tens*."

21. To express ten, twenty, thirty, etc., we write 1, 2, 3, etc., denoting the number of tens, to the left of 0. Thus,

Ten,	10	Forty,	40	Seventy,	70
Twenty,	20	Fifty,	50	Eighty,	80
Thirty,	30	Sixty,	60	Ninety,	90

Why are 1, 2, 3, etc., called significant figures? What is the cipher sometimes called? How high numbers can be expressed by a single figure? How may all numbers be denoted? What name is given to nine and one more? To ten and one, etc.? How do we express ten, twenty, thirty, etc.?

where 0 denotes the absence of *ones*, or units of the *first order*, and makes other figures express tens, or units of the *second* order.

22. To express the whole numbers intermediate between ten and twenty, twenty and thirty, thirty and forty, etc., we write the figures denoting the number of tens, to the left of 1, 2, 3, etc., expressing the ones, or units of the first order. Thus,

Eleven,	11	Sixteen,	16	Twenty-two,	22
Twelve,	12	Seventeen,	17	Twenty-three,	23
Thirteen,	13	Eighteen,	18	Twenty-four,	24
Fourteen,	14	Nineteen,	19	Twenty-five,	25
Fifteen,	15	Twenty-one,	21	Twenty-six,	26

and so on.

23. Ten tens are called ONE HUNDRED, which forms a unit of the *third* order, and is written 100.

Therefore, to express one hundred, two hundred, three hundred, etc., we write the figure denoting their number with two ciphers at the right. Thus,

Two hundred,	200	Six hundred,	600
Three hundred,	300	Seven hundred,	700
Four hundred,	400	Eight hundred,	800
Five hundred,	500	Nine hundred,	900

24. In expressing a whole number by three figures, we place the figure denoting the *hundreds* in the *third*, the figure denoting the *tens* in the *second*, and the figure denoting the *units* in the *first place* from the right. Thus,

Two hundred and sixty-three, or 2 hundreds, 6 tens, and 3 units, is written 263.

Five hundred and seven, or 5 hundreds, 0 tens, and 7 units, is written 507.

How do we express numbers between ten and twenty? What are ten tens called? What forms a unit of the third order? In writing a number expressed by three figures, how are the figures placed?

Exercises.

Write in figures arranged in columns:

1. The numbers between forty-five and sixty-three.
2. The numbers between ninety-one and one hundred.
3. One hundred and one, one hundred and eleven.
4. Eighty-eight, eight, eight hundred and eighty.
5. Thirteen, thirty-one, three hundred and one.
6. Six hundred and five, five hundred and sixty-six.
7. Eleven, seventy-seven, seven hundred and eleven.

THOUSANDS.

25. Ten hundreds are called ONE THOUSAND, which forms a unit of the *fourth* order, and is written 1000.

Therefore, to express one thousand, two thousand, three thousand, etc., we write the figures denoting their number, with three ciphers at the right. Thus, we write,

Two thousand,	2000	Six thousand,	6000
Three thousand,	3000	Seven thousand,	7000
Four thousand,	4000	Eight thousand,	8000
Five thousand,	5000	Nine thousand,	9000

26. Ten thousands are called ONE TEN-THOUSAND, which forms a unit of the *fifth* order, and is written 10000. Also,

Two ten-thousands, or twenty thousand, is written 20000.

Three ten-thousands, or thirty thousand, is written 30000, and so on.

27. Ten ten-thousands are called ONE HUNDRED-THOUSAND, which forms a unit of the *sixth* order, and is written 100000. Also,

Two hundred thousand is written 200000; three hundred thousand is written 300000; and so on.

What are ten hundreds called? What unit does one thousand form? How are one thousand, two thousand, etc., written? What are ten-thousands called? What unit do ten-thousands form? What are ten ten-thousands called? What unit do ten ten-thousands form?

28. The first six ORDERS OF UNITS, beginning with the units, are named: UNITS, *tens*, *hundreds*, THOUSANDS, *ten-thousands*, *hundred-thousands*.

The ORDERS OF FIGURES are the positions they occupy with reference to each other, when written side by side.

Each figure expresses an unvarying number of units, and the order of the figure determines the size or name of the units.

For convenience, in reading numbers expressed by figures, their orders are separated into groups, of three figures each, called PERIODS. Each period takes its name from its right hand order. Thus, we have,

Second Period. THOUSANDS.				First Period. UNITS.		
Hundred-thousands	Ten-thousands	Thousands		Hundreds	Tens	Units
3	0	6	,	5	7	4

where the figures express 3 hundred-thousands 0 ten-thousands 6 *thousands* 5 hundreds 7 tens 4 *units*, or three hundred six thousand five hundred and seventy-four.

29. In general, in writing a number by figures, we write each of the figures in the order of its units, and note the absence of a significant figure, in an order, by a cipher. Thus,

Five thousand and twenty, or 5 thousands 0 hundreds 2 tens 0 units, is written 5,020.

Name the first six orders of units. What are orders of figures? To what do they correspond? How are orders of figures separated for convenience in reading? From what does each period take its name? Name the first two periods. How do we write figures in expressing numbers?

Exercises.

Write in figures and read:

1. Three units of the fourth order, with no units of the third order, two units of the second, and one unit of the first order.

Ans. 3,021; read, three thousand and twenty-one.

2. 5 hundred-thousands 3 ten-thousands and 4 thousands 6 hundreds and 4 tens.

Ans. 534,640; read, five hundred thirty-four thousand six hundred and forty.

3. 415 in the second period, and 405 in the first period.

Ans. 415,405; read, four hundred and fifteen thousand four hundred and five.

4. 207 in the second period and 0 in each order of the first period.

5. 38 thousands 3 hundreds 5 tens and 2 units.

6. 6 hundred-thousands 5 ten-thousands with seven units of the first order.

30. The Arabic system of notation is based upon the following

GENERAL PRINCIPLES.

1. *Numbers may be expressed by writing figures so as to denote their orders of units.*

2. *Ciphers written with other figures denote or mark the orders in which units are omitted.*

3. *Ten units of any lower order are always equal to one of the next higher.*

That is, ten units make one ten, ten tens make one hundred, ten hundreds make one thousand, and so on. Hence.

4. *Each removal of a figure an order towards the left, makes the value expressed ten-fold.*

Give the first general principle. The second. The third. The fourth.

SCALE OF NUMBERS.

31. A **Scale of Numbers** is the number or numbers expressing the law of relation between their different units.

32. In numbers, where ten units of any lower order always make one of the next higher, the scale is *ten*, and *uniform*.

For this reason, the system of numbers in general use has been called from *decem*, the Latin for *ten*, the DECIMAL SYSTEM OF NUMBERS.

33. The common, or French method of Numeration, is exhibited in the following

Numeration Table.

6th Period. QUADRILLIONS.			5th Period. TRILLIONS.			4th Period. BILLIONS.			3d Period. MILLIONS.			2d Period. THOUSANDS.			1st Period. UNITS.		
18th. Hundred-quadrillions.	17th. Ten-quadrillions.	16th. Quadrillions.	15th. Hundred-trillions.	14th. Ten-trillions.	13th. Trillions.	12th. Hundred-billions.	11th. Ten-billions.	10th. Billions.	9th. Hundred-millions.	8th. Ten-millions.	7th. Millions.	6th. Hundred-thousands.	5th. Ten-thousands.	4th. Thousands.	3d. Hundreds.	2d. Tens.	1st. Units.
4	3	1,	5	6	2,	7	9	3,	1	5	4,	2	0	5,	6	3	8;

where the figures express four hundred thirty-one QUADRILLIONS, five hundred sixty-two TRILLIONS, seven hundred ninety-three BILLIONS, one hundred fifty-four MILLIONS, two hundred five THOUSANDS, six hundred and thirty-eight.

A dot (.), called the Decimal Point, is used to mark the units' place, by being written at the right of the units' figure. Thus,

8. is read eight units.

What is a Scale of Numbers? What is the scale of numbers in which ten units of a lower order always make one of the next higher? Beginning at the right, name the periods in the table. The orders. The value expressed.

The periods above Quadrillions, in their order, are Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, Septendecillions, Octodecillions, Novendecillions, Vigintillions, etc.

34. To read numbers expressed by figures.

1. Let it be required to read 7693254.

Pointing off the given figures into periods, beginning at the units' place, we have 7,693,254.

The third period is 7 *millions*, the second period is 693 *thousands*, the first period is 254 *units*; therefore, the whole reads, seven million six hundred ninety-three thousand two hundred and fifty-four. Hence, the

RULE. *Beginning at the units' place, point off the given expression into as many periods as possible, of three figures each.*

Then, begin at the left and read each period, giving after each, excepting the last, the name of the period.

The name of units' period is not given in reading figures, since it is readily understood; and the decimal point, when not written, is also understood.

Examples.

Point off and read the following:

2.	604	11.	80600	20.	998081
3.	98	12.	51155	21.	19167700
4.	704	13.	99666	22.	1324567
5.	1132	14.	80080	23.	82000310
6.	5005	15.	110011	24.	17634042
7.	7230	16.	33333	25.	40040061
8.	990	17.	520404	26.	7892316
9.	6606	18.	700776	27.	43448801
10.	17711	19.	144189	28.	50040030

29. 2001002003004 | 30. 5700800990442736

Name the periods in their order above Quadrillions. Repeat the Rule. Why is not the name of units' period given in reading figures?

25. To write numbers in figures.

1. Let it be required to write in figures, seven million six hundred ninety-three thousand two hundred and four.

Writing the 7 millions as the only order of the third period, the 693 thousands as the orders of the second period, and the 204 as the orders of the first period, we have 7,693,204. Hence, the following

RULE. *Beginning with the highest period to be expressed, write the figures belonging to each period, in their orders, observing to mark the omission of any order of units with a cipher.*

Examples.

Write in figures the following numbers:

2. 8 tens and 7 units. *Ans.* 87.
3. 3 units of the third order and 1 of the first. *Ans.* 301.
4. One hundred and twenty-five.
5. 7 hundreds, nine tens, and 6 units.
6. 8 units of the second order and 9 of the first.
7. Nine hundred and ninety-seven.
8. Five thousand and sixty-two.
9. Fifty-five thousand and five hundred.
10. One hundred six thousand.
11. 90 thousands and nine hundreds. *Ans.* 90,900.
12. 100 thousands 7 hundreds 6 tens and 4 units.
13. One hundred thousand four hundred and fifteen.
14. Thirty-six thousand and forty-six.
15. One million one hundred thousand and one hundred.
16. One hundred fifty-one millions.
17. 3 billions, 4 millions, 14 thousands, 4 hundreds, 5 tens, and 6 units. *Ans.* 3,004,014,456.
18. Sixteen trillions seven hundred forty-one billions two hundred twenty-three millions one hundred seventy-eight thousand.

Repeat the Rule.

ADDITION.

36. 1. Arthur has 4 books and his sister has 3; how many have both together?

SOLUTION. *They have, together, as many books as are equal to 4 books and 3 books, which are 7 books. Therefore, they both together have 7 books.*

2. Paid 8 cents for a pencil and 2 cents for a pen-holder; how much did I pay for the whole?

3. How many are 3 and 6? 5 and 4? 7 and 5?

4. John has 2 apples, Edward 7, and Henry 9; how many have they together?

5. How many are 3 and 1 and 8? 2 and 0 and 6?

6. In a yard are 10 peach trees, 5 apple trees, and 4 plum trees; how many trees are there in all?

The preceding operations are called ADDITION. Hence,

37. Addition is the process of finding a number equal to two or more given numbers of the same kind.

The SUM or AMOUNT is the result of the addition; and contains as many units as there are in all the numbers added.

38. A SIGN is a mark used to denote an operation to be performed, or to shorten an expression.

The SIGN OF ADDITION is an erect cross, $+$, called *plus*. Thus, $3 + 2$, read *three plus two*, denotes that 3 and 2 are to be added.

The SIGN OF EQUALITY is two short parallel horizontal lines, and is read *equals*, or *are equal to*. Thus, $3 + 2 = 5$, is read, three plus two are equal to five.

What is Addition? The Sum or Amount? A Sign? The Sign of Addition? The Sign of Equality?

Addition Table.

1 and	2 and	3 and	4 and	5 and
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 " 3	2 " 4	2 " 5	2 " 6	2 " 7
3 " 4	3 " 5	3 " 6	3 " 7	3 " 8
4 " 5	4 " 6	4 " 7	4 " 8	4 " 9
5 " 6	5 " 7	5 " 8	5 " 9	5 " 10
6 " 7	6 " 8	6 " 9	6 " 10	6 " 11
7 " 8	7 " 9	7 " 10	7 " 11	7 " 12
8 " 9	8 " 10	8 " 11	8 " 12	8 " 13
9 " 10	9 " 11	9 " 12	9 " 13	9 " 14
10 " 11	10 " 12	10 " 13	10 " 14	10 " 15
6 and	**7 and**	**8 and**	**9 and**	**10 and**
1 are 7	1 are 8	1 are 9	1 are 10	1 are 11
2 " 8	2 " 9	2 " 10	2 " 11	2 " 12
3 " 9	3 " 10	3 " 11	3 " 12	3 " 13
4 " 10	4 " 11	4 " 12	4 " 13	4 " 14
5 " 11	5 " 12	5 " 13	5 " 14	5 " 15
6 " 12	6 " 13	6 " 14	6 " 15	6 " 16
7 " 13	7 " 14	7 " 15	7 " 16	7 " 17
8 " 14	8 " 15	8 " 16	8 " 17	8 " 18
9 " 15	9 " 16	9 " 17	9 " 18	9 " 19
10 " 16	10 " 17	10 " 18	10 " 19	10 " 20

39. The process of Addition is based upon the following

PRINCIPLES.

1. *Like numbers, and units of the same order, alone, can be added.* Thus,

Dollars and dollars can be added, but not dollars and days; also, units and units, tens and tens; but not units and tens.

Repeat the column 1 and 1. 2 and 1. 3 and 1. 4 and 1, etc. What is the first Principle?

2. *The sum of two or more numbers is the same in whatever order they are added.* Thus,

The sum of 2, 5, and 3 is 10, and the sum of 5, 3, and 2, or of 3, 2, and 5, is 10.

3. *The sum and the numbers added must be like numbers.* Thus,

The sum of 4 dollars and 6 dollars is 10 *dollars*, not 10 pounds.

40. To add numbers.

1. Let it be required to add 236, 541, and 102.

OPERATION.
236
541
102
Sum, 879

For convenience, we write the given numbers so that all the figures of the same order stand in the same column, and begin with units to add.

2, 1, and 6 units are 9 *units*, which we write. 0, 4, and 3 tens are 7 *tens*, which we write. 1, 5, and 2 hundreds are 8 *hundreds*, which we write.

Therefore, the sum is 8 hundreds, 7 tens, and 9 units, or 879.

2. Let it be required to find the sum of 595, 361, and 723.

OPERATION.
595
361
723
Sum, 1679

For convenience, we write, as before, the figures of the same order in the same column, and begin with units to add.

3, 1, and 5 units are 9 *units*, which we write. 2, 6, and 9 tens are 17 *tens*, or 1 hundred and 7 tens; we write the 7 tens and add the 1 hundred in with hundreds.

1, 7, 3, and 5 hundreds are 16 *hundreds*, or, 1 thousand and 6 hundreds, which we write.

Therefore, the sum is 1 thousand, 6 hundreds, 7 tens, and 9 units, or 1679.

In practice, it is sufficient to name only results. Thus, in the operation, we may say: three, four, *nine*,—write 9; two, eight, *seventeen*,—write 7 and add 1 with next column; eight, eleven, *sixteen*,—write 16; answer, 1679.

What is the second Principle? The third?

RULE. *Write the numbers to be added so that figures of the same order shall stand in the same column.*

Begin at the right, add the numbers expressed by the figures of each column separately, and write the sum underneath, if less than ten of the order added.

If, however, the sum is ten or more, write the right-hand figure underneath, and add the number expressed by the other figure or figures with the numbers of the next column.

Write the whole sum of the last column.

PROOF. Add the numbers a second time, in the same manner as at the first, except in an opposite direction, and, if the result agrees with that first obtained, the work is supposed to be correct. Or,

Separate the given numbers into parts; add each of the parts, and then add their sums; and, if the result agrees with that first obtained, the work is supposed to be correct.

The test, by either proof, consists in doing the work twice, in a different manner.

Examples.

	(3.)	(4.)		(5.)	Proof.
	16	191			
	812	98		4161	
	407	803		3140	7301
				5908	
Sum,	1235	1092		5020	10928
Proof,	1235	1092	Sum,	18229	18229

(6.)	(7.)	(8.)	(9.)	(10.)
100	1071	233	814	4444
673	341	511	816	1234
207	5001	179	31	5561

Repeat the Rule. What is the first Proof? The second Proof?

(11.)	(12.)	(13.)	(14.)	(15.)
Dollars.	Pounds.	Yards.	Men.	
36415	656	1786	110	1191
1234	315	314	517	818
510	708	516	819	110
89	190	1710	140	618

16. What is the sum of 423, 567 and 385? *Ans.* 1375.
17. What is the sum of 98, 483, 956 and 85? *Ans.* 1622.
18. What is the sum of 15, 603, 1145 and 6342?
19. What is the sum of $376+493+102+315$?
20. What is the sum of $4753+6378+9257+2896$?
21. $974+65+376+487+598+88=$ what?
22. $1000+100+10000+100000+11=$ what?
23. $7865+3586+4321+8576=$ how many? *Ans.* 24348.
24. $8+105+1006+810501+111+3+4400=$ how many? *Ans.* 816134.
25. Find the sum of 4080, 715, 13634, 29 and 5. *Ans.* 18463.
26. Find the sum of 6037, 2480, 2651 and 333. *Ans.* 11501.
27. Find the sum of $17896+570937+784947+9678$.
28. Find the sum of $16304+119+18913+306204$.
29. How many are $127+646+300+29$? *Ans.* 1102.
30. How many are $614034+783420+10316$?
31. How many are $3616+250+40460+18+3101+9$?
32. How many are $856+9193+8765+4287+6696+9185+979$? *Ans.* 39961.
33. $1031+125+9+641+10+449+6072=$ what?
34. Add five hundred sixty-seven thousand three hundred and seven, eighty-eight thousand and eight, nine hundred and sixty-two, and nineteen. *Ans.* 656296.

Review Questions. What is a Unit? (1) A Quantity? (2) A Number? (3) The Unit of a Number? (4) Like Numbers? (5)

APPLICATIONS.

1. A farmer sold a horse for 250 dollars, a yoke of oxen for 175 dollars, some cows for 324 dollars, and some hay for 60 dollars; how much did he get for the whole? *Ans.* 809 dollars.

SOLUTION. *If he sold a horse for* 250 *dollars, a yoke of oxen for* 175 *dollars, some cows for* 324 *dollars, and some hay for* 60 *dollars, he received for the whole, the sum of* 250 + 175 + 324 + 60 *dollars or* 809 *dollars. Therefore, he received for the whole* 809 *dollars.*

2. A merchant bought molasses for 1430 dollars, and sold it at a gain of 620 dollars; for what sum did he sell it? *Ans.* 2050 dollars.

3. A man commenced trade with three thousand five hundred and twenty-five dollars; after trading for some time, he had gained three hundred and nineteen dollars; how much had he then? *Ans.* 3844 dollars.

4. Going out to collect money, I had in my pocket 1500 dollars, and received from one person 17 dollars, from another 132 dollars, from another 527 dollars, and from a fourth as much as I started out with; how many dollars had I then?

5. Bought a carriage for 163 dollars, a pair of horses for 260 dollars, and a harness for 84 dollars; how much was paid for the whole? *Ans.* 507 dollars.

6. Figures were used by the Arabs in the year 890, and decimal fractions were invented 574 years later; in what year were they invented? *Ans.* 1464.

7. In one book are 513 pages, in another 144, and in another as many as in the other two; how many pages in the three books? *Ans.* 1314 pages.

8. A merchant bought beef for 644 dollars, pork for 450 dollars, and fish for 226 dollars; for how much must the whole be sold that the gain may be 240 dollars?

REVIEW QUESTIONS. What is an Operation? (8) An Answer? (9) A Solution? (10) A Rule? (11) An Example? (12) An Exercise? (13)

9. A butcher has five fat oxen; the first weighs 1124 pounds, the second 1235 pounds, the third 1300 pounds, the fourth 1420 pounds, and the fifth 1525 pounds; what is the weight of the whole number? *Ans.* 6604 pounds.

10. Purchased a farm for 19620 dollars, paid for repair of fences 820 dollars, for having a barn built 516 dollars, and sold it so as to gain 632 dollars; how much was obtained for it?

11. A farmer has live stock as follows: on one farm 5 horses, 20 cows, and 116 sheep; on a second farm 16 oxen and 264 sheep; and on a third farm 2 horses, 12 cows, and 8 calves. How many head has he in all? *Ans.* 443 head.

12. An army was furnished at one time with 16500 rations, at another time with 63000 rations, and at a third time with 72545 rations; how many rations were furnished altogether?

13. If the Atlantic slope contains 967576 square miles, and the Mississippi Valley exceeds the Atlantic slope by 269535 square miles, what is the area of the Mississippi Valley? *Ans.* 1237111 square miles.

14. If the area of Maine is 30000 square miles, of New Hampshire 9280, of Vermont 9056, of Massachusetts 7800, of Rhode Island 1306, of Connecticut 4674, and the area of Missouri 5264 square miles greater than that of the six States named, what is the area of Missouri? *Ans.* 67380 square miles.

15. Bought a lot of ground for 675 dollars; erected a house upon the same, at the cost for carpenters' work 2540 dollars, masons' work 637 dollars, painters' work 242 dollars, and for grading the lot 293 dollars; what was the cost of the whole?

16. If the area of Illinois is 55405 square miles, of Wisconsin 53924, of Minnesota 83000, and of Iowa 50914, what is the area of all these States? *Ans.* 243243 square miles.

17. Bought wheat for 6500 dollars, corn for 10500 dollars, and oats for 1062 dollars, and sold the wheat at a profit of 650

REVIEW QUESTIONS. What is Arithmetic? (14) Practical Arithmetic? (15)

dollars, the corn at a profit of 1050 dollars, and the oats at cost; what sum was received for the whole? *Ans.* 19762 dollars.

18. If all your debts to different persons are as follows: 2556 dollars, 1200 dollars, 5 dollars, 537 dollars, 495 dollars, and 5730 dollars, how much do you owe? *Ans.* 10523 dollars.

19. A certain year Ohio produced 59078695 bushels of corn, Indiana 52964363 bushels, Kentucky 58672591 bushels, and Tennessee 52276223 bushels; what was the amount produced by all these States? *Ans.* 222991872 bushels.

20. According to Weimer, Europe contains 3807195 square miles, Asia 17805146, Africa 11647428, America 13542400, and Oceanica 3347840; what does this make the extent of land on the surface of the globe? *Ans.* 50150009 square miles.

21. Bought five loads of hay, which weighed as follows: 2500 pounds, 2364 pounds, 3156 pounds, 1965 pounds, and 1831 pounds; what was the weight of the whole?

22. The skull has 8 bones, the face 14, the ear 4, the tongue 1, the teeth 32, the trunk 53, the shoulders 4, the arms 6, the wrists 16, the hands 38, legs 8, ankles 14, and feet 38; required the number in the whole body?

23. The number of regular soldiers furnished by each of the States in the Revolution was as follows:

New Hampshire,	12497	Delaware,	2386
Massachusetts,	67907	Maryland,	13912
Rhode Island,	5908	Virginia,	26678
Connecticut,	31939	North Carolina,	7263
New York,	17781	South Carolina,	6417
New Jersey,	10726	Georgia,	2679
Pennsylvania,	25678		

What was the whole number furnished by them? *Ans.* 231771.

REVIEW QUESTIONS. What is Notation? (16) Numeration? (17) What are figures? (18) What is the method of expressing numbers by figures called? (18) What is meant by orders of figures? (28) To what do they correspond? (28) How many orders are taken for a period? (28)

SUBTRACTION.

41. 1. Mary had 6 cents and gave away 2 cents; how many had she left?

SOLUTION. *She had left as many as are equal to 6 cents less 2 cents, and 2 cents from 6 cents leave 4 cents. Therefore, she had 4 cents left.*

2. James had 5 apples and gave his brother 3 of them; how many had he left?

3. A hawk having taken 4 chickens from a brood of 9, how many remain?

4. How many does 4 from 8 leave? 7 from 10?

5. How many does 6 from 9 leave? 4 from 11? 2 from 7? 5 from 12?

6. Henry had 10 cents and spent 5 cents; how much had he left?

7. John had 13 marbles and lost 8; how many had he left?

8. How many must be added to 8 to make 13?

The preceding operations are called SUBTRACTION. Hence,

42. Subtraction is the process of finding the difference between two given numbers of the same kind.

The SUBTRAHEND is the number subtracted.

The MINUEND is the number subtracted from.

The DIFFERENCE, or REMAINDER, is the result of the subtraction.

When the two given numbers are equal, either may be taken as the minuend, and the difference is 0.

43. The SIGN of SUBTRACTION is a short horizontal line, —, called *minus*. Thus, 6 — 4, read *six minus four*, denotes that 4 is to be subtracted from 6.

What is Subtraction? The Subtrahend? The Minuend? The Difference, or Remainder? What is the difference when the minuend and subtrahend are equal? The Sign of Subtraction?

Subtraction Table.

1 from	2 from	3 from	4 from	5 from
1 leaves 0	2 leaves 0	3 leaves 0	4 leaves 0	5 leaves 0
2 " 1	3 " 1	4 " 1	5 " 1	6 " 1
3 " 2	4 " 2	5 " 2	6 " 2	7 " 2
4 " 3	5 " 3	6 " 3	7 " 3	8 " 3
5 " 4	6 " 4	7 " 4	8 " 4	9 " 4
6 " 5	7 " 5	8 " 5	9 " 5	10 " 5
7 " 6	8 " 6	9 " 6	10 " 6	11 " 6
8 " 7	9 " 7	10 " 7	11 " 7	12 " 7
9 " 8	10 " 8	11 " 8	12 " 8	13 " 8
10 " 9	11 " 9	12 " 9	13 " 9	14 " 9
11 " 10	12 " 10	13 " 10	14 " 10	15 " 10
6 from	7 from	8 from	9 from	10 from
6 leaves 0	7 leaves 0	8 leaves 0	9 leaves 0	10 leaves 0
7 " 1	8 " 1	9 " 1	10 " 1	11 " 1
8 " 2	9 " 2	10 " 2	11 " 2	12 " 2
9 " 3	10 " 3	11 " 3	12 " 3	13 " 3
10 " 4	11 " 4	12 " 4	13 " 4	14 " 4
11 " 5	12 " 5	13 " 5	14 " 5	15 " 5
12 " 6	13 " 6	14 " 6	15 " 6	16 " 6
13 " 7	14 " 7	15 " 7	16 " 7	17 " 7
14 " 8	15 " 8	16 " 8	17 " 8	18 " 8
15 " 9	16 " 9	17 " 9	18 " 9	19 " 9
16 " 10	17 " 10	18 " 10	19 " 10	20 " 10

44. The process of Subtraction is based upon the following

PRINCIPLES.

1. *Like numbers, and units of the same order, alone, can be subtracted, one from the other.* Thus,

Dollars can be subtracted from dollars, but not dollars from days; also, units from units; but not units from tens, or tens from units.

Repeat the column 1 from 1. 2 from 2. 3 from 3. 4 from 4, etc. What is the first Principle?

2. *The difference, minuend and subtrahend, must be like numbers.* Thus,

The difference between 6 dollars and 4 dollars is 2 *dollars*, and not 2 yards.

3. *The difference and subtrahend, taken together, must equal the minuend.* Thus,

The difference, 5, between 13 and 8, added to 8, equals 13.

45. To subtract one number from another.

1. Let it be required to subtract 325 from 958.

OPERATION.	
Minuend,	958
Subtrahend,	325
Difference,	633

For convenience, we write the subtrahend under the minuend, so that figures of the same order stand in the same column, and begin at the right to subtract.

5 units from 8 units leave 3 units, which we write.

2 tens from 5 tens leave 3 tens, which we write.

3 hundreds from 9 hundreds leave 6 hundreds, which we write.

Therefore, the difference is 6 hundreds, 3 tens, and 3 units, or 633.

2. Let it be required to find the difference between 652 and 423.

OPERATION.	
Minuend,	652
Subtrahend,	423
Difference,	229

For convenience, we write, as before, figures of the same order in the same column, and begin with units to subtract.

We cannot take 3 units from 2 units; but we can take 1 ten from the 5 tens, leaving 4 tens; and the 1 ten taken is 10 units, which added to the 2 units make 12 units; 3 units from 12 units leave 9 units, which we write.

2 tens from 4 tens leave 2 tens, which we write.

4 hundreds from 6 hundreds leave 2 hundreds, which we write.

Therefore, the difference is 2 hundreds, 2 tens, and 9 units, or 229.

The operation may be performed another way:

As the 3 units cannot be taken from 2 units, we add 10 units to the 2 units; and 3 units from 12 units leave 9 units, which we write.

What is the second Principle? The third?

To balance the 10 units added to the 2 units, we add 1 ten to the 2 tens, in accordance with the principle that *if any two numbers are equally increased their difference remains the same;* and say, 3 tens from 5 tens leave 2 tens, which we write.

4 hundreds from 6 hundreds leave 2 hundreds, which we write.

Therefore, the difference is 229.

In practice, a brief explanation is sufficient. Thus, we may say: 3 from 2, impossible; but 3 from 12 leaves 9, which we write; 2 from 4 leaves 2, which we write; 4 from 6 leaves 2, which we write. Answer, 229.

RULE. *Write the less number under the greater, so that figures of the same order shall stand in the same column.*

Begin with units, subtract the number expressed by each figure of the subtrahend from the number expressed by the figure above it, and write the difference underneath.

If the upper figure expresses a less number than the lower, conceive that number increased by TEN, *subtract, and write the difference, and considering either the number expressed by the next upper figure* ONE LESS, *or that expressed by the next lower figure* ONE GREATER, *proceed as before.*

PROOF. Add the difference to the subtrahend, and, if the work is correct, the sum will equal the minuend. Or,

Subtract the difference from the minuend, and, if the work is correct, what is left will equal the subtrahend.

Examples.

	(3.)	(4.)	(5.)
Minuend,	406	395	6000
Subtrahend,	154	288	1234
Difference,	252	107	4766
Proof,	406	288	6000

In example 5, we cannot subtract 4 units from no units, and there

Repeat the Rule. What is the Proof?

are no tens and no hundreds; so that we cannot take one of the tens and call it ten units, or one of the hundreds and call it 9 *tens* and 10 *units;* but, we can take one of the 6 thousands, leaving 5 thousands, and call it 9 *hundreds*, 9 *tens*, and 10 *units*. Then, we can take 4 units from the 10 units, 3 tens from the 9 tens, etc.

	(6.)	(7.)	(8.)	(9.)
From	623	700	1120	4789
Subtract	515	465	744	1987

	(10.)	(11.)	(12.)	(13.)
	Tons.	Sheep.	Yards.	Bushels.
From	912	1060	2360	809
Subtract	453	343	1272	87

14. From 854 take 578. *Ans.* 276.
15. From 1799 take 1732. *Ans.* 67.
16. Find the difference between 8641 and 1904.
17. From 5496 subtract 1492. *Ans.* 4004.
18. From 1584 subtract 920. *Ans.* 664.
19. From 5672 subtract 2356. *Ans.* 3316.
20. From 74760 subtract 39817. *Ans.* 34943.
21. 52365 — 15423 = how many? *Ans.* 36942.
22. 78567 — 32782 = how many?
23. 9736214 — 8878946 = how many? *Ans.* 857268.
24. What is the difference between 900000 and 123454?
25. How much larger is 38607 than 3867? *Ans.* 34740.
26. How much smaller is 34730 than 38607?
27. How much must be taken from 2483 to leave 391?
28. How much must be added to 2082 to make 2483?
29. From 7630005 take 3270006. *Ans.* 4359999.
30. The larger of two numbers is 10640 and the less 9535; what is their difference? *Ans.* 1105.

REVIEW QUESTIONS. Upon what General Principles is the Arabic Notation based? (30) What is a Scale of Numbers? (31) What is the Scale in numbers expressed according to the Arabic Notation?

31. Subtract two thousand one hundred and nineteen from five thousand two hundred and twelve. *Ans.* 3093.

32. If one be taken from one hundred thousand, what will remain? *Ans.* 99999.

33. What is the difference between nine units and ninety-nine millions?

34. From four hundred fifty thousand and ninety-four take ninety-nine thousand nine hundred and nine.

35. From 35 billions 63 millions and 9 thousand take 7 billions 103 millions and 9. *Ans.* 27960008991.

APPLICATIONS.

1. A farmer raised 420 bushels of wheat, and sold 198 bushels; how many bushels had he left?

SOLUTION. *If he raised* 420 *bushels and sold* 198 *bushels, he must have left as many bushels as the difference between* 420 *and* 198 *bushels, or* 222 *bushels. Therefore, he had* 222 *bushels left.*

2. A boy had 526 apples, and gave away 411 of them; how many had he left? *Ans.* 115 apples.

3. Albert Smith borrowed 1090 dollars, and soon after paid 909 dollars; how much of the sum borrowed did he then owe? *Ans.* 181 dollars.

4. How many years have elapsed since the first planting of cotton in this country in 1769?

5. How many years from the beginning of the revolutionary war in 1775 to the beginning of the late war in 1861?

6. A merchant sold for 3697 dollars goods which cost him 2807 dollars; how much did he gain?

7. A man owns property to the amount of five thousand eight hundred and twenty-five dollars, and owes one thousand three hundred and forty dollars; how much will he be worth when his debts are paid? *Ans.* 4485 dollars.

REVIEW QUESTIONS. Give the names of the orders in the Numeration *Table. (33) What is the* Rule for reading numbers? (34)

8. Bought a ship for 42650 dollars and sold it for 49000 dollars; what did I gain? *Ans.* 6350 dollars.

9. A gentleman gave 12462 dollars for a house and some land; the house alone was worth 9375 dollars; what was the the value of the land? *Ans.* 3087 dollars.

10. A lumberman, having 650000 feet of boards, sold 162372 feet of them; how many feet then remained? *Ans.* 487628 feet.

11. The battle of Gettysburg, in 1863, was 48 years after the battle of New Orleans; in what year was the latter?

12. A man having 100000 dollars, gave away 365 dollars; how much had he left? *Ans.* 99635 dollars.

13. A merchant owns property to the amount of 45563 dollars, and owes 21209 dollars; how much is he worth more than he owes?

14. If two candidates for office received in the aggregate 73462 votes, and the successful one had 45309 votes, how many did the other have? *Ans.* 28153 votes.

15. Illinois contains 55405 square miles and Iowa 50914 square miles; how many more square miles does the one contain than the other?

16. Mount Sorata, in South America, is 25380 feet high, and 19146 feet higher than Mount Washington in New Hampshire; how high is Mount Washington? *Ans.* 6234 feet.

17. Girard College, in Philadelphia, is said to have cost 1422800 dollars, and Trinity Church, in New York City, 338000 dollars; how much more did the one cost than the other?

18. If the value of the annual products of the industry of Massachusetts is 266000000 dollars, and that of Pennsylvania is 285500000 dollars, how much do the products of the one State exceed those of the other? *Ans.* 19500000 dollars.

19. If the population of Ohio was 45365 in 1800, and 2339502 in 1860, how much was the increase? *Ans.* 2294137.

REVIEW QUESTIONS. What is Addition? (37) What principles are to be observed in Addition? (39)

REVIEW EXERCISES.

1. $573 + 6$ thousand $+ 6$ million $+ 507963 + 1245 =$ how many? *Ans.* 6515781.

2. If the minuend is eight million six hundred seventy-three thousand four hundred and one, and the subtrahend six million seven hundred twenty thousand seven hundred and thirty, what is the difference? *Ans.* 1952671.

3. If the larger of two numbers is 100101 and their difference 9902, what is the smaller number? *Ans.* 90199.

4. A man owning 4605 acres of land, gave to one of his sons 1420 acres and to another 1280 acres; how many acres had he remaining?

SOLUTION. *If he gave to one son* 1420 *acres and to another* 1280 *acres, he must have given to both the sum of* 1420 *acres and* 1280 *acres, or* 2700 *acres.*

If he had 4605 *acres and gave away* 2700 *acres, he must have had remaining the difference between* 4605 *acres and* 2700 *acres, or* 1905 *acres. Therefore, he had remaining* 1905 *acres.*

5. A grain dealer bought 6000 bushels of wheat; he afterwards sold to one man 1575 bushels, and to another 3600 bushels; how many bushels remain unsold?

6. A man died leaving 24000 dollars, of which he gave his wife 8000 dollars, one daughter 3500 dollars, another 4500 dollars, and the residue to his son; what was the son's portion?

7. Mr. Jones had in a bank 16830 dollars, drew out 9460 dollars, and afterward put in 2000 dollars; how much had he then in the bank? *Ans.* 9370 dollars.

8. A farmer had a horse worth 275 dollars, and exchanged it for a yoke of oxen and two cows; the oxen he sold for 125 dollars, one of the cows for 75 dollars, and the other for 58 dollars. How much did he lose by the trade? *Ans.* 17 dollars.

REVIEW QUESTIONS. What is Subtraction? (42) What principles are to be observed in Subtraction? (44)

MULTIPLICATION.

46. 1. How many dollars will 6 tons of coal cost, at 7 dollars a ton?

SOLUTION. *Since* 1 *ton of coal costs* 7 *dollars*, 6 *tons must cost* 6 *times* 7 *dollars, which are* 42 *dollars. Therefore*, 6 *tons of coal, at* 7 *dollars a ton, will cost* 42 *dollars.*

2. How many cents will buy 5 pencils, at 8 cents each?

3. When berries are 8 cents a quart, how much must be paid for 4 quarts?

4. If a boy can walk 3 miles in an hour, how many miles can he walk in 5 hours?

5. A farmer had 10 cows in each of 3 pastures; how many had he in all of them?

6. If 1 horse will eat 4 tons of hay in a given time, how many tons will 7 horses eat in the same time?

The preceding operations are called MULTIPLICATION. Hence,

47. Multiplication is the process of finding the result of taking one of two given numbers as many times as there are units in the other.

The MULTIPLICAND is the number to be taken.

THE MULTIPLIER is the number denoting how many times the multiplicand is to be taken.

The PRODUCT is the result of the multiplication.

The FACTORS OF THE PRODUCT are the multiplicand and multiplier.

48. The SIGN OF MULTIPLICATION is an inclined cross, $\times$, read *multiplied by*. Thus, 5×4 is read, 5 multiplied by 4.

What is Multiplication? The Multiplicand? The Multiplier? The Product? The Factors of the Product? The Sign of Multiplication?

Multiplication Table.

Once	2 times	3 times	4 times	5 times	6 times
1 is 1	1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 “ 2	2 “ 4	2 “ 6	2 “ 8	2 “ 10	2 “ 12
3 “ 3	3 “ 6	3 “ 9	3 “ 12	3 “ 15	3 “ 18
4 “ 4	4 “ 8	4 “ 12	4 “ 16	4 “ 20	4 “ 24
5 “ 5	5 “ 10	5 “ 15	5 “ 20	5 “ 25	5 “ 30
6 “ 6	6 “ 12	6 “ 18	6 “ 24	6 “ 30	6 “ 36
7 “ 7	7 “ 14	7 “ 21	7 “ 28	7 “ 35	7 “ 42
8 “ 8	8 “ 16	8 “ 24	8 “ 32	8 “ 40	8 “ 48
9 “ 9	9 “ 18	9 “ 27	9 “ 36	9 “ 45	9 “ 54
10 “ 10	10 “ 20	10 “ 30	10 “ 40	10 “ 50	10 “ 60
11 “ 11	11 “ 22	11 “ 33	11 “ 44	11 “ 55	11 “ 66
12 “ 12	12 “ 24	12 “ 36	12 “ 48	12 “ 60	12 “ 72
7 times	**8 times**	**9 times**	**10 times**	**11 times**	**12 times**
1 are 7	1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 “ 14	2 “ 16	2 “ 18	2 “ 20	2 “ 22	2 “ 24
3 “ 21	3 “ 24	3 “ 27	3 “ 30	3 “ 33	3 “ 36
4 “ 28	4 “ 32	4 “ 36	4 “ 40	4 “ 44	4 “ 48
5 “ 35	5 “ 40	5 “ 45	5 “ 50	5 “ 55	5 “ 60
6 “ 42	6 “ 48	6 “ 54	6 “ 60	6 “ 66	6 “ 72
7 “ 49	7 “ 56	7 “ 63	7 “ 70	7 “ 77	7 “ 84
8 “ 56	8 “ 64	8 “ 72	8 “ 80	8 “ 88	8 “ 96
9 “ 63	9 “ 72	9 “ 81	9 “ 90	9 “ 99	9 “ 108
10 “ 70	10 “ 80	10 “ 90	10 “ 100	10 “ 110	10 “ 120
11 “ 77	11 “ 88	11 “ 99	11 “ 110	11 “ 121	11 “ 132
12 “ 84	12 “ 96	12 “ 108	12 “ 120	12 “ 132	12 “ 144

Any number of times 0 is 0, and 0 times any number is 0. Thus,

$$0 \times 1 = 0,\ 0 \times 2 = 0, \text{ etc.};\ 1 \times 0 = 0,\ 2 \times 0 = 0, \text{ etc.}$$

49. The process of Multiplication is based upon the following

---

Repeat the column once 1 is 1. 2 times 1 are 2. 3 times 1 are 3, etc. What is any number of times 0? 0 times any number?

PRINCIPLES.

1. *The product and multiplicand must be like numbers.* Thus,

4 times 6 *men* are 24 *men*. 3 times 7 *cents* are 21 *cents*.

2. *The multiplier must always be regarded as an abstract number.* Thus,

In finding the cost of 6 tons of coal at 7 dollars a ton, the 7 dollars are taken 6 times, and not multiplied by 6 tons

3. *The product of two or more factors is the same in whatever order they are taken.* Thus,

The product of 6×3, or 3×6, is 18, and the product of $5 \times 3 \times 2$, or $2 \times 3 \times 5$, or $3 \times 5 \times 2$, is 30.

50. From the definition of Multiplication, it follows, that

Multiplication, when the *size* or *value* of a single thing, or unit, is given, enables us to find the *size* or *value* of any number of things of the same kind.

51. To multiply one number by another.

1. Let it be required to multiply 564 by 7.

OPERATION.	
Multiplicand,	564
Multiplier,	7
Product,	3948

For convenience, we write the multiplier under the units in the multiplicand, and begin with units to multiply.

7 times 4 units are 28 units, which equal 2 tens and 8 units; we write the 8 units, and reserve the 2 tens to add to the next product.

7 times 6 tens are 42 tens, which with the 2 tens added are 44 tens, or 4 hundreds and 4 tens, we write the 4 tens, and reserve the 4 hundreds to add to the next product.

7 times 5 hundreds are 35 hundreds, which with the 4 hundreds added are 39 hundreds, or 3 thousands and 9 hundreds; which we write.

Therefore, the product is 3 thousand 9 hundred and 48, or 3948.

What is the first Principle? The second? The third? What does Multiplication enable us to find?

In practice, the name of the order of units may be omitted. Thus, in the operation we can say: 7 times 4 are 28; we write the 8, and add the 2 to the next product: 7 times 6 are 42, and 2 are 44; we write 4, and add 4 to the next product; 7 times 5 are 35, and 4 are 39; answer, 3948.

2. Let it be required to find the product of 736 by 206.

OPERATION.	
Multiplicand,	736
Multiplier,	206
Partial Products,	4416
	14720
Product,	151616

For convenience, we write the multiplier under the multiplicand, so that figures of the same order stand in the same column; and multiplying by the units, as in the preceding operation, we obtain 4416.

There being 0 tens, we write a cipher in the order of tens underneath, and pass to the hundreds' figure of the multiplier.

2 hundreds are 2 hundred units, and 2 hundred times 6 units are 12 hundreds, or 1 thousand and 2 hundreds. We write the 2 hundreds, and reserve the 1 thousand to add to the next product.

2 hundred times 3 tens are 6 hundred tens or 6 thousands, and the 1 thousand added are 7 thousands, which we write.

2 hundred times 7 hundreds are 14 hundred-hundreds, equal 1 hundred-thousand, and 4 ten-thousands, which we write, and obtain 1472 hundreds, or, with the cipher on the right, 14720 tens.

Adding the two partial products we have for the entire product 151616.

Therefore, the product of 736 by 206 is 151616.

RULE. *Write the multiplier under the multiplicand, so that units may stand under units, tens under tens,* etc.

If the multiplier contains but one order of units, beginning at the right multiply each order of the multiplicand by it, writing the right-hand figure of each product underneath, adding the numbers expressed by the other figures, if any, to the next product, observing to write all the figures of the last product.

If the multiplier contains more than one order of units,

REVIEW QUESTIONS. What is a Sign? (38) The Sign of Addition? (38) Of Equality? (38) Of Subtraction? (43) Of Multiplication? (48)

multiply by each of the orders, successively, writing the right-hand figure of each partial product under the order used. The sum of the partial products will be the entire product.

PROOF. Multiply the multiplier by the multiplicand; and, if the product is the same as that first obtained, the work is supposed to be correct. Or,

Separate the multiplier into parts and make each of them a multiplier, and, if the sum of the products equals the first product, the work is supposed to be correct.

Exercises.

3. Find the product of 78 by 37.

OPERATION.

$$\begin{array}{rl} 78 & \\ 37 & \\ \hline 546 & = 78 \times 7 \\ 234 & = 78 \times 30 \\ \hline \textit{Ans. } 2886 & = 78 \times 37 \end{array}$$

Proof.

$$\begin{array}{rl} 37 & \\ 78 & \\ \hline 296 & = 37 \times 8 \\ 259 & = 37 \times 70 \\ \hline 2886 & = 37 \times 78 \end{array}$$

4. Find the product of 316 by 13.

OPERATION.

$$\begin{array}{rl} 316 & \\ 13 & \\ \hline 948 & = 316 \times 3 \\ 316 & = 316 \times 10 \\ \hline \textit{Ans. } 4108 & = 316 \times 13 \end{array}$$

$$\text{Proof.}\left\{\begin{array}{rl} 316 & \\ 11 & \\ \hline 316 & = 316 \times 1 \\ 316 & = 316 \times 10 \\ 632 & = 316 \times 2 \\ \hline 4108 & = 316 \times 13 \end{array}\right.$$

	(5.)	(6.)	(7.)	(8.)
Multiply	416	3102	1091	4467
By	4	8	7	9
Product,	1664	24816	7637	40203

What is the Rule? The Proof?

	(9.)	(10.)	(11.)	(12.)
Multiply	196	7781	4465	5532
By	6	5	3	8

Multiply

13. 1306 by 3. *Ans.* 3918.
14. 971 by 5. *Ans.* 4855.
15. 4137 by 6.
16. 46002 by 4. *Ans.* 184008.
17. 1190 by 7. *Ans.* 8330.
18. 60441 by 5.
19. 9943 by 2. *Ans.* 19886.
20. 6453 by 5. *Ans.* 32265.
21. 978609 by 1.
22. 1706 by 11. *Ans.* 18766.
23. 19774 by 10. *Ans.* 197740.
24. 8320 by 13. *Ans.* 108160.
25. 2538 by 11. *Ans.* 27918.
26. 1223 by 8. *Ans.* 9784.
27. 67812 by 8.
28. 12091 by 7. *Ans.* 84637.
29. 7090 by 12. *Ans.* 85080.
30. 5009 by 17.
31. 387 by 22. *Ans.* 8114.
32. 664 by 19. *Ans.* 12616.
33. 315 by 23.
34. 1782 by 61. *Ans.* 108602.
35. 909 by 29. *Ans.* 26361.
36. 81201 by 73. *Ans.* 5927673.
37. $75452 \times 47 =$ how many? *Ans.* 3546244.
38. $54302 \times 89 =$ how many? *Ans.* 4832878.
39. $784 \times 203 =$ how many? *Ans.* 159152.
40. What is the product of 137 by 35? *Ans.* 4795.
41. What is the product of 567 by 108? *Ans.* 61236.
42. What is the product of 5, 25, and 37? *Ans.* 4625.
43. What is the product of 3, 17, and 111?
44. How many are 1234 times 7013? *Ans.* 8654042.
45. Multiply 486 by 259. *Ans.* 125874.
46. Multiply 34618 by 259. *Ans.* 8966062
47. Multiply 80704 by 432. *Ans.* 34864128.
48. Multiply thirty-one thousand three hundred and eleven by one thousand two hundred and thirteen.
49. Multiply ninety-three thousand one hundred and eighty-six by four thousand four hundred and fifty-five. *Ans.* 415143630.

REVIEW QUESTIONS. What is the answer called in Addition? (37) In *Subtraction*? (42) In Multiplication? (47)

52. When there are ciphers *between* significant figures in the multiplier, *the operation may be shortened by passing over each* 0 *of the multiplier.*

50. Multiply 4236 by 2007.

OPERATIONS.

$$\left.\begin{array}{r} 4236 \\ 2007 \\ \hline 29652 \\ 847200 \\ \hline 8501652 \end{array}\right\} \quad \text{or,} \quad \left\{\begin{array}{r} 4236 \\ 2007 \\ \hline 29652 \\ 8472 \\ \hline 8501652 \end{array}\right.$$

51. Multiply 15607 by 3094. *Ans.* 48288058.
52. Multiply 60121 by 3108.

53. When the multiplier consists of two significant figures, with or without intervening ciphers, and begins or ends with 1, we may *consider the multiplicand as a product by the* 1, *and write the other partial product as many orders to the right or left as is required by the multiplier.*

53. Multiply 251 by 31.

OPERATION.

$$\begin{array}{rl} 251 & = 251 \times 1 \\ 753 & = 251 \times 30 \\ \hline 7781 & = 251 \times 31 \end{array}$$

54. Multiply 1235 by 1004.

OPERATION.

$$\begin{array}{rl} 1235 & = 1235 \times 1000 \\ 4940 & = 1235 \times 4 \\ \hline 1239940 & = 1235 \times 1004 \end{array}$$

In example 56, the one partial product is units and the other tens, and in example 57, the one partial product is thousands and the other units; and they are so written that, in each case, the sum of the partial products may be the required product.

55. Multiply 3403 by 501. *Ans.* 1704903.

How may you multiply when there are ciphers between the significant figures of the multiplier? When either of the two significant figures of the multiplier is 1?

56. Multiply 5121 by 1002. *Ans.* 5131242.

57. Multiply 61303 by 701. *Ans.* 42973403.

54. When the multiplier is 10, 100, 1000, etc., *the product may be obtained, at once, by annexing to the multiplicand as many ciphers as there are in the multiplier, and regarding the decimal point as removed an equal number of places to the right.*

For, the value expressed by figures is made tenfold by each removal of them an order to the left. (Art. 30.) Thus, $2 \times 10 = 20$, $2 \times 100 = 200$, etc.

58. Multiply 619 by 100. *Ans.* 61900.

59. Multiply 11644 by 1000. *Ans.* 11644000.

60. $4568 \times 1000000 =$ how many? *Ans.* 4568000000.

55. When there are ciphers on the right of either or both of the factors, *we may multiply without reference to them, and annex to the product as many ciphers as there are on the right of both factors.*

61. Find the product of 2050 by 1300.

OPERATIONS.

$$\left.\begin{array}{r} 2050 \\ 1300 \\ \hline 615000 \\ 2050 \\ \hline 2665000 \end{array}\right\} \text{ or, } \left\{\begin{array}{r} 2050 \\ 1300 \\ \hline 615 \\ 205 \\ \hline 2665000 \end{array}\right.$$

The second operation is evidently the same as the first, except that the ciphers on the right are not written until the partial products are added.

62. Multiply 485 by 240. *Ans.* 116400.

63. Multiply 36500 by 730. *Ans.* 26645000.

64. Multiply six hundred seventy-four thousand and two hundred by two thousand one hundred and four.

Ans. 1418516800.

When the multiplier is 10, 100, 1000, etc.? How do you multiply when there are ciphers at the right of either or both of the factors?

APPLICATIONS.

1. If a man travel 212 miles a week, how far will he travel in 52 weeks?

SOLUTION. *If a man travel* 212 *miles in a week, he will travel in* 52 *weeks* 52 *times* 212 *miles, or* 11024 *miles. Therefore, if a man travel* 212 *miles in a week, he will travel in* 52 *weeks* 11024 *miles.*

2. If an orchard containing 313 trees produce 15 bushels of apples to a tree, how many bushels is the produce of the whole orchard? *Ans.* 4695.

3. How many lights of glass in 18 windows, if each window contains 24?

4. How many bushels of corn will grow on 160 acres, at the average rate of 45 bushels to an acre? *Ans.* 7200 bushels.

5. What cost 2463 barrels of flour at 9 dollars a barrel?

6. If a man can earn 83 dollars in one month, how many can he earn in 12 months? *Ans.* 996 dollars.

7. What will be the cost of a farm containing 684 acres, at 57 dollars per acre? *Ans.* 38988 dollars.

8. A certain field has 625 hills of potatoes, and each hill will average 8 potatoes; how many potatoes at that rate in the field?

9. If 17 men can do a piece of work in 91 days, how long will it take one man alone to do it? *Ans.* 1547 days.

10. An army consists of 6 brigades, each brigade of 4 regiments, and each regiment of 613 men, rank and file; required the number of men in the army? *Ans.* 14712 men.

11. If a saw-mill can produce 4360 feet of boards a day, how many can it produce in 106 days? *Ans.* 462160 feet.

12. If the earth moves around the sun at the rate of 68000 miles an hour, how far will it move in 365 days of 24 hours each? *Ans.* 595680000 miles.

REVIEW QUESTIONS. What is the Rule in Addition? (40) In Subtraction? (45) In Multiplication? (51)

DIVISION.

56. 1. If 4 cents will buy one orange, how many oranges will 12 cents buy?

SOLUTION. *If* 4 *cents will buy one orange,* 12 *cents will buy as many oranges as* 4 *cents are contained times in* 12 *cents, or* 3. *Therefore, if* 4 *cents will buy one orange,* 12 *cents will buy* 3 *oranges.*

2. At 7 cents a pound, how many pounds of rice can be bought for 21 cents?

3. If 2 boys share equally 8 marbles, how many will each have?

SOLUTION. *If* 2 *boys share equally* 8 *marbles, each will have* 1 *of the* 2 *equal parts of* 8 *marbles;* 1 *of the* 2 *equal parts of* 8 *marbles is* 4 *marbles. Therefore, if* 2 *boys share equally* 8 *marbles, each will have* 4 *marbles.*

One of the *two* equal parts of a number is called *one half* of the number; one of the *three* equal parts, *one third* of the number; one of the *four* equal parts, *one fourth* of the number; one of the *five* equal parts, *one fifth* of the number; one of the *ten* equal parts, *one tenth* of the number; one of the *hundred* equal parts, *one hundredth* of the number, and so on. Hence,

A *whole* number has two halves, three thirds, four fourths, five fifths, ten tenths, one hundred hundredths, etc.

4. When 4 apples cost 16 cents, what is the cost of one apple?

SOLUTION. *When the cost of* 4 *apples is* 16 *cents, the cost of one apple is one fourth of* 16 *cents, or* 4 *cents. Therefore, when* 4 *apples cost* 16 *cents, the cost of one apple is* 4 *cents.*

5. If 20 dollars are shared equally between 5 boys, how many does each boy receive?

The preceding operations illustrate what is called DIVISION. Hence,

---

What is one of two equal parts of a number called? One of three equal parts? One of four equal parts?

57. Division is the process of finding how many times one number is contained in another; or of finding one of the equal parts of a number.

The DIVIDEND is the number to be divided.

The DIVISOR is the number by which we divide.

The QUOTIENT is the result or number obtained by the division.

The REMAINDER is that part of the dividend which is left after finding the exact whole number of the quotient. Thus,

3 is contained in 7, 2 times and 1 as a remainder.

The division is said to be *exact*, when there is no remainder.

58. The SIGN OF DIVISION is a short horizontal line with a dot above and below it, $\div$, read, *divided by*. Thus,

$6 \div 2$ is read, six divided by two.

Sometimes, in place of the dots, the number divided is written above the line, and the number which divides it is written below. Thus,

$\frac{6}{2}$ is read six divided by two.

Division, also, may be indicated by a curved line,), the divisor being written before, and the dividend after it. Thus,

2) 6 indicates that 6 is to be divided by 2, and is so read.

59. One or more equal parts of a unit are called *fractions*, to distinguish them from unbroken or whole numbers, which are called *integers*, or *integral numbers*.

$\frac{1}{2}$, read one divided by two, or one half,
$\frac{1}{3}$, read one divided by three, or one third,
$\frac{2}{3}$, read two divided by three, or two thirds, etc.,

are *fractional* expressions.

What is Division? The Dividend? The Divisor? The Quotient? The Remainder? When is the division said to be exact? What are the Signs of Division? What are one or more equal parts of a unit called?

Division Table.

1 in	2 in	3 in	4 in	5 in
1, 1 time	2, 1 time	3, 1 time	4, 1 time	5, 1 time
2, 2 times	4, 2 times	6, 2 times	8, 2 times	10, 2 times
3, 3 "	6, 3 "	9, 3 "	12, 3 "	15, 3 "
4, 4 "	8, 4 "	12, 4 "	16, 4 "	20, 4 "
5, 5 "	10, 5 "	15, 5 "	20, 5 "	25, 5 "
6, 6 "	12, 6 "	18, 6 "	24, 6 "	30, 6 "
7, 7 "	14, 7 "	21, 7 "	28, 7 "	35, 7 "
8, 8 "	16, 8 "	24, 8 "	32, 8 "	40, 8 "
9, 9 "	18, 9 "	27, 9 "	36, 9 "	45, 9 "
10, 10 "	20, 10 "	30, 10 "	40, 10 "	50, 10 "
6 in	**7 in**	**8 in**	**9 in**	**10 in**
6, 1 time	7, 1 time	8, 1 time	9, 1 time	10, 1 time
12, 2 times	14, 2 times	16, 2 times	18, 2 times	20, 2 times
18, 3 "	21, 3 "	24, 3 "	27, 3 "	30, 3 "
24, 4 "	28, 4 "	32, 4 "	36, 4 "	40, 4 "
30, 5 "	35, 5 "	40, 5 "	45, 5 "	50, 5 "
36, 6 "	42, 6 "	48, 6 "	54, 6 "	60, 6 "
42, 7 "	49, 7 "	56, 7 "	63, 7 "	70, 7 "
48, 8 "	56, 8 "	64, 8 "	72, 8 "	80, 8 "
54, 9 "	63, 9 "	72, 9 "	81, 9 "	90, 9 "
60, 10 "	70, 10 "	80, 10 "	90, 10 "	100, 10 "

60. The process of Division is based upon the following

PRINCIPLES.

1. *The quotient will be an abstract number, when the divisor and dividend are like numbers.*

For, the quotient will denote *how many times* the divisor is contained in the dividend.

2. *The divisor must be regarded as an abstract number, when the dividend is concrete, and the divisor not a like number.*

Repeat the line 1 in 1. 2 in 2. 3 in 3. 4 in 4. 5 in 5, etc. What is the first Principle? The second?

For, the divisor must then denote *the number of equal parts* into which the dividend is to be divided. Hence,

3. *If the divisor and dividend are not like numbers, the quotient and dividend will be like numbers.*

For, the quotient will denote *one of the equal parts* into which the dividend is divided.

4. *The remainder and dividend must always be like numbers.*

For, the remainder is evidently *a part* of the dividend.

5. *Division may be regarded as the reverse of multiplication.*

For, the dividend corresponds to the product, and the divisor and quotient to the two factors.

61. From the foregoing principles, it follows that there may be two kinds of division:

First kind — when the *size* or *value* of the equal parts of a quantity is given, to find their *number*; and

Second kind — when the *number* of equal parts of a quantity is given, to find their *size* or *value*.

CASE I.

62. To divide by Short Division.

1. Let it be required to divide 1702 by 7.

OPERATION.

Divisor, 7) 1702 Dividend.

243$\frac{1}{7}$ Quotient.

For convenience, we write the divisor at the left of the dividend, and begin at the left to divide.

7 is contained in 1 thousand no thousands times; therefore, there will be no thousands in the quotient. Try 17 hundreds; 7 is contained in 17 hundreds, 2 hundreds times, and 3 hundreds, equal to 30 tens, remaining. We write the 2 hundreds, and add the 30 tens to the 0 tens, making 30 tens.

What is the third Principle? The fourth? The fifth? Name the two kinds of Division.

7 in 30 tens, 4 tens times, and 2 tens, equal to 20 units, remaining. We write the 4 tens, and add the 20 units to the 2 units, making 22 units.

7 in 22 units, 3 units times, and 1 unit remaining. We write the 3 units, and indicating the division of the 1 unit, we annex the fractional expression, $\frac{1}{7}$ unit, to the integral part of the quotient.

Therefore, 1702 divided by 7 is equal to $243\frac{1}{7}$, read two hundred forty-three and one seventh.

The solution by the second kind of Division is:

One seventh of 17 hundreds is 2 hundreds, and a remainder of 3 hundreds, equal to 30 tens. We write the 2 hundreds, and add the 30 tens to the 0 tens, making 30 tens.

One seventh of 30 tens is 4 tens, and a remainder of 2 tens, equal to 20 units. We write the 4 tens, and add the 20 units to the 2 units, making 22 units.

One seventh of 22 units is 3 units, and a remainder of 1 unit. We write the 3 units, and indicating a seventh of the 1 unit, we annex the expression, $\frac{1}{7}$, to the integral part of the quotient.

Therefore, 1702 divided by 7 is $243\frac{1}{7}$.

2. Let it be required to obtain the quotient of 763 divided by 7.

OPERATION.

Divisor, 7) 763 Dividend.

109 Quotient.

For convenience, we write the divisor, and begin to divide, as in the preceding operation.

7 in 7 hundreds, 1 hundreds time, which we write.

7 in 6 tens 0 tens times, and 6 tens, equal to 60 units, remaining. We write the 0 tens, and add the 60 units to the 3 units, making 63 units.

7 in 63 units, 9 units times, which we write.

In practice we may say, 7 in 7, 1, which we write; 7 in 6, 0, which we write; prefix the 6 to the figure 3; 7 in 63, 9, which we write; answer, 109.

When, as in the above operations, the dividing is performed mentally, except in writing the quotient figures, the process is called SHORT DIVISION.

Give the solution by the first kind of Division. The second. When is the process called Short Division?

Rule. *Write the divisor at the left of the dividend.*

Begin at the left, divide the numbers expressed by each figure of the dividend by the divisor, and write the result beneath.

If there be a remainder after any division, regard it as prefixed to the figure of the next lower order, and divide as before.

If any partial dividend be less than the divisor, write a cipher in the quotient, and prefix such dividend to the next figure, if any, for a new dividend.

If there be a final remainder, write it, with the divisor beneath, after the integral part of the quotient.

Proof. Multiply the integral number of the quotient by the divisor, and to the product add the remainder, if any; and the result will equal the dividend, if the work is right.

Examples.

	(3.)	(4.)	(5.)
	4) 8852	5) 90651	7) 220523
Ans.	2213	$18130\frac{1}{5}$	$31503\frac{2}{7}$
	4	5	7
Proof.	8852	90651	220523

(6.)	(7.)	(8.)
2) 1006	8) 17626	3) 61273

Divide

9. 1896 by 4. *Ans.* 474.
10. 6819 by 5. *Ans.* $1363\frac{4}{5}$.
11. 60104 by 2.
12. 321001 by 7. *Ans.* $45857\frac{2}{7}$.
13. 447078 by 8.
14. 85765 by 3. *Ans.* $28588\frac{1}{3}$.
15. 65548 by 9. *Ans.* $7283\frac{1}{9}$.
16. 578096 by 8.
17. 161413 by 6. *Ans.* $26902\frac{1}{6}$.
18. 9080706 by 5.

Repeat the Rule. What is the Proof?

Required

19. One third of 189. *Ans.* 63.
20. One fifth of 1790. *Ans.* 358.
21. One eighth of 9872.
22. One ninth of 8011. *Ans.* $890\frac{1}{9}$.
23. $870621 \div 2 =$ how many? *Ans.* $435310\frac{1}{2}$.
24. How many are $\frac{341625}{7}$? *Ans.* $48803\frac{4}{7}$.

APPLICATIONS.

1. How many cords of wood can be bought for 1965 dollars, at 5 dollars a cord?

SOLUTION. *Since the cost of* 1 *cord is* 5 *dollars, as many cords can be bought for* 1965 *dollars as* 5 *dollars are contained times in* 1965 *dollars, or* 393. *Therefore, there can be bought* 393 *cords of wood for* 1965 *dollars, at* 5 *dollars a cord.*

2. If 4 bushels of wheat make 1 barrel of flour, how many barrels will 9650 bushels make?

3. At 7 cents a pound, how many pounds of rice can be bought for 363 cents? *Ans.* $51\frac{6}{7}$ pounds.

4. If 9 horses cost 2025 dollars, how much must 1 horse cost?

SOLUTION. *If* 9 *horses cost* 2025 *dollars,* 1 *horse must cost one ninth of* 2025 *dollars, or* 225 *dollars. Therefore, if* 9 *horses cost* 2025 *dollars, one horse must cost* 225 *dollars.*

5. When 3168 dollars are paid for 6 bales of cloth, how much is paid for 1 bale? *Ans.* 528 dollars.

6. How many cords of wood, at 5 dollars a cord, can be bought for 1965 dollars? *Ans.* 393 cords.

7. When a carpenter is paid 581 dollars for 7 months' labor, how much is that a month?

8. 7 times a certain number is equal to 22134; what is the number? *Ans.* 3162.

REVIEW QUESTIONS. What is a Concrete Number? (6) An Abstract Number? (7)

CASE II.

63. To divide by Long Division.

1. Let it be required to divide 34531 by 15.

OPERATION.

```
           Dividend.
Divisor 15 ) 34531 ( 2302 1/15 Quotient.
             30
             --
              45
              45
              --
                31
                30
                --
                 1 Remainder.
```

For convenience, we write the divisor at the left and the quotient at the right of the dividend, and begin to divide as in Short Division.

15 is contained in 3 ten-thousands 0 ten-thousands times; therefore, there will be 0 ten-thousands in the quotient. Take 34 thousands; 15 is contained in 34 thousands, 2 thousands times; we write the 2 thousands in the quotient. 15 $\times$ 2 thousands $=$ 30 thousands, which, subtracted from 34 thousands, leaves 4 thousands $=$ 40 hundreds. Adding the 5 hundreds, we have 45 hundreds.

15 in 45 hundreds, 3 hundreds times; we write the 3 hundreds in the quotient. 15 $\times$ 3 hundreds $=$ 45 hundreds, which subtracted from 45 hundreds, leaves nothing. Adding the 3 tens, we have 3 tens.

15 in 3 tens, 0 tens times; we write 0 tens in the quotient. Adding to the three tens, which equal 30 units, the 1 units, we have 31 units.

15 in 31 units, 2 units times; we write the 2 units in the quotient. 15 $\times$ 2 units $=$ 30 units, which, subtracted from 31 units, leaves 1 unit as a remainder. Indicating the division of the 1 unit, we annex the fractional expression, $\frac{1}{15}$ unit, to the integral part of the quotient.

Therefore, 34531 divided by 15 is equal to $2302\frac{1}{15}$.

In practice, we may say: 15 in 34, 2 times; write 2 in the quotient; $15 \times 2 = 30$, which from 34 leaves 4. Bring down 5; 15 in 45, 3 times; write 3 in the quotient; $15 \times 3 = 45$, which from 45 leaves 8. Bring down 3; 15 in 3, 0 times; write 0 in the quotient. Bring down 1; 15 in 31, 2 times; $15 \times 2 = 30$, which from 31 leaves 1. Answer, $2302\frac{1}{15}$.

Explain the operation.

When, as above, the work of dividing is mostly written out, the process is called LONG DIVISION.

RULE. *Write the divisor at the left of the dividend.*

Begin at the left, divide the number expressed by the fewest figures of the dividend that will contain the divisor, and write the quotient at the right of the dividend.

Multiply the divisor by this quotient; subtract the product from the part of the dividend used, and to the remainder bring down the next figure of the dividend.

Divide as before, till all the figures of the dividend have been used.

If there be a final remainder, write it, with the divisor beneath, after the integral part of the quotient.

PROOF. The same as in Short Division.

Examples.

2. How many times is 24 contained in 7816?

OPERATION.	PROOF.
24) 7816 ($325\frac{16}{24}$ Quotient.	325
72	24
61	1300
48	650
136	7800
120	16 Remainder.
16 Remainder.	7816

Divide

3. 8687 by 7. *Ans.* 1241.	7. 86009 by 63. *Ans.* $1365\frac{14}{63}$.
4. 1618 by 9.	8. 18570 by 34.
5. 15702 by 11. *Ans.* $1427\frac{5}{11}$.	9. 5783 by 108. *Ans.* $53\frac{59}{108}$.
6. 25620 by 12. *Ans.* 2135.	10. 98701 by 75.

When is the process called Long Division? Repeat the Rule. What is the Proof?

11. 17354 by 86. *Ans.* $201\frac{68}{86}$.
12. 7012 by 52. *Ans.* $134\frac{44}{52}$.
13. 91028 by 109.
14. 974932 by 365.
15. $13354 \div 17 =$ how many? *Ans.* $785\frac{9}{17}$.
16. $3406 \div 62 =$ how many? *Ans.* $54\frac{58}{62}$.
17. $10000 \div 35 =$ how many?
18. $10064 \div 110 =$ how many? *Ans.* $91\frac{54}{110}$.
19. $45078 \div 73 =$ how many? *Ans.* $617\frac{37}{73}$.
20. $111111 \div 222 =$ how many?
21. $60702 \div 51 =$ how many? *Ans.* $1190\frac{12}{51}$.
22. $13415 \div 55 =$ how many? *Ans.* $243\frac{50}{55}$.
23. What is the value of $\frac{45630}{121}$?
24. Divide 23218 by 60. *Ans.* $386\frac{58}{60}$.
25. Divide 63125 by 123. *Ans.* $513\frac{26}{123}$.
26. Divide 1554768 by 216 *Ans.* 7198.
27. Divide 200204 by 81.
28. Divide 100000 by 102. *Ans.* $980\frac{40}{102}$.
29. Divide 40060 by 1023. *Ans.* $39\frac{153}{1023}$.
30. Divide 8317 by 27. *Ans.* $308\frac{1}{27}$.
31. Divide 6421284 by 642. *Ans.* 10002.
32. Divide 120345 by 3102. *Ans.* $38\frac{2469}{3102}$.

33. Divide six million three hundred forty-six thousand two hundred and sixty-nine by one thousand two hundred and sixty-nine. *Ans.* 5001.

34. Divide two million nine hundred fifty-three thousand and seventy-nine by one thousand seven hundred and twenty-eight. *Ans.* $1708\frac{1655}{1728}$.

64. When the divisor is 10, 100, 1000, etc., *the quotient may be obtained, at once, by removing the decimal point in the dividend, as many places to the left as there are ciphers in the divisor.*

For, since the value denoted by figures is multiplied by 10 by removing the decimal point one place to the right, by 100 by removing it two places, etc. (Art. 54); and as division is the reverse of multiplication (Art. 60), removing the decimal point in the dividend one place to the left divides it by 10, two places divides it by 100, etc.

REVIEW QUESTIONS. What is Multiplication? (47) Division? (57.) How may the quotient be obtained when the divisor is 10, 100, 1000, etc.?

The integral part of the quotient will be on the left of the decimal point, and *the remainder will be the part on the right of the point.* Thus,

$1243 \div 10 = 124.3 = 124\frac{3}{10}$, read, one hundred twenty-four units and three tenths; $1243 \div 100 = 12.43 = 12\frac{43}{100}$, read, twelve units and forty-three hundredths; $5004 \div 1000 = 5.004 = 5\frac{4}{1000}$, read, five units and four thousandths, etc. That is,

The first order at the right of the decimal point expresses tenths; the second, hundredths; the third, thousandths, etc.

Divide

35. 1463 by 10. *Ans.* $146\frac{3}{10}$.
36. 6700 by 100.
37. 16301 by 100. *Ans.* $163\frac{1}{100}$.
38. 85761 by 1000.
39. 60013 by 1000. *Ans.* $60\frac{13}{1000}$.
40. 33444 by 10000.
41. 80000 by 1000. *Ans.* 80.

65. When the divisor has any number of significant figures with ciphers on the right, *the work may be abridged by cutting off the ciphers at the right of the divisor and an equal number of figures from the right of the dividend, and then dividing the remaining part of the dividend by the remaining part of the divisor, and, if there be a remainder, prefixing it to the figures that were cut off from the dividend for the entire remainder.*

42. Find how many times 1700 is contained in 39792.

OPERATION.

```
17|00 ) 397|92 ( 23 692/1700, Ans.
        34
        ---
         57
         51
         ---
         692
```

17 hundreds is contained in 397 hundreds, 23 times, with a remainder of 6 hundreds, which, with 92, makes the entire remainder 600 + 92, or 692. Therefore, 1700 is contained in 39792, $23\frac{692}{1700}$ times.

What will mark the parts of the quotient? What will the part on the right of the point be? How may the work be abridged when the divisor has any number of significant figures with ciphers on the right? In the operation, what is the remainder found by using the hundreds in dividing? What is taken to form the entire remainder?

43. How many times 70 in 22120? *Ans.* 316.
44. How many times 900 in 829800?
45. How many times 1900 in 40220? *Ans.* $21\frac{320}{1900}$.
46. How many times 1600 in 137000? *Ans.* $85\frac{1000}{1600}$.
47. Divide 89952 by 500. *Ans.* $179\frac{452}{500}$.
48. Divide 131127 by 12000.
49. Divide 4590000 by 306000 *Ans.* 15.
50. Divide 13834500 by 120300. *Ans.* 115.
51. Divide 803402 by 400000.
52. Divide 11579112 by 890000. *Ans.* $13\frac{9112}{890000}$.
53. Divide 3678900 by 326100. *Ans.* $11\frac{91800}{326100}$.

APPLICATIONS.

1. James Brown bought a farm of 387 acres for 8514 dollars; how much did it cost him per acre? *Ans.* 22 dollars.

2. An army of 9870 men lost one-fifteenth of its whole number in storming a fort; how many were lost? *Ans.* 658 men.

3. If a field of 109 acres produces 3379 bushels of wheat, how much is the yield per acre? *Ans.* 31 bushels.

4. If a field yielding at the rate of 31 bushels to the acre produces 3379 bushels of wheat, how many acres are there in the field?

5. How long should 17 men subsist upon a supply of provisions which would suffice for one man 6205 days?

6. A teamster removed 31340 bricks at 20 loads; how many did he remove at a load? *Ans.* 1567 bricks.

7. If the valuation of a certain town of 7500 inhabitants is 2625000 dollars, what is the average to each individual? *Ans.* 350 dollars.

8. How many years must a person labor to accumulate 23100 dollars, by saving 1100 dollars a year?

9. If light moves at the rate of 192000 miles a second, how many seconds is it in coming from the moon to the earth, the distance being 240000 miles? *Ans.* $1\frac{48000}{192000}$ seconds.

10. Certain States and the District of Columbia furnished in the late war 2653062 soldiers; how many days would it take a person to count them at the rate of 15000 a day?

Ans. $176\frac{13062}{15000}$ days.

REVIEW EXERCISES.

1. If the multiplicand is 15607 and the multiplier 3094, what is the product? *Ans.* 48288058.

2. If the dividend is 48288058 and the the divisor 3094, what is the quotient?

3. If two men start from the same point and travel in opposite directions, the one at the rate of 42 miles and the other 45 miles a day, how far apart will they be at the end of 12 days?

SOLUTION. *If one man travel at the rate of* 42 *miles a day, he will travel in* 12 *days* 12 *times* 42 *miles, or* 504 *miles.*

If the other man travel at the rate of 45 *miles a day, he will travel in* 12 *days* 12 *times* 45 *miles, or* 540 *miles.*

If the one travel 504 *miles and the other* 540 *miles, and in opposite directions, they must be apart, the sum of* 504 *miles and* 540 *miles, or* 1044 *miles.*

Therefore, if two men start from the same point and travel in opposite directions, the one at the rate of 42 *miles and the other* 45 *miles a day, they will be* 1044 *miles apart at the end of* 12 *days.*

4. If two men start from the same point and travel in the same direction, the one at the rate of 512 miles and the other 540 miles a week, how far apart will they be at the end of 8 weeks?

REVIEW QUESTIONS. What is the Rule for Short Division? (62) What is the Rule for Long Division? (63)

5. I have two farms; the first contains 160 acres, worth 80 dollars an acre, and the second 220 acres, worth 65 dollars an acre; how much are both worth? *Ans.* 27100 dollars.

6. What will be the cost of 103 barrels of flour at 7 dollars a barrel? *Ans.* 721 dollars.

7. A merchant bought 30 hogsheads of molasses at 45 dollars each, and paid 800 dollars down, and gave his note for the balance; for what amount was the note?

8. If a man sell 90 acres of land at 38 dollars an acre, and divide the money equally among his 5 children, what is each child's share?

SOLUTION. *If a man sell 90 acres of land at 38 dollars an acre, he will receive for it 90 times 38 dollars, or 3420 dollars.*

If he divide 3420 dollars equally among his 5 children, each child will receive as a share one-fifth of 3420 dollars, or 684 dollars.

Therefore, if a man sell 90 acres of land at 38 dollars, and divide the money equally among his 5 children, each child's share is 684 dollars.

9. If a man having 5500 dollars to invest should purchase 15 United States bonds, at 105 dollars each, how many shares of railroad stock, at 157 dollars each, could he purchase with the balance? *Ans.* 25 shares.

10. William Miller bought some land for 18050 dollars. He sold 50 acres of it for 60 dollars an acre, and then found that the remainder cost him 50 dollars an acre; how many acres were there of the remainder? *Ans.* 301 acres.

11. Bought 5 cows at 50 dollars each, and 7 horses at half the price each of the entire cost of the cows; how much was the cost of both? *Ans.* 1125 dollars.

12. Smith has 168 acres of land, Johnson 4 times as much and 35 acres, and Wade 3 times as much as both of them less 1200 acres; how many acres in all have they?

REVIEW QUESTIONS. What is the Proof in Addition? (40) In Subtraction? (45) In Multiplication? (51) In Division? (62) What is the answer in Multiplication called? (47) In Division? (57)

GENERAL PRINCIPLES AND APPLICATIONS.

66. The **Fundamental Operations** or **Processes** of Arithmetic, or those upon which all others depend, are based upon Notation, and are

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION.

67. The Signs used to indicate processes, or to abbreviate expressions, are called

SYMBOLS.

$+$, read plus, or added to.	$=$, read equals, or equal to.
$-$, read minus, or less.	$\therefore$, read therefore, hence.
$\times$, read multiplied by.	$\because$, read since, because.
$\div$, read divided by.	( ), parenthesis.

68. Numbers in a parenthesis, or under a vinculum, $\overline{\quad}$, are to be regarded as all subject to the same operation. Thus,

$16 - (3 \times 2))$, denotes that the *product* of 3 multiplied by 2 is to be subtracted from 16.

$\overline{16 - (5 + 3)} \times 5$, denotes 5 times the difference between 16 and the sum of 5 added to 3.

Exercises.

1. What is the value of $(31 \times 6) - 86$?

SOLUTION. 31×6 *equals* 186, *and* $186 - 86$ *equals* 100. *Therefore*, 31 *multiplied by* 6, *in parenthesis*, *less* 86, *equals* 100.

2. $(18 \div 6) + 13 =$ how many? *Ans.* 16.
3. $(6 \times 6) \div (4 \times 3) =$ how many? *Ans.* 3.
4. $(\overline{8 + 2} \times 5) - 20 =$ how many? *Ans.* 30.
5. $\dfrac{(160 - 70) + 18}{12} =$ how many? *Ans.* 9.
6. $(3 + 9) \times (13 - \overline{5 \times 2}) =$ how many?

Upon what are the Fundamental Operations of Arithmetic based? Name them. What are Symbols of Operation? How are numbers in a parenthesis, or under a vinculum, to be regarded?

FORMULAS.

69. An Arithmetical **Formula** is an arithmetical expression of a general rule.

70. The following formulas, which include the fundamental operations of Arithmetic, follow from the preceding definitions, principles, and illustrations:

1. *The* SUM = *all the parts added.*
2. *The* DIFFERENCE = *the Minuend* — *the Subtrahend.*
3. *The* MINUEND = *the Subtrahend* + *the Difference.*
4. *The* SUBTRAHEND = *the Minuend* — *the Difference.*
5. *The* PRODUCT = *the Multiplicand* × *the Multiplier.*
6. *The* MULTIPLICAND = *the Product* ÷ *the Multiplier.*
7. *The* MULTIPLIER = *the Product* ÷ *the Multiplicand.*
8. *The* QUOTIENT = *the Dividend* ÷ *the Divisor.*
9. *The* DIVIDEND = *the Quotient* × *the Divisor.*
10. *The* DIVISOR = *the Dividend* ÷ *the Quotient.*

These *ten Formulas,* from their general nature and importance, may be regarded as *Fundamental.*

The ninth and tenth formulas are general, as will hereafter appear; but, when there is a *Remainder* to be considered, they may for present applications be given thus:

11. *The* DIVIDEND = *(the integral part of the Quotient* × *the Divisor)* + *the Remainder;*

12. *The* DIVISOR = *(the Dividend* — *the Remainder)* ÷ *the integral part of the quotient.*

What is an Arithmetical Formula? To what is the Sum equal? The Difference? The Minuend? The Subtrahend? The Product? The Multiplicand? The Multiplier? The Quotient? The Dividend? The Divisor? Which of the formulas may be regarded as fundamental? When there is a Remainder to be regarded, how may the ninth and tenth formulas be given?

The sixth and seventh formulas furnish reliable methods of proving Multiplication by Division. Thus, the multiplication is proved, when,

13. *The Product* $\div$ *the Multiplier* = *the Multiplicand*; or,

14. *The Product* $\div$ *the Multiplicand* = *the Multiplier.*

SOME PRINCIPLES OF DIVISION.

71. *Multiplying the dividend, or dividing the divisor, by any number, multiplies the quotient by the same number.* Thus,

$$\frac{16}{4} = 4;\ \text{and}\ \frac{16 \times 2}{4} = 8,\ \text{or}\ \frac{16}{4 \div 2} = 8.$$

72. *Dividing the dividend, or multiplying the divisor, by any number, divides the quotient by the same number.* Thus,

$$\frac{16}{4} = 4;\ \text{and}\ \frac{16 \div 2}{4} = 2,\ \text{or}\ \frac{16}{4 \times 2} = 2.$$

73. *Dividing or multiplying both the dividend and divisor by the same number will not change the quotient.* Thus,

$$\frac{16}{4} = 4;\ \text{and}\ \frac{16 \div 2}{4 \div 2} = 4,\ \text{or}\ \frac{16 \times 2}{4 \times 2} = 4.$$

REVIEW EXERCISES.

1. If the items of a certain debt are 12 dollars, 106 dollars, and 112 dollars, what is its entire sum?

SOLUTION. *Since the sum is equal to all the parts added, if the items of a certain debt are* 12 *dollars,* 106 *dollars, and* 112 *dollars, its entire sum must be equal to* 12 *dollars* + 106 *dollars* + 112 *dollars, or* 230 *dollars.*

Which formulas furnish a proof of Multiplication? What is the first Principle of Division? The illustration? The second Principle? The illustration? The third Principle? The illustration?

2. The larger of two numbers is 1685, and the smaller is 768; what is their difference?

3. The difference between two numbers is 146 and the less is 467; what is the larger?

4. If 1406 be the difference between two numbers, and the greater number is 4879, what is the less number?

5. The two factors of a number are 27 and 36; what is the number? *Ans.* 972.

6. The product of A's and B's ages is 3200 years, and B's age is 50 years; what is the age of A?

7. The product of two numbers is 515522, and the larger number is 1601; what is the smaller? *Ans.* 322.

8. If 168465 pounds of beef be divided equally between 11231 soldiers, how many pounds will each receive?

Ans. 15 pounds.

9. If a quantity of beef was divided equally between 11231 soldiers, and each received for his share 15 pounds, what quantity was divided?

10. The product of two numbers is 890368, and the larger number is 3478; what is the smaller? *Ans.* 256.

11. 168465 pounds of beef having been equally divided between a number of soldiers, each received as his share 15 pounds; what was the number of soldiers?

12. If the integral part of a quotient be 23, the remainder 692, and the dividend 39792, what is the divisor?

Ans. 1700.

13. John Reed says his number of sheep is such that if he should separate the flock so as to put 115 in each of 5 fields, there would be 4 sheep remaining. How many sheep has he?

Ans. 579 sheep.

14. A farmer having 579 sheep, on putting an equal number of them into each of five fields, had 4 remaining. How many did he put into each of the fields?

ARITHMETICAL ANALYSIS.

74. The process of solving questions, by considering their elements, or by comparing their numbers and reasoning from them to a required number, is called *Analysis*. Hence,

75. **Arithmetical Analysis** is the process of solving a question, by considering its elementary parts, and obtaining a required result by a course of reasoning.

In analyzing a simple question, the reasoning is usually from a given number to a unit of the same name, and then from this unit to the required number.

Exercises.

1. If 7 acres of land produce 525 bushels of wheat, how many bushels will 11 acres produce?

ANALYSIS. *If* 7 *acres of land produce* 525 *bushels of wheat*, 1 *acre will produce one seventh of* 525 *bushels, or* 75 *bushels, and* 11 *acres will produce* 11 *times* 75 *bushels, or* 825 *bushels. Therefore, etc.*

2. If 11 acres of land produce 825 bushels of corn, how many bushels will 7 acres produce?

3. When 120 barrels of flour cost 1560 dollars, what will 139 barrels cost? *Ans.* 1807 dollars.

4. If 139 barrels of flour cost 1807 dollars, what will 120 barrels cost?

5. If a man can travel 2440 miles in 4 weeks, how many miles can he travel in 9 weeks? *Ans.* 5490 miles.

6. If 20 men can do a piece of work in 11 days, how many days will it take 17 men to do it?

ANALYSIS. *If* 20 *men can do a piece of work in* 11 *days, it will take* 1 *man* 20 *times* 11 *days, or* 220 *days, to do it, and* 17 *men* 1 *seventeenth of* 220 *days, or* $12\frac{16}{17}$ *days. Therefore, etc.*

What is Arithmetical Analysis?

7. When 13 men can hoe a field in 21 days, in what time can 39 men hoe it? *Ans.* 7 days.

8. If 19 cords of wood will pay for 57 yards of cloth at 2 dollars a yard, what is the wood a cord? *Ans.* 6 dollars.

9. When 540 tons of coal will pay for 30 tons of iron, at 90 dollars a ton, what is the coal a ton? *Ans.* 5 dollars.

10. If 14 horses can be bought for 2240 dollars, how many can be bought for 3040 dollars?

ANALYSIS. *If* 14 *horses can be bought for* 2240 *dollars,* 1 *horse can be bought for* 1 *fourteenth of* 2240 *dollars, or* 160 *dollars; and if* 1 *horse can be bought for* 160 *dollars, as many horses can be bought for* 3040 *dollars as* 160 *dollars are contained times in* 3040 *dollars, or* 19 *horses. Therefore, etc.*

11. If 19 horses can be bought for 3040 dollars, how many can be bought for 2240 dollars?

12. When 23 men earn 1380 dollars in a month, how many men will earn 1980 dollars in the same time? *Ans.* 33 men.

13. If 31 horses will consume 93 tons of hay in a certain time, how many horses will consume 87 tons in the same time? *Ans.* 29 horses.

14. How many hogsheads of molasses can be bought for 17670 dollars, if 16 hogsheads can be bought for 912 dollars? *Ans.* 310 hogsheads.

15.* Daniel White bought an equal number of cows and oxen for 2880 dollars, each cow at 45 dollars, and each ox at 75 dollars; how many of each did he buy?

ANALYSIS. *Since* 1 *cow cost* 45 *dollars and* 1 *ox* 75 *dollars,* 1 *cow and* 1 *ox together were bought for* 45 *dollars plus* 75 *dollars, or* 120 *dollars; and since the number of each was equal, he bought as many of each as* 120 *dollars are contained times in* 2880 *dollars, or* 24. *Therefore, etc.*

REVIEW QUESTIONS. Which are the Fundamental Operations of Arithmetic? (66) What are called Symbols of Operation? (67) What is an Arithmetical Formula? (69) How many of them are called Fundamental? (70)

* Optional.

16.* I have 11900 dollars in National currency consisting of an equal number of 5 dollar, 10 dollar, and 20 dollar bills; how many have I of each? *Ans.* 340.

17. Frank Homer bought a farm for 11200 dollars, giving 60 dollars an acre for one half of it and 80 dollars an acre for the other half; how many acres in each half? *Ans.* 80 acres.

18. A farmer sold an equal number of colts at 90 dollars each, and horses at 175 dollars each, for 3445 dollars; how much did he get for each kind?

Ans. Colts, 1170 dollars; horses, 2275 dollars.

19. If A and B together have 1625 dollars, and B 825 dollars more than A, how much is the money of each?

SOLUTION. *If A and B together have* 1625 *dollars, and B* 825 *dollars more than A,* 1625 *dollars less* 825 *dollars, or* 800 *dollars, must be twice A's money.*

If 800 *dollars be twice A's money, one half of* 800 *dollars, or* 400 *dollars, is A's money; and, since B has* 825 *dollars more than A, B's money is* 400 *dollars plus* 825 *dollars, or* 1225 *dollars.*

Therefore, etc.

20. At a certain election A and B were candidates; the whole number of votes cast for them was 5963, of which A had 321 more than B; how many votes did each receive?

Ans. A 3142; B 2821.

21. Two men having met on a journey, found that they had travelled 1200 miles, and that one had travelled 360 miles more than the other; what distance had each travelled?

22. Albert Smith gave his three sons 1350 dollars, Samuel receiving 50 dollars more than Edmund, and Ernest 150 more than Samuel; how much did each receive?

Ans. Edmund $366\frac{2}{3}$ dollars, Samuel $416\frac{2}{3}$ dollars, and Ernest $566\frac{2}{3}$ dollars.

REVIEW QUESTIONS. How is the sum of two or more numbers found? (37) How is the difference of two numbers found? (42)

* This page is optional.

UNITED STATES MONEY.

76. United States Money is the legal currency of the United States.

The name of its different orders, or denominations, beginning with mills, is shown in the following

Table.

10	mills	make	1	cent,	marked	c.
10	cents,	"	1	dime,	"	d.
10	dimes,	"	1	dollar,	"	$
10	dollars,	"	1	eagle,	"	E.

COINS.

77. Coins are pieces of metal converted into money by legal stamping.

The COINS of the United States are of gold, silver, nickel, and bronze, as follows:

GOLD.			SILVER.		
Double eagle,	value	$20	Dollar,	value	$1.
Eagle,	"	10	Half-dollar,	"	50 c.
Half-eagle,	"	5	Quarter-dollar,	"	25 c.
Three dollar,	"	3	Dime,	"	10 c.
Quarter-eagle,	"	2½	Half-dime,	"	5 c.
Dollar,	"	1	Three-cent,	"	3 c.

NICKEL.

Three-cent, value 3 c. | Five-cent, value 5 c.

BRONZE.

Two-cent, value 2 c. | Cent, value 1 c.

What is United States Money? Repeat the Table. What are Coins? Name the gold coins. The silver coins. The nickel coin. The bronze.

The *Standard* of gold coin of the United States is 9 parts pure metal, and 1 part silver and copper; and of silver coin, is 9 parts pure metal, and 1 part copper.

The bronze coin is 95 parts copper, and 5 parts tin and zinc.

The dollar mark, $, may be considered as the letter U written upon an S, denoting U. S., the initials of United States.

NOTATION AND NUMERATION.

78. The Dollar is the *unit* of United States Money.

In accounts, eagles are written as *tens* of dollars, and indicated by the dollar mark ($); and they are usually read as a number of dollars. Thus,

5 eagles are written $50, and read fifty dollars.

Dimes are written as *tenths*, cents as *hundredths*, and mills as *thousandths* of a dollar, and separated from dollars by the decimal point (.). Dimes are usually read as a number of cents, and mills are sometimes read as parts of a cent. Thus,

3 eagles, 3 dollars, 3 dimes, 3 cents, and 3 mills are written,

$$\$33.333,$$

and read thirty-three dollars, thirty-three cents, and three mills; and,

4 eagles, 2 dimes, and 5 mills may be written,

$$\$40.20\tfrac{1}{2},$$

and read forty dollars, twenty and one half cents.

What is the standard of gold coin? Of silver coin? Of what is the bronze coin composed? What may the dollar mark be considered? What is the unit of United States Money? How are eagles written and usually read? How are dimes, cents, and mills written? How are dimes usually read? Mills sometimes?

79. In United States Money, 10 of a lower denomination make 1 of the next higher, according to the Decimal System (Art. 32). Hence,

United States Money may be written, added, subtracted, multiplied, and divided, by preceding rules.

Exercises.

Copy and read:

1. $6.15	4. $21.06	7. $108.12½
2. $9.87	5. $37.70	8. $1813.19
3. $8.625	6. $93.708	9. $35.005

Write in figures:

10. Forty-two dollars, eighteen cents. *Ans.* $42.18.
11. Thirteen dollars, twelve and a half cents. *Ans.* $13.12½.
12. Seventy-three dollars, eleven cents.
13. Sixty-six dollars, six dimes. *Ans.* $66.60.
14. Five eagles, five cents. *Ans.* $50.05.
15. One hundred nine dollars, nine cents.
16. Six hundred dollars, ten cents, five mills. *Ans.* $600.105.
17. One thousand three hundred fifty dollars.
18. Five hundred dollars, eighty-seven cents, five mills.

REDUCTION.

80. Reduction is the process of changing one number into another of a different denomination, but of equal value.

The reduction is called *descending*, when the change is into a number of a lower denomination; and is called *ascending*, when the change is into a number of a higher denomination.

How may United States Money be written, added, etc.? What is Reduction?

CASE I.

81. To reduce a number to another of a lower denomination.

1. In 59 cents how many mills?

SOLUTION. *Since in* 1 *cent there are* 10 *mills, in* 59 *cents there must be* 59 *times* 10 *mills, or* 590 *mills.*

2. In 8 dollars how many cents?

SOLUTION. *Since in* 1 *dollar there are* 100 *cents, in* 8 *dollars there must be* 8 *times* 100 *cents, or* 800 *cents.*

3. In 8 dollars how many mills?

SOLUTION. *Since in* 1 *dollar there are* 100 *cents and in each cent* 10 *mills, there must be in* 1 *dollar* 100 *times* 10 *mills, or* 1000 *mills, and in* 8 *dollars* 8 *times* 1000 *mills, or* 8000 *mills.*

RULE. *To reduce cents to mills, multiply by* 10; *dollars to cents, multiply by* 100; *or dollars to mills, multiply by* 1000.

When there is no decimal point in the number, the multiplication may be performed by annexing ciphers (Art. 54), omitting the dollar mark, if used.

When there is a decimal point in the number, dollars and cents may be reduced to cents, and dollars, cents, and mills to mills, by simply removing the dollar mark and the decimal point. For, multiplying by 10, 100, etc., has the same effect as removing the decimal point as many places to the right as there are ciphers in the multiplier (Art. 30).

Examples.

4. In $162 how many mills? *Ans.* 162000 mills.
5. In $1.62 how many cents? *Ans.* 162 cents.
6. In $31 how many cents?
7. In $6.008 how many mills? *Ans.* 6008 mills.
8. In 87 cents how many mills? *Ans.* 870 mills.
9. In $36.03 how many mills?
10. Reduce $160.90 to cents. *Ans.* 16090 cents.

How many mills make 1 cent? How many cents make 1 dollar? How many mills make 1 dollar? Repeat the Rule. When there is no decimal point in the number, how may the multiplication be performed? How when there is a decimal point in the number?

CASE II.

82. To reduce a number to another of a higher denomination.

1. In 590 mills how many cents?

SOLUTION. *Since in* 10 *mills there is* 1 *cent, there must be in* 590 *mills as many cents as* 10 *mills are contained times in* 590 *mills, or* 59 *cents.*

2. In 800 cents how many dollars?

SOLUTION. *Since in* 100 *cents there is* 1 *dollar, in* 800 *cents there are as many dollars as* 100 *cents are contained times in* 800 *cents, or* $8.

3. In 8000 mills how many dollars?

SOLUTION. *Since in* 1000 *mills there is* 1 *dollar, in* 8000 *mills there are as many dollars as* 1000 *mills are contained times in* 8000 *mills, or* $8.

RULE. *To reduce mills to cents, divide by* 10; *cents to dollars, divide by* 100; *or mills to dollars, divide by* 1000.

Cents or mills may be reduced directly to dollars by placing the decimal point as many places to the left as there are ciphers in the divisor (Art. 64).

Examples.

4. In 691000 mills how many dollars? *Ans.* $691.
5. In 162 cents how many dollars?
6. In 3100 cents how many dollars? *Ans.* $31.
7. In 970 mills how many cents? *Ans.* 97 cents.
8. In 36030 mills how many dollars?
9. Reduce 875 mills to cents. *Ans.* $87\frac{1}{2}$ cents.
10. Reduce 16090 cents to dollars. *Ans.* $160.90.
11. Reduce 18734 mills to dollars.
12. Reduce 18734 mills to cents. *Ans.* $1873\frac{4}{10}$ cents.

Repeat the Rule. How may cents or mills be reduced directly to dollars?

ADDITION.

83. 1. Add $21.38, $1.82, and $31.015.

OPERATION.

$21.38
1.82
31.015
$54.215

For convenience, we write the numbers so that figures of the same order stand in the same column, and begin at the right to add, observing to place the decimal point between dollars and cents.

2. Add $164.00, $561.45, $902.03, $31.61, and $.95. *Ans.* $1660.04.

3. Add $80.375, $103.16, $1500.81, and $100.87½.

4. Find the sum of $3064.05, $10091.63, $601, and $111.62½. *Ans.* $13868.30½.

5. $809 + $310.05 + $1701.12 + $10003.14 = how many?

6. $10806.17½ + $9850.37 + $6345 + $916032 + $82.12½ = how much? *Ans.* $943115.67.

SUBTRACTION.

84. 1. Subtract $109.02½ from $963.03.

OPERATION.

$963.030
109.025
$854.005

For convenience, we write the subtrahend under the minuend, so that figures of the same order stand in the same column, and begin at the right to subtract, observing to place the decimal point between dollars and cents.

Here, in the subtrahend we write the ½ cent as 5 mills, and in the minuend note the place of mills by 0.

2. From $98.175 take $19.18. *Ans.* $78.995.

3. From $46.95 take $31.008.

4. From $136 take $1.63. *Ans.* $134.37.

5. From one thousand dollars take three cents and one mill. *Ans.* $999.969.

How do you write the numbers to be added in Addition of United States Money? Where do we begin to add? Where do we place the decimal point? How are the numbers written for subtracting? Where do we begin to subtract? How is the ½ cent written? Why is a 0 annexed to the minuend?

6. From 13000 dollars, 93 cents, and 4 mills, take 1300 dollars, 9 cents, and 5 mills. *Ans.* $11700.839.

MULTIPLICATION.

85. 1. Multiply $513.125 by 7.

OPERATION.

$513.125 7 ——— $3591.875	We begin at the right to multiply, observing to place the decimal point between dollars and cents in the product.

2. Multiply $306.07 by 13. *Ans.* $3978.91.
3. Multiply $365.155 by 15. *Ans.* $5477.325.
4. Multiply $97.38 by 19.
5. Multiply $39.67 by 100. *Ans.* $3967.
6. Multiply 10 dollars 25 cents by 27.
7. Multiply one hundred thirteen dollars, five cents, and five mills, by one hundred and five. *Ans.* $11870.775.

DIVISION.

86. 1. Divide 653 dollars by 4.

OPERATIONS.

4) $653 ——— $163¼	or,	4) $653.00 ——— $163.25

Dividing the dollars, we have a remainder, which we write as a part of a dollar, in the first operation.

In the second operation, the division is extended to cents by supplying the two places of cents by ciphers in the dividend. The two answers, although a little different in form, express the same value, since ¼ of a dollar and 25 cents are equal.

In like manner, if it had been required, the division could have been extended to mills, by supplying an additional cipher.

How do we begin to multiply? Where do we place the decimal point in the result? When there is a remainder, after dividing dollars, how may the division be extended to cents? To mills?

2. Divide \$254.04 by 12. *Ans.* \$21.17.

3. Divide \$562 by 5. *Ans.* \$112.40.

4. Divide \$181.125 by 9.

5. Divide \$97.44 by 1624. *Ans.* \$.06.

6. Divide \$61.205 by 8. *Ans.* $\$7.650\frac{5}{8}$.

Here, there is a remainder after dividing mills, which is expressed in the answer; but in such cases it is customary to omit the remainder, and to add +, to indicate that the division is not exact, thus, \$7.650 +.

7. Divide one thousand dollars and sixty cents by one hundred and ninety-nine.

87. When divisor and dividend both express sums of money, but of different denominations, they must be brought to the same denomination before dividing.

8. How many times is \$1.25 contained in \$40.

OPERATION.

```
125 ) 4000 ( 32
      375
      ---
       250
       250
```

In \$1.25 there are 125 cents; in \$40 there are 4000 cents.

125 cents are contained in 4000 cents 32 times.

9. Divide \$954 by 25 cents. *Ans.* 3816

10. Divide \$141 by 75 cents.

11. Divide \$444 by 370 mills. *Ans.* 1200.

12. How many times seven mills in twenty-nine dollars? *Ans.* $4142\frac{6}{7}$.

13. How many times twelve cents and five mills in three hundred and seventy dollars? *Ans.* 2960.

14. How many times one dollar and twenty-five cents in thirteen eagles seven dollars and five dimes? *Ans.* 110.

What is customary when there is a remainder after dividing mills? What must be done before dividing when the divisor and dividend are sums of money, but of different denominations? To what must dollars be reduced in order to divide them by cents?

APPLICATIONS.

1. Bought of C. Washburne, flour for $13.25, sugar $3.75, tea $9.27, pepper $.17, starch $.12½, and kerosene $1.87½; what was the whole amount? *Ans.* $28.44.

2. Sold a horse for $300, a carriage for $375.50, and a saddle for $15.75; how much was received for the whole?

3. Bought goods for $9635, but the same receiving injury, I was content to sell them at a loss of $367.87½; what did I receive for them? *Ans.* $9267.12½.

4. Nathan Soule bought a house for $6167, and sold it for $5375.75; how much was his loss?

5. I paid for a horse $375, for a yoke of oxen $263, for a cow $75.50, for a yoke $7.37½, and sold the whole at a profit of $13.12½; how much did I get for them? *Ans.* $734.

6. A young lady went "a shopping." She purchased a silk dress for $43, some velvet for $9.75, a shawl for $25, a bonnet for $19.87, and some pins for $.15. If she started with a hundred dollar bill, how much change should she bring back? *Ans.* $2.23.

7. If a barrel of flour is worth $13.65, how much are 110 barrels worth? *Ans.* $1501.50.

8. When flour is $13.65 a barrel, how many barrels can be bought for $1501.50?

9. When coal is $4.25 a ton, what will 1000 tons cost? *Ans.* $4250.

10. When 1000 tons of coal can be bought for $4250, what is the price of one ton?

11. If a farm of 47 acres can be bought for $1774.25, what is the price of one acre?

12. What cost 316 bushels of wheat at $1.63 a bushel? *Ans.* 515.08.

13. What cost 316 bales of cotton, at $260.50 a bale?

REVIEW QUESTIONS. What is a Rule? (11) What is an Arithmetical Formula? (69) Arithmetical Analysis? (74)

14. If 519 bushels of potatoes cost $194.625, what will one bushel cost? *Ans.* $.37½.

15. Bought 65 yards of cloth at 27 cents a yard, and 15 yards more at $6.50 a yard; what did the whole amount to? *Ans.* $115.05.

16. By receiving $3.25 a day, and paying out for expenses 50 cents a day, in how many days will a man earn $825? *Ans.* 300 days.

17. If 200 pounds of pork cost $25, how much is it a pound?

18. At $.125 a pound, how many pounds of pork can be bought for $25?

19. When 5 bales of cotton, weighing each 312 pounds, can be bought for $491.40, what is the price a pound? *Ans.* $.315.

20. Tobey Wasteful spends each day for beer 15 cents, for cigars 12½ cents, and for oysters 25 cents; at that rate, how much does he spend in 365 days? *Ans.* $191.62½.

EXCHANGE OF COMMODITIES.

88. Exchange of Commodities is trade or traffic, by passing goods or wares from one party to another for an equivalent, in goods or money.

Exercises.

1. If I bought an arithmetic for 63 cents, a reader for $1.25, a slate for 38 cents, a globe for $15.50, and gave in payment a ten-dollar and two five-dollar bills, how much change should I receive?

SOLUTION. *If I bought an arithmetic for* 63 *cents, a reader for* $1.25, *a slate for* 38 *cents, and a globe for* $15.50, *I bought in amount,* $.63 + $1.25 + $.38 + $15.50, *or* $17.76.

If I gave in payment a ten-dollar and two five-dollar bills, I gave $10 + $5 + $5, *or* $20; *and should receive in change, the difference between* $20 *and* $17.76, *or* $2.24. *Therefore, etc.*

What is Exchange of Commodities?

2. A farmer buys a bill of goods amounting to $235.25, and pays down $37.50, and agrees to furnish wheat for the balance at the rate of $1.75 a bushel; how many bushels must he furnish? *Ans.* 113 bushels.

3. I have 12 cords of wood, worth $8 a cord, and 17 hundred rails, worth $6 a hundred; if I should exchange them for a carriage, worth $200, how much should I gain by the operation?

4. Joseph Bryant exchanged 150 spelling-books at 25 cents each, for arithmetics at 50 cents each; how many arithmetics did he receive? *Ans.* 75 arithmetics.

5. If I give 600 pairs of shoes, at $1.25 a pair, for 160 pairs of boots, what are the boots a pair?

6. Joseph Howland had 300 tons of coal, at $7.25 a ton, for which Frank Dunmore gave $1515 in cash, and the balance in wood at $4.40 per cord; how many cords were required? *Ans.* 150 cords.

7. If I should exchange 50 bushels of corn, at $.65 a bushel, for 120 yards of muslin at 15 cents a yard, how much balance would there be in my favor? *Ans.* $14.50.

8. How many pounds of butter, at $.28 a pound, must be given for 14 yards of gingham at 32 cents a yard?

9. How many barrels of flour, at $9.50 a barrel, can be exchanged for 475 pounds of cotton at 30 cents a pound, and 76 gallons of molasses at 50 cents a gallon? *Ans.* 19 barrels.

BILLS AND INVOICES.

89. A **Bill** is a written statement of merchandise bought or sold, or of services rendered.

90. An **Invoice** is a bill of merchandise sent by the seller to the purchaser.

REVIEW QUESTIONS. What is United States Money? (76) Coins? (77) The Unit of United States Money? (79)

What is a Bill? An Invoice?

To make out a bill or invoice, we find the cost of each of the items, and the amount of the whole.

91. A bill is *receipted* when its payment is acknowledged in writing, by the party in whose favor it is, or by some one authorized to sign for him.

The first bill in the following Exercises is receipted by A. C. Lombard & Co., and the third bill, by Robert T. Gould, by George Boyd, who is authorized to sign for Gould.

Exercises.

Reckon and find the amount of the following bills, or invoices:

(1.)

New York, May 16, 1866.

Mr. George A. Crawford,

Bought of A. C. Lombard & Co.

100 lb. Tea,	*a*	*$*	*.50*
150 " Sugar,	*"*		*.14*
60 " Coffee,	*"*		*.42*
132 gal. Molasses,	*"*		*.60*
10 bbl. Flour,	*"*		*12.50*
			$300.40

Received Payment,

A. C. Lombard & Co.

How do we make out a bill or invoice? When is a bill receipted? What does the "*a*" in the bill mean? *Ans.* At.

(2.)

Chicago, July 5, 1866.

Messrs. Thomas, Blakeman, & Co.

Bought of Alfred Shaw,

210 bu. Corn,	*a*	*$ .63*	
500 " Wheat,	*"*	*1.50*	
250 " Oats,	*"*	*.40*	
150 " Barley,	*"*	*.60*	
50 bbl. Flour, extra,	*"*	*9.50*	
			$1547.30

Received Payment,

Alfred Shaw.

(3.)

Boston, Dec. 7, 1866.

Benjamin Richardson, Esq.

To Robert T. Gould, Dr.

To 25 days' Work, on cellar,	*a*	*$4.25*	
" 30 " Work, on wall,	*"*	*4.25*	
" 20 " Work of Apprentice,	*"*	*1.25*	
" 3000 Brick,	*"*	*7.50*	
" 15 tons Stone,	*"*	*10.00*	
			$

Received Payment,

Robert T. Gould.

By George Boyd.

How is the 3d bill receipted? What does the "Dr." mean? *Ans.* Debtor.

ACCOUNTS.

92. An **Account** is a written statement of debt and credit, between two parties.

The party owing is the *Debtor*, and the party owed is the *Creditor.*

93. In the settlement of an account, it is required to find the difference due, or balance.

94. A DUE-BILL is a written acknowledgment that a certain amount is due, and is often given in making settlements, when it is not convenient to make immediate payment.

Exercises.

Reckon and find the balance of the following accounts.

(1.)

Philadelphia, Dec. 12, 1866.

Mr. George W. Grimshaw,

To Cummings, Simons, & Co., Dr.

June 10.	For 31 *yd.*	Muslin,	*a* $.15		
" 13.	" 20 "	Flannel,	" .50		
Aug. 15.	" 16 "	Broadcloth,	" 5.50		
Oct. 31.	" 33 "	Gingham,	" .45		$117.50

Cr.

Sept. 25.	By Cash,	$100.00	
" 17.	" Merchandise,	13.50	113.50
	Balance due C. S. & Co.,		$4.00

Received Payment,

Cummings, Simons, & Co.

2. Baltimore, Nov. 16, 1866. James McClintock owed Andrew Saulsbury for 110 bu. of corn, at 75 cents a bushel,

What is an Account? What party is the Debtor? The Creditor? What is required in the settlement of an account? What is a Due-Bill?

bought Oct. 1; 3 bbl. of flour, at $7.50 a barrel, bought Oct. 7; and 62 bu. of oats, at 43 cents a bushel, bought Nov. 5; and Mr. Saulsbury owed him for 6 thousand of extra shingles, at $6 a thousand, delivered Oct. 5; for cash $60, paid Nov. 1; for bill of labor, amounting to $8.66, rendered Nov. 10. The account was settled Nov. 16 by giving due-bill, amounting to $27, for the balance due A. S. Make out the account, and settle it in your own name for Andrew Saulsbury.

LEDGER COLUMNS.

95. A **Ledger** is the book in which merchants enter a summary of accounts.

The items thus brought together often make long columns.

Accountants, by practice, are enabled to add rapidly the numbers of two or more columns at one operation.

Exercises.

(1.)	(2.)	(3.)	(4.)
$14.56	$321.00	$264.10	$310.06
31.00	123.54	100.78	152.70
78.00	70.25	88.90	31.62
19.15	34.80	142.33	79.08
26.05	11.19	66.08	10.56
17.04	13.33	72.39	180.61
3.36	55.44	67.02	282.39
13.54	72.69	30.50	67.31
14.38	2.18	50.22	55.25
21.00	13.44	17.82	178.90
6.47	7.56	11.12	300.00
2.03	9.00	44.56	65.88
1.12	10.13	14.23	13.44
$247.70			

What is a Ledger? How are accountants enabled to add rapidly?

Here, with Exercise 1, we may illustrate the manner of adding the numbers of two columns at one operation, thus: taking first tens and then units, 12 plus 3 = 15, plus 40 = 55, plus 7 = 62, plus 30 = 92, plus 8 = 100, plus 50 = 150, plus 4 = 154, plus 30 = 184, plus 6 = 190, plus 4 = 194, plus 5 = 199, plus 10 = 209, plus 5 = 214, plus 50 = 264, plus 6 = 270 cents, or \$2.70; we write the 70 cents, and add the \$2 with the dollars.

2 plus 1 = 3, plus 2 = 5, plus 6 = 11, plus 20 = 31, plus 1 = 32, plus 10 = 42, plus 4 = 46, plus 10 = 56, plus 3 = 59, plus 3 = 62, plus 10 = 72, plus 7 = 79, plus 20 = 99, plus 6 = 105, plus 10 = 115, plus 9 = 124, plus 70 = 194, plus 8 = 202, plus 30 = 232, plus 1 = 233, plus 10 = 243, plus 4 = 247 dollars, which we write. Answer, \$247.70.

The process may be abridged by simply naming results: 15; 55, 62; 92, 100; 150, 154; 184, 190; 194; 199; 209, 214; 264, 270; we write the 70.

3; 5; 11; 31, 32; 42, 46; 56, 59; 62; 72, 79; 99, 105; 115, 124; 194, 202; 232, 233; 243, 247; we write the 247. Answer, \$247.70.

FACTORING.

96. An **Integer, or Integral Number,** is a number that contains the unit 1 an exact number of times.

97. An **Exact Divisor, or Measure,** of a number, is any number, that gives an integer for a quotient. Thus,

2, 4, and 8 are exact divisors of 16.

98. An **Integral Factor** of a number is any integer which is an exact divisor of the number.

99. A **Prime Number** is a number that has no integral factor, except itself and 1. Thus,

1, 2, 3, 5, 7, and 11 are prime numbers.

100. A **Composite Number** is a number that has other integral factors besides itself and 1. Thus,

4, 6, 8, 9, 10, and 12 are composite numbers.

Illustrate, by Exercise 1, the process of adding two columns at one operation. What is an Integer? An Exact Divisor? A Factor? A Prime *Number? A Composite* Number?

101. A **Prime Factor** is a factor that is a prime number.

102. A Composite Number is equal to the product of all its prime factors. Thus,

$$8 = 2 \times 2 \times 2, \text{ and } 12 = 2 \times 2 \times 3.$$

103. The number of times a number is taken as a factor, is sometimes denoted by a small figure, called an *exponent*, written at the right of the figures expressing the factor, and above the line. Thus.

$$2^3 = 2 \times 2 \times 2, \text{ and } 18 = 3^2 \times 2.$$

104. Two or more numbers are said to be *prime with respect to each other*, when they have no common integral factor, except 1. Thus,

4 and 9 are prime with respect to each other.

105. Factoring is the process of finding the factors of a composite number.

EXACT DIVISORS.

106. *Two* is an exact divisor of every even number, but of no odd number. Thus,

2 is an exact divisor of 2, 4, 6, etc., but not of 3, 5, 7, etc.

107. *Three* is an exact divisor of any number, when it is an exact divisor of the sum of the units expressed by its figures. Thus,

3 is an exact divisor of 546.

108. *Four* is an exact divisor of any number, when it is an exact divisor of its tens and units. Thus,

4 is an exact divisor of 572, 1928.

109. *Five* is an exact divisor of any number whose unit figure is 5 or 0. Thus,

5 is an exact divisor of 15, 20, 25, 30.

A Prime Factor? To what is a composite number equal? When are two or more numbers said to be prime to each other? What is factoring? Of *what numbers is two an* exact divisor? Three? Four? Five?

110. *Six* is an exact divisor of any even number of which 3 is an exact divisor. Thus,

6 is an exact divisor of 12, 18, 24, 30.

111. *Nine* is an exact divisor of any number, when it is an exact divisor of the sum of the units expressed by its figures. Thus,

9 is an exact divisor of 7542, as 9 is an exact divisor of $7 + 5 + 4 + 2 = 18$.

PRIME NUMBERS.

112. No direct method of detecting prime numbers has been discovered.

After 2, there can be no even number prime, since 2 is an exact divisor of every even number (Art. 106).

In a series of odd numbers beginning with 1, it has been found by trial, that every *third* number after the prime 3 has 3 as a factor, every *fifth* number after the prime 5 has 5 as a factor, and so on.

113. Hence, we have a practical method of detecting prime numbers by sifting out those that are not prime, as follows:

Write the odd numbers from 1 *to any desirable limit.*

Begin with the first prime number after 2, *which is* 3, *and mark every third number after the* 3 *by writing* 3 *over it, every fifth number after the* 5 *by writing* 5 *over it, every seventh number after the* 7 *by writing* 7 *over it, and so on.*

Then, all that remain are prime numbers; and those marked are composite, with factors over them. Thus,

$1,\ 3,\ 5,\ 7,\ \overset{3}{9},\ 11,\ 13,\ \overset{3,5}{15},\ 17,\ 19,\ \overset{3,7}{21},\ 23,\ \overset{5}{25},$

$\overset{3}{27},\ 29,\ 31,\ \overset{3,11}{33},\ \overset{5,7}{35},\ 37,\ \overset{3,13}{39},\ 41,\ 43,\ \overset{3,5}{45},\ 47,\ \overset{7}{49},\ \overset{3,17}{51},$

$53,\ \overset{5,11}{55},\ \overset{3,19}{57},\ 59,\ 61,\ \overset{3,7}{63},\ \overset{5,13}{65},\ 67,\ \overset{3,23}{69},\ 71,\ 73,\ \overset{3,5}{75},$ etc.

Six? Nine? What is said with regard to detecting prime numbers? What is the only even prime number? Give the practical method of detecting prime numbers.

114. Prime numbers to 1009 are included in the following

Table of Prime Numbers.

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	1009

Every prime number, except 2 and 5, has 1, 3, 7, or 9 for its unit figure.

FACTORING OF NUMBERS.

115. To resolve or separate a number into its prime factors.

1. Resolve or separate 84 into its prime factors.

OPERATIONS.

$$\begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \text{or,} \qquad \begin{aligned} 84 &= 2 \times 42 \\ 42 &= 2 \times 21 \\ 21 &= 3 \times 7 \end{aligned}$$

Ans. $84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7.$

What has every prime number, except 2 and 5, for its unit figure?

By trial, we find that 84 is composed of two factors, 2 and 42; of which 2 is prime and 42 is composite.

The 42 we find composed of two factors, 2 and 21; of which 2 is prime and 21 is composite.

The 21 is composed of two factors, 3 and 7, both prime.

Therefore, the prime factors of 84 are 2, 2, 3, and 7, and may be written 2^2, 3, and 7.

Rule. *Divide the given number by any prime number greater than* 1, *that is an exact divisor, and the quotient, if composite, in the same manner; and thus continue until the quotient is prime. The divisors and the last quotient will be the prime factors required.*

Proof. The product of the prime factors will equal the given number, if the work is right (Art. 102).

Since 1 is a factor of every number, it is not commonly specified as such.

Examples.

What are the prime factors

2. Of 252?		7. Of 6409?	
3. Of 1011?	*Ans.* 3, 337.	8. Of 707?	*Ans.* 7, 101.
4. Of 6381?		9. Of 7854?	
5. Of 1000?	*Ans.* $2^3, 5^3$.	10. Of 4350?	*Ans.* $2, 3, 5^2, 29$.
6. Of 11368?	*Ans.* $2^3, 7^2, 29$.	11. Of 32320?	*Ans.* $2^6, 5, 101$.

116. To find the prime factors common to two or more numbers, we may

Take out the common factors, by dividing the given numbers by any prime number greater than 1, *that is an exact divisor of them all, and treat the quotients in the same manner, until quotients shall be obtained prime to each other.*

12. What are the prime factors common to 28 and 56?

What is the Rule? The Proof? How may the prime factors common to two or more numbers be found?

OPERATION.

$$\begin{array}{r|l} 2 & 28,\ 56 \\ \hline 2 & 14,\ 28 \\ \hline 7 & 7,\ 14 \\ \hline & \quad\ \ 2 \end{array}$$

Ans. 2, 2, and 7, or 2^2 and 7.

By trial, we find 2 to be an exact divisor of all the numbers, and we therefore take it as one of the common factors, and have left of 28 the factor 14, and of 56 the factor 28.

We find 2 to be an exact divisor of the factors 14 and 28, and we take it as another common factor of the given numbers, and have left of 28 the factor 7, and of 56 the factor 14.

We find 7 is an exact divisor of the factors 7 and 14, and we take it as a common factor of the given number, and we have left only a factor 2 of 56, and which cannot be common to 28. Therefore, 2, 2, and 7 are the common prime factors of 28 and 56.

What are the prime factors common,

13. To 45 and 75.
14. To 99, 165, and 330. *Ans.* 3 and 11.
15. To 60, 210, and 390. *Ans.* 2, 3, and 5.

MULTIPLICATION BY FACTORS.

117. 1. Multiply 452 by 35, using factors.

OPERATION.

$$\begin{array}{r} 452 \\ 7 \\ \hline 3164 \\ 5 \\ \hline \end{array}$$

Ans. 15820

35 is equal to 7 times 5; hence 35 times 452 is equal to 5 times 7 times 452.

7 times 452 is equal to 3164, and 5 times 7 times 452, or 5 times 3164, is equal to 15820.

Therefore, 452 multiplied by 35 is equal to 15820.

Had 35 contained any other convenient set of factors, they could have been used in like manner.

RULE. *Separate the multiplier into convenient factors. Multiply the multiplicand by one of these factors, and the product by another, and so on, until all the factors have been used. The last product will be the one required.*

Repeat the Rule.

Examples.

Multiply, using factors,

2. 165 by 24. *Ans.* 3960.	5. 876 by 28. *Ans.* 24528.
3. 3405 by 56.	6. 11350 by 81.
4. 4031 by 63. *Ans.* 253953.	7. 1130 by 54. *Ans.* 61020.

8. Multiply 30451 by 70, or by 10×7. *Ans.* 2131570.

Here, we annex a cipher to the multiplicand for the product of it by 10 (Art. 54), and then multiply by the 7; that is, *we annex the cipher to the multiplicand, and multiply by the number expressed by the significant figure.*

9. Multiply 40067 by 50, or by 10×5. *Ans.* 2003350.

DIVISION BY FACTORS.

118. 1. Divide 15820 by 35, using factors.

OPERATION.

$$\begin{array}{r} 5\,)\,15820 \\ \hline 7\,)\,3164 \\ \hline 452 \end{array}$$

Since 35 times a number is equal to 7 times 5 times the number (Art. 117), one thirty-fifth of the number must equal one seventh of one fifth of the number.

One fifth of 15820 is 3164, and one seventh of one fifth of 15820, or one seventh of 3164, is 452.

Therefore, 15820 divided by 35 is equal to 452.

Had the divisor contained any other convenient set of factors than 5 and 7, they could have been used with like result.

2. Divide 6103 by 15, using factors.

OPERATION.

$$\begin{array}{rl} 3\,)\,6103 & \\ \hline 5\,)\,2034, & \text{Rem. 1 unit} = 1 \\ \hline 406, & \text{" 4 threes} = 12 \\ & \overline{} \\ \end{array}$$

Ans. $406\frac{13}{15}$. True Remainder $= 13$

Dividing by the factor 3, we obtain 2034 *threes*, and a remainder of 1 *unit*. Dividing by the factor 5, we obtain 406 (five times threes, or *fifteens*), and a remainder of 4 *threes*. 1 *unit* + 4 *threes*, or 13, equal the entire remainder.

What is 35 times a number equal to?

RULE. *Separate the divisor into convenient factors.*

Divide the dividend by one of these factors, and the quotient by another, and so on, until all the factors have been used. The last quotient will be the one required.

Should there be one or more remainders, multiply each remainder by the divisors, if any, preceding the one that produced it, and the sum of the products plus the remainder left by the first division, if any, will be the true remainder.

Examples.

Divide, using factors,

3. 31344 by 24. *Ans.* 1306.
4. 24528 by 28.
5. 190680 by 56. *Ans.* 3405.
6. 3766 by 25. *Ans.* $150\frac{16}{25}$.
7. 290516 by 63.
8. 26406 by 42. *Ans.* $628\frac{30}{42}$.
9. Divide 134607 by $700 = 100 \times 7$.
10. Divide 376875 by $315 = 5 \times 7 \times 9$. *Ans.* $1196\frac{135}{315}$.

119. Since dividing both the dividend and divisor by the same number will not change the quotient (Art. 73), it is often possible to shorten arithmetical operations *by rejecting equal factors from both dividend and divisor, and using only the remaining factors.*

The process has been called CANCELLATION, from the rejected factors being usually noted by being *crossed* or *canceled.*

11. Divide 3 times 90 by 54.

OPERATION.

$$\frac{90 \times 3}{54} = \frac{\cancel{18} \times 5 \times \cancel{3}}{\underset{1}{\cancel{18} \times \cancel{3}}} = 5, \textit{Ans.}$$

Indicating the division (Art. 58), and factoring, the dividend becomes $18 \times 5 \times 3$, and the divisor 18×3.

Canceling the factors 18 and 3, common to both dividend and divisor, we have $\frac{5}{1}$, or 5, which is the quotient.

Repeat the Rule. How is it often possible to shorten arithmetical operations? What is the process of rejecting factors called?

12. Divide 16 times 21 by 8 times 14.

OPERATION.

$$\frac{\overset{2}{\cancel{16}} \times \overset{3}{\cancel{21}}}{\cancel{8} \times \underset{\underset{1}{\cancel{7}}}{\cancel{14}}} = 3, \textit{ Ans.}$$

Canceling the factor 8, common to both divisor and dividend, we have left in the dividend 2, in place of 16; canceling the factor 2, common to both divisor and dividend, we have left in the divisor 7 in place of 14; and canceling the factor 7, common to the dividend and divisor, we have left $\frac{3}{1}$, or 3, which is the quotient.

When any factor is canceled, 1 is understood to remain, and need be written only when the last of all other factors in the dividend or divisor is canceled.

13. Divide 9 times 40 by 15 times 24. *Ans.* 1.

14. Divide $75 \times 25 \times 7$ by 50×3. *Ans.* $87\frac{1}{2}$.

15. $(36 \times 63 \times 12) \div (54 \times 40 \times 10) =$ how many?

16. $(510 \times 63 \times 4) \div (680 \times 84) =$ how many? *Ans.* $2\frac{1}{4}$.

17. Divide $400 \times 189 \times 33 \times 5$ by $320 \times 126 \times 11 \times 5$. *Ans.* $5\frac{5}{8}$.

APPLICATIONS.

1. How many horses, worth \$132 each, must be given for 1476 sheep, worth \$11 each?

OPERATION.

$$\frac{\cancel{11} \times \cancel{1476}}{\underset{\cancel{12}}{\cancel{132}}} = 123, \textit{ Ans.}$$

If one sheep is worth \$11, 1476 sheep must be worth \$11 × 1476, and as many horses, worth each \$132, must be given for \$11 × 1476, as \$132 are contained times in \$11 × 1476, or 123.

2. How many pounds of butter, at 35 cents a pound, can be bought for 105 yards of muslin, at 21 cents a yard? *Ans.* 63 pounds.

3. At 14 cents a pound, how much sugar can be bought for 2 cords of wood, at \$5.60 a cord?

When a factor is canceled, what is understood to remain? When, only, need the 1 be written?

4. How many loads of hay, of 18 hundreds each, at 75 cents a hundred, will pay for 162 bushels of oats, at 50 cents a bushel? *Ans.* 6 loads.

5. William Marsh sold 360 pounds of beef, at 14 cents a pound, for 3 firkins of butter, each weighing 56 pounds. How much was the butter a pound? *Ans.* 30 cents.

6. How many days must a carpenter work, at $2.50 a day, to pay for the services of a farmer for 40 days at $1.50 a day?

7. Sold 164 dozen school readers, at $9 a dozen, and received in payment quarto dictionaries, at $12 apiece. How many dictionaries did I receive? *Ans.* 123 dictionaries.

8. When $40.50 is paid for 30 barrels of apples, each containing 3 bushels, how much are they a bushel? *Ans.* $.45.

9. How many bales of goods, each bale containing 60 pieces, and each piece 49 yards, worth 75 cents a yard, must be given for 80 government bonds, worth $110.25 each? *Ans.* 4 bales.

GREATEST COMMON DIVISOR.

120. A **Common Divisor** of two or more numbers is any exact divisor (Art. 97) of each of them. Thus,

2, 3, and 6 are common divisors of 6 and 12.

121. The **Greatest Common Divisor** of two or more numbers is the greatest exact divisor of each of them. Thus,

4 is the greatest common divisor of 8 and 12.

But 4 is equal to the product of 2 and 2, the only common prime factors of 8 and 12 (Art. 116). Hence, the principle,

The greatest Common Divisor of two or more numbers is equal to the product of all their common prime factors.

REVIEW QUESTIONS. What is an Exact Divisor? (97) A Factor? (98) A Prime Number? (99) A Composite Number? (100) — What is a Common Divisor of two or more numbers? What is the Greatest Common Divisor of two or more numbers? The Principle?

122. To find the Greatest Common Divisor of two or more Numbers.

1. What is the greatest common divisor of 8, 12, and 20?

OPERATIONS.

$$\begin{array}{r|rrr} 2 & 8, & 12, & 20 \\ \hline 2 & 4, & 6, & 10 \\ \hline & 2, & 3, & 5 \end{array} \quad \text{or,} \quad \begin{aligned} 8 &= 2 \times 2 \times 2 \\ 12 &= 2 \times 2 \times 3 \\ 20 &= 2 \times 2 \times 5 \end{aligned}$$

2×2, or 4, *Ans.*

Taking out the common prime factors of the given numbers (Art. 116), we find them to be 2 and 2; and therefore the greatest common divisor of 8, 12, and 20, is 2×2, or 4.

In the second operation, we resolve the given numbers into their prime factors (Art. 115), and find the common prime factors to be 2 and 2, and take their product, with the same result as before.

RULE. *Find the common prime factors of the numbers, and take their product. Or,*

Resolve the numbers into their prime factors, and take the product of those which are common.

Numbers prime with respect to each other (Art. 104), having no common factor, except 1, are said to have no common divisor.

Examples.

What is the greatest common divisor

2. Of 45, 72, and 81. *Ans.* 9.
3. Of 66 and 165?
4. Of 720 and 960? *Ans.* 240.
5. Of 54 and 258? *Ans.* 6.
6. Of 323 and 425?
7. Of 30, 110, 140, and 680?

123. Another method, and often the most convenient, is based upon the principle, that

The greatest common divisor of two numbers is likewise the greatest common divisor of the smaller and of the remainder after division.

8. Find the greatest common divisor of 247 and 323.

Repeat the Rule. When are two or more numbers said to have no *common divisor?* *What is the* principle of another method?

OPERATIONS.

$$
\begin{array}{l}
247)\,323\,(1 \\
\quad\;\; 247 \\
\quad\;\; \overline{\;\;76)\,247\,(3} \\
\qquad\quad\;\; 228 \\
\qquad\quad\;\; \overline{\;\;19)\,76\,(4} \\
\qquad\qquad\quad\; 76 \\
\qquad\qquad\quad\; \overline{\;\;\;\;}
\end{array}
\qquad \text{or,} \qquad
\begin{array}{r}
323 \\
247 \times 1 = 247 \\
228 = 3 \times \;\; 76 \\
19 \times 4 = \;\; 76
\end{array}
$$

Ans. 19.

Since 247 is the greatest divisor of itself, if it is an exact divisor of 323, it will be the greatest common divisor of 247 and 323. It is not an exact divisor of 323, for there is a remainder, 76.

If 76, which is the greatest divisor of itself, is an exact divisor of 247, it must be the greatest common divisor of 76 and 247, and likewise of 247 and 323. It is not an exact divisor of 247, for there is a remainder, 19.

If 19, which is the greatest divisor of itself, is an exact divisor of 76, it is the greatest common divisor of 19 and 76, likewise of 76 and 247, and of 247 and 323. It is found to be an exact divisor of 76, and, therefore, 19 is the greatest common divisor of 247 and 323.

The two operations are exactly the same, except that the second is written in a more compact form than the first.

124. Hence, to find the greatest common divisor of two numbers, we may

Divide the greater number by the less, and the divisor by the remainder, and so on, until there is no remainder; the last divisor will be the greatest common divisor.

9. Find the greatest common divisor of 308 and 506. *Ans.* 22.

10. Find the greatest common divisor of 3252 and 4248.

11. What is the greatest common divisor of 2145 and 3471? *Ans.* 39.

Explain the operations. How may we find the greatest common divisor of two numbers?

125. When there are more than two of the given numbers, *find the greatest common divisor of any two of them, and then of this divisor and a third number, and so on, until all the numbers have been taken.*

12. Find the greatest common divisor of 492, 744, and 1044. *Ans.* 12.

13. What is the greatest common divisor of 1326, 3094, and 4420? *Ans.* 442.

APPLICATIONS.

1. There is a certain field 788 rods long and 356 rods wide; what is the length of the longest chain that will exactly measure both its length and breadth? *Ans.* 4 rods.

2. What must be the width of carpeting to fit three rooms, the first being 15 feet, the second 18 feet, and the third 21 feet wide? 3

3. I have 3 tracts of land, the first containing 375, the second 450, and the third 525 acres; if I should divide it into the largest possible farms, having the same number of acres in each, how many acres would there be in each farm? *Ans.* 75 acres.

4. A farmer has 1008 bushels of summer wheat, 1152 bushels of winter wheat, and 720 bushels of barley. Required the capacity of the largest bins of equal size that will exactly contain the whole, without mixing. *Ans.* 144 bushels.

5. A has $679, B $5901, and C $6734; they agree to lay it out for sheep, at the highest price per head that will allow each to exactly invest his money; how much can they pay a head, and how many can each purchase?

Ans. $7 a head; A 97 sheep, B 843 sheep, and C 962 sheep.

How may we find the greatest common divisor when there are more than two given numbers?

LEAST COMMON MULTIPLE.

126. A **Multiple** of a number is any number which has that number as an exact divisor. Thus,

8 and 12 are multiples of 4.

127. A **Common Multiple** of two or more numbers is any number which has each of those numbers as an exact divisor. Thus,

24 and 48 are common multiples of 4, 8, and 12.

The **Least Common Multiple** of two or more numbers is the *least* number which has each of those numbers as an exact divisor. Thus,

24 is the least common multiple of 4, 8, and 12.

But 24 contains only the prime factors, 2, 2, 2, and 3, which alone are required to produce 4, 8, and 12. Hence, the principles,

1. *The least common multiple of two or more numbers is a number containing all the prime factors of each number, and no others.*

2. *The least common multiple of two or more numbers which are prime to each other, is equal to the product of the numbers.*

128. To find the Least Common Multiple of two or more numbers.

1. What is the least common multiple of 6, 14, and 49?

OPERATION.

$6 = 2 \times 3$
$14 = 2 \times 7$
$49 = 7 \times 7$
$7 \times 7 \times 2 \times 3 = 294$, *Ans.*

Since a multiple of 49 must contain 49, it must contain all its prime factors; we take the factors 7×7.

These are all the prime factors of 49 and 14 except 2, which we take, and have $7 \times 7 \times 2$.

These are all the prime factors of 49, 14, and 6, except 3, which we take, and have $7 \times 7 \times 2 \times 3$.

These are all the prime factors of each of the given numbers, and no others; therefore, their product, or 294, is the least common multiple required.

What is a Multiple of a number? A Common Multiple of two or more numbers? The Least Common Multiple? The first principle? The second? What must a multiple of 49 contain?

RULE. *Resolve the numbers into their prime factors, and take the product of all the different prime factors, using each the greatest number of times it occurs in any one of the given numbers.*

Examples.

Find the least common multiple

2. Of 30 and 55. *Ans.* 330.
3. Of 9, 12, and 42.
4. Of 3, 5, 7, and 21. *Ans.* 105.
5. Of 25, 60, 84, and 15. *Ans.* 2100.
6. Of 36, 56, 75, and 72.
7. Of 81, 27, 45, and 18. *Ans.* [illegible]
8. What is the least common multiple of 5, 19, and 21? (Art. 127, prin. 2.) *Ans.* 1995.

129. To find the least common multiple of two or more numbers, we may, often most conveniently,

Divide by any prime number greater than 1, *that is an exact divisor of two or more of the given numbers.*

Divide the quotients and undivided numbers, if any, in like manner, and so continue, until there is no exact divisor, greater than 1, *of any two of them. Take the product of the divisors and the final quotients, for the least common multiple of the numbers required.*

9. What is the least common multiple of 18, 20, and 30?

OPERATION.

$$\begin{array}{r|l} 2 & 18,\ 20,\ 30 \\ \hline 3 & 9,\ 10,\ 15 \\ \hline 5 & 3,\ 10,\ \ 5 \\ \hline & 3,\ \ 2. \end{array}$$

$2 \times 3 \times 5 \times 3 \times 2 = 180$, *Ans.*

Taking out the prime factor 2, common to the given numbers, we have left of 18 the factor 9, of 20 the factor 10, and of 30 the factor 15.

Taking out the factor 3, common to 9 and 15, we have left of 18 the factor 3, of 20 the factor 10, and of 30 the factor 5.

Taking out the factor 5, common to 10 and 5, we have left of 18 the factor 3, of 10 the factor 2, and of 30 no factor, and these factors are prime.

What is the Rule? How may the least common multiple of two or more numbers be often most conveniently found?

Therefore, 2, 3, 5, 3, and 2, are all the prime factors of the given numbers; hence their product, or 180, is the least common multiple required.

10. What is the least common multiple of 15, 35, 16, and 56? *Ans.* 1680.

11. Find the least common multiple of 39, 26, 65, and 15?

12. Find the least common multiple of 12, 36, 60, and 72?

OPERATION.

$$\begin{array}{r|l} 12 & \cancel{12},\ \cancel{36},\ 60,\ 72 \\ \hline & \qquad\quad 5,\ \ 6 \end{array}$$

$12 \times 5 \times 6 = 360$, *Ans.*

Here, since a multiple of 72 must be a multiple of its exact divisors, 12 and 36, we cancel those numbers.

Again, since 12, although not prime, is an exact divisor of *all* the remaining numbers, all its factors must be factors of the multiple, and taking it out, we have left of 60 the factor 5, and of 72 the factor 6, which factors are prime to each other; therefore, $12 \times 5 \times 6 = 360$, is the multiple required.

13. Find the least common multiple of 35, 105, 210, and 750?

14. What is the least common multiple of 54, 378, 486, and 540? *Ans.* 34020.

APPLICATIONS.

1. What is the least sum of money for which I could purchase a number of sheep at \$3, \$4, \$5, or \$6 each, and just expend the whole? *Ans.* \$60.

2. Four men start at the same time and place to walk around a race-course, in the same direction. A can go around in 10 minutes, B in 12 minutes, C in 8 minutes, and D in 18 minutes; in what time will they all be again together at the point of starting? 360

3. What is the smallest sum of money for which John Fuller can hire a number of men for one month, at either \$12, \$18, \$30, or \$36 each, and what will be the number of men that can be employed at each rate?

Ans. \$180; 15 men at \$12, 10 men at \$18, 6 men at \$30, or 5 men at \$36.

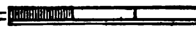
Why, in the operation, can we cancel the 12 and 36? Why can we take out 12 *from the remaining* numbers as a factor of the required multiple?

COMMON FRACTIONS.

130. If a unit, as one inch, be divided into two equal parts, one of these parts is *one half.* Thus,

$\frac{1}{2}$ of 1 inch = 

If a unit be divided into three equal parts, one of these parts is *one third;* two of them *two thirds,* etc. Thus,

$\frac{1}{3}$ of 1 inch = , $\frac{2}{3}$ of 1 inch = , etc.

If a unit be divided into four equal parts, one of these parts is *one fourth;* two of them *two fourths,* etc. Thus,

$\frac{1}{4}$ of 1 inch = , $\frac{3}{4}$ of 1 inch = ; etc.

In like manner, if a unit be divided into five equal parts, the parts are *fifths;* if into six equal parts, *sixths;* and so on. Such parts of a unit are called *fractions.* Hence,

131. A **Fraction** is a part of a unit, consisting of one or more of the equal parts, which compose the unit.

The UNIT of a fraction is the unit or thing divided.

A FRACTIONAL UNIT is one of the equal parts into which the unit of a fraction is divided.

132. The DENOMINATOR of a fraction is the number which shows into how many equal parts the unit is divided. Thus,

Three is the denominator of two thirds.

133. The NUMERATOR of a fraction is the number which shows how many of the equal parts of the unit are taken. Thus,

Two is the numerator of two thirds.

NOTATION AND NUMERATION.

134. A **Common Fraction** is a fraction expressed by writing the numerator above, and the denominator below, a dividing line; as,

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What is a Fraction? The Unit of a fraction? A Fractional Unit? The *Denominator of* a Fraction? The Numerator? How is a Common Fraction expressed?

One half,	written	$\frac{1}{2}$	Two fifths,	written	$\frac{2}{5}$
One third,	"	$\frac{1}{3}$	Three fifths,	"	$\frac{3}{5}$
Two thirds,	"	$\frac{2}{3}$	Four fifths,	"	$\frac{4}{5}$
One fourth,	"	$\frac{1}{4}$	One seventh,	"	$\frac{1}{7}$
Two fourths,	"	$\frac{2}{4}$	Three eighths,	"	$\frac{3}{8}$
Three fourths,	"	$\frac{3}{4}$	Five ninths,	"	$\frac{5}{9}$
One fifth,	"	$\frac{1}{5}$	Seven tenths,	"	$\frac{7}{10}$

135. A PROPER FRACTION is one whose numerator is less than the denominator; as $\frac{3}{7}$, $\frac{7}{8}$.

136. An IMPROPER FRACTION is one whose numerator is not less than the denominator; as $\frac{5}{5}$, $\frac{9}{8}$.

137. A MIXED NUMBER is a whole number with a fraction; as $3\frac{1}{4}$; read three and one fourth.

138. The TERMS of a fraction are the numerator and denominator.

The numerator expresses the *number* of fractional units in the fraction, and the denominator their *name* or *denomination*.

139. A whole number may be expressed in a fractional form, by writing 1 under it, for a denominator. Thus,

2	may be written	$\frac{2}{1}$,	and read,	2 ones.
3	"	$\frac{3}{1}$,	"	3 ones.
7	"	$\frac{7}{1}$,	"	7 ones.
10	"	$\frac{10}{1}$,	"	10 ones.

140. A fractional expresssion may be explained by showing what is denoted by its terms.

Exercises.

1. Read and explain $\frac{3}{5}$.

SOLUTION. *Three fifths; the denominator* 5 *denoting that the name or denomination of fractional units expressed is fifths, and the numerator* 3 *that three of those units are taken.*

What is a Proper Fraction? An Improper Fraction? A Mixed Number? What are the Terms of a fraction? What do they express? How may a whole number be expressed in a fractional form? How may a fractional expression be explained?

Read and explain,

2. $\frac{7}{8}$.	5. $\frac{10}{11}$.	8. $\frac{25}{35}$.	11. $\frac{19}{108}$.
3. $\frac{9}{10}$.	6. $\frac{2}{7}$.	9. $\frac{8}{195}$.	12. $\frac{166}{379}$.
4. $\frac{13}{15}$.	7. $\frac{17}{20}$.	10. $\frac{11}{173}$.	13. $\frac{634}{10000}$.

14. Express by figures seven ninths.

Solution. *Since the name or kind of fractional units to be expressed is ninths, we write 9 as the denominator, and since the number of these units to be taken is seven, we write 7 as the numerator, and have as the required expression $\frac{7}{9}$.*

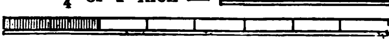
Express by figures,

15. 6 ninths. *Ans.* $\frac{6}{9}$.
16. 17 twenty-fifths.
17. Two fiftieths.
18. Eleven twenty-seconds.
19. 28 thirty-firsts.
20. 19 sixty-seconds.
21. One one hundred fifteenths.
22. 98 three hundredths.
23. 51 six hundred fortieths.
24. Sixteen thousandths.
25. 167 nine hundred twenty-ninths.
26. Seventy-two thousand twenty-firsts.

141. A fraction may be regarded as indicated division (Art. 58), the numerator answering to the dividend, and the denominator to the divisor. Hence, the

GENERAL PRINCIPLES.

1. *The value expressed by a fraction is the quotient of the numerator divided by the denominator.* Thus,

$\frac{2}{4}$ of 1 inch =  = $\frac{1}{4}$ of 2 inches = . By Art. 71–3, it follows, that

2. *Multiplying the numerator or dividing the denominator multiplies the fraction.*

3. *Dividing the numerator or multiplying the denominator divides the fraction.*

How may a fraction be regarded? What is the value expressed by a fraction? How is the value of a fraction affected by multiplying its numerator or by dividing its denominator? What effect has dividing the numerator or multiplying the denominator of a fraction?

4. *Multiplying or dividing both terms of a fraction by the same number does not change its value.*

Mental Exercises.

1. If a unit be divided into 4 equal parts, what will be the name or denomination of the parts?

SOLUTION. *If a unit be divided into 4 equal parts, each of the equal parts will be $\frac{1}{4}$. Therefore, the name or denomination of the parts will be fourths.*

2. If a unit be divided into 5 equal parts, what will be the name or denomination of the parts? If into 9 equal parts? 13? 17? 62? 92? 150?

3. If a unit be divided into 63 equal parts, what will 1 of the equal parts be called? 3 of the equal parts? 9? 19? 25? 41? 62?

4. If a unit be divided into 19 equal parts, what is 1 of the fractional units called? 7 of the fractional units? 11? 16? 17?

5. How much is $\frac{1}{2}$ of 12? $\frac{1}{3}$ of 12? $\frac{1}{4}$? $\frac{1}{6}$? $\frac{1}{12}$?

6. How much is $\frac{1}{4}$ of 8? Of 12? Of 16? Of 20?

7. How much is $\frac{2}{3}$ of 18?

SOLUTION. *Since* 1 *third of* 18 *is* 6, 2 *thirds of* 18 *are* 2 *times* 6, *or* 12. *Therefore,* $\frac{2}{3}$ *of* 18 *is* 12.

8. How much is $\frac{3}{4}$ of 24? Of 36? Of 40? Of 48?

9. How much is $\frac{5}{7}$ of 21? Of 28? Of 35? Of 49?

10. How much is $\frac{7}{9}$ of 18? Of 36? Of 45? Of 72?

11. Which is the greater, $\frac{1}{2}$ of 12, or $\frac{1}{3}$ of 12?

SOLUTION. *Since* $\frac{1}{2}$ *of* 12 *is* 6 *and* $\frac{1}{3}$ *of* 12 *is* 4, $\frac{1}{2}$ *of* 12 *is greater than* $\frac{1}{3}$ *of* 12 *by the difference between* 6 *and* 4, *or* 2. *Therefore,* $\frac{1}{2}$ *of* 12 *is greater than* $\frac{1}{3}$ *of* 12 *by* 2.

12. Which is the greater, $\frac{1}{3}$ of 30, or $\frac{1}{5}$ of 30? $\frac{1}{2}$ of 24, or $\frac{1}{8}$ of 24?

What effect has multiplying or dividing both terms of a fraction? Which is the *greater, a fourth* or a half of any thing? A third or a tenth?

13. Which is the greater, $\frac{1}{3}$ of 72 or $\frac{1}{9}$ of 72? $\frac{1}{2}$ of 100 or $\frac{1}{10}$ of 100?

14. Which is the greater, $\frac{2}{3}$ of 15 or $\frac{2}{5}$ of 15? $\frac{3}{7}$ of 42 or $\frac{3}{6}$ of 42?

15. How many fourths in a half?

SOLUTION. *Since there are 4 fourths in 1 unit, in 1 half of 1 unit there is one half of 4 fourths, or 2 fourths. Therefore, in 1 half there are 2 fourths.*

16. How many sixths in a third? How many twelfths?

17. How many eighths in a fourth? How many sixteenths?

18. How many tenths in a fifth? How many twentieths?

19. How many thirds in $\frac{4}{6}$?

SOLUTION. *Since in 1 unit there are 6 sixths, in 1 third of 1 unit there is 1 third of 6 sixths, or 2 sixths, and in 4 sixths as many thirds as 2 sixths are contained times in 4 sixths, which are 2. Therefore, in $\frac{4}{6}$ there are 2 thirds.*

20. How many halves in $\frac{2}{4}$? In $\frac{3}{6}$? In $\frac{4}{8}$?

21. How many fourths in $\frac{6}{8}$? Fifths in $\frac{4}{10}$? Sixths in $\frac{8}{12}$?

22. How many sevenths in $\frac{10}{14}$? Tenths in $\frac{6}{20}$?

23. What part of 7 is 2?

SOLUTION. *Since 1 is 1 seventh of 7, 2 is 2 times 1 seventh of 7, or 2 sevenths of 7. Therefore, 2 is $\frac{2}{7}$ of 7.*

24. What part of 9 is 5? 7? 2? 8?

25. What part of 11 is 4? 6? 9? 10?

26. What part of 17 is 7? 11? 13? 16?

27. What part of 19 is 8? 10? 14? 16?

28. 8 is 1 fifth of what number?

SOLUTION. *If 1 fifth of some number is 8, 5 fifths, or the number itself, is 5 times 8, or 40.*

29. 7 is $\frac{1}{6}$ of what number? $\frac{1}{7}$ of what number?

30. 10 is $\frac{1}{3}$ of what number? $\frac{1}{4}$ of what number?

REVIEW QUESTIONS. What is a Fraction? (131) How is a Common Fraction expressed? (134) What are the Terms of a fraction? (138) How may a fraction be explained? (140)

31. 12 is $\frac{1}{4}$ of what number? $\frac{1}{8}$ of what number?
32. 11 is $\frac{1}{2}$ of what number? $\frac{1}{9}$ of what number?
33. 8 is $\frac{2}{3}$ of what number?

SOLUTION. *If $\frac{2}{3}$ of some number is 8, 1 third of that number is $\frac{1}{2}$ of 8, or 4; and 3 thirds of that number are 3 times 4, or 12.*

34. 16 is $\frac{4}{5}$ of what number? $\frac{2}{3}$ of what number?
35. 18 is $\frac{2}{3}$ of what number? $\frac{3}{8}$ of what number?
36. 21 is $\frac{3}{8}$ of what number? $\frac{7}{8}$ of what number?
37. 30 is $\frac{5}{11}$ of what number? $\frac{6}{7}$ of what number?
38. 40 is $\frac{5}{8}$ of what number? $\frac{5}{12}$ of what number?

REDUCTION.

142. Reduction of Fractions is the process of changing their name or denomination, without changing the value expressed.

CASE I.

143. To reduce a fraction to its lowest or smallest terms.

A fraction is expressed in its *lowest*, or *smallest* terms, when the numerator and denominator are prime to each other, or have no common divisor.

1. Reduce $\frac{10}{12}$ to its smallest terms.

Since in 1 unit there are 12 *twelfths*, in 1 *sixth* of 1 unit there is $\frac{1}{6}$ of 12 *twelfths*, or 2 *twelfths*, and in 10 *twelfths* there are as many *sixths* as 2 *twelfths* are contained times in 10 *twelfths*, which are 5. Now, in $\frac{5}{6}$ the terms are prime to each other. Therefore, $\frac{10}{12}$ in its smallest terms is $\frac{5}{6}$.

2. Reduce $\frac{15}{20}$ to its smallest terms.
3. Reduce $\frac{18}{30}$ to its smallest terms.

OPERATIONS.

$$\frac{18}{30}=\frac{\cancel{2}\times\cancel{3}\times 3}{\cancel{2}\times\cancel{3}\times 5}=\frac{3}{5}, \text{ or } \frac{18}{30}=\frac{18\div 6}{30\div 6}=\frac{3}{5}.$$

Since dividing both terms of a fraction by the same number does not change the value expressed (Art. 141), we divide both terms by

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What is *Reduction of Fractions*? When is a fraction expressed in its *lowest or smallest terms.*

the prime factors common to them, by canceling, and obtain $\frac{3}{5}$, whose terms have no common divisor. Therefore, $\frac{18}{30}$ in its smallest terms is $\frac{3}{5}$.

In the second operation, we divide both terms of $\frac{18}{30}$ by their greatest common divisor, with the same result, $\frac{3}{5}$.

RULE. *Cancel in the numerator and denominator all the factors common to both.* Or,

Divide both terms of the fraction by their greatest common divisor.

Examples.

Reduce to their smallest terms:

4. $\frac{48}{56}$.	*Ans.* $\frac{6}{7}$.	8. $\frac{27}{45}$.	*Ans.* $\frac{3}{5}$.	12. $\frac{176}{192}$.	*Ans.* $\frac{11}{12}$.
5. $\frac{126}{210}$.		9. $\frac{315}{378}$.	*Ans.* $\frac{5}{6}$.	13. $\frac{270}{375}$.	
6. $\frac{84}{170}$.	*Ans.* $\frac{42}{85}$.	10. $\frac{108}{144}$.		14. $\frac{182}{196}$.	*Ans.* $\frac{13}{14}$.
7. $\frac{1296}{3024}$.	*Ans.* $\frac{3}{7}$.	11. $\frac{1296}{2268}$.	*Ans.* $\frac{4}{7}$.	15. $\frac{9876}{88884}$.	*Ans.* $\frac{1}{9}$.

16. Express in its simplest form 114 divided by 285. *Ans.* $\frac{2}{5}$.

17. Express in its simplest form 1980 divided by 3168. *Ans.* $\frac{5}{8}$.

CASE II.

144. To reduce an improper fraction to an equivalent whole or mixed number.

1. Reduce $\frac{19}{5}$ to an equivalent whole or mixed number.

Since in 5 *fifths* there is 1 unit, in 19 *fifths* there are as many units as 5 *fifths* are contained times in 19 *fifths*, which are $3\frac{4}{5}$. Therefore, $\frac{19}{5}$ is equal to $3\frac{4}{5}$.

2. Reduce $\frac{25}{6}$ to an equivalent whole or mixed number.

3. Reduce $\frac{41}{8}$ to an equivalent whole or mixed number.

OPERATION.

$$\frac{41}{8} = 41 \div 8 = 5\tfrac{1}{8}, \textit{ Ans.}$$

RULE. *Divide the numerator by the denominator.*

What is the Rule for reducing fractions to their lowest or smallest terms? What is the Rule for reducing an improper fraction to a whole or mixed number?

Examples.

Reduce to an equivalent whole or mixed number,

4. $\frac{125}{9}$.	*Ans.* $13\frac{8}{9}$.	9. $\frac{103}{4}$.	*Ans.* $25\frac{3}{4}$.
5. $\frac{132}{12}$.		10. $\frac{2880}{23}$.	
6. $\frac{141}{17}$.	*Ans.* $8\frac{5}{17}$.	11. $\frac{1210}{55}$.	*Ans.* 22.
7. $\frac{119}{7}$.	*Ans.* 17.	12. $\frac{3127}{43}$.	*Ans.* $72\frac{31}{43}$.
8. $\frac{3848}{21}$.	*Ans.* $183\frac{5}{21}$.	13. $\frac{4641}{221}$.	*Ans.* 21.

CASE III.

145. To reduce a mixed number to an equivalent improper fraction.

1. Reduce $3\frac{1}{4}$ to an equivalent improper fraction.

Since in 1 there are four *fourths*, in 3 there are 3 times 4 *fourths*, or 12 *fourths*, and 12 *fourths* and 1 *fourth* added are 13 *fourths*, or $\frac{13}{4}$. Therefore, $3\frac{1}{4}$ is equal to $\frac{13}{4}$.

2. Reduce $4\frac{3}{8}$ to an equivalent improper fraction.
3. Reduce $5\frac{2}{3}$ to an equivalent improper fraction.

OPERATION.

$$5\tfrac{2}{3}=\frac{5\times 3}{3}+\frac{2}{3}=\frac{15+2}{3}=\frac{17}{3},\ \textit{Ans.}$$

Rule. *Multiply the whole number by the denominator of the fraction, to the product add the numerator, and write the sum over the denominator.*

Examples.

Reduce to equivalent improper fractions,

4. $15\frac{3}{4}$.	*Ans.* $\frac{63}{4}$	8. $13\frac{15}{17}$.	*Ans.* $\frac{236}{17}$.
5. $12\frac{11}{12}$.	*Ans.* $\frac{155}{12}$.	9. $37\frac{15}{37}$.	*Ans.* $\frac{1384}{37}$.
6. $13\frac{8}{9}$.		10. $72\frac{31}{43}$.	
7. $200\frac{27}{50}$.	*Ans.* $\frac{10027}{50}$.	11. $128\frac{11}{18}$.	*Ans.* $\frac{2315}{18}$.

12. In $115\frac{13}{15}$ how many fifteenths? *Ans.* $\frac{1738}{15}$.
13. In $719\frac{11}{12}$ how many twelfths? *Ans.* $\frac{8639}{12}$.

What is the Rule for reducing mixed numbers to an equivalent improper fraction?

146. To reduce a whole number to an equivalent improper fraction having a given denominator.

Multiply the whole number by the given denominator, and take the product for the numerator of the fraction.

14. Reduce 17 to fifths. *Ans.* $\frac{85}{5}$.

15. Reduce 110 to an equivalent fraction whose denominator is 9.

16. Reduce 306 to an equivalent fraction whose denominator is 13. *Ans.* $\frac{3978}{13}$.

COMMON DENOMINATOR.

147. Fractions have a COMMON DENOMINATOR when they have the same number for a denominator.

148. A common denominator of several fractions is a *common multiple* of their denominators. Hence,

The LEAST COMMON DENOMINATOR *of several fractions is the least common multiple of their denominators.*

149. To reduce fractions to equivalent fractions having a common denominator.

1. Reduce $\frac{2}{3}$ to fifteenths.

Since there are 15 *fifteenths* in 1 unit, in $\frac{1}{3}$ of 1 unit there is $\frac{1}{3}$ of 15 *fifteenths*, or 5 *fifteenths*, and in $\frac{2}{3}$ of 1 unit there are 2 times 5 *fifteenths*, or 10 *fifteenths*. Therefore, $\frac{2}{3}$ is equal to $\frac{10}{15}$.

2. Reduce $\frac{3}{4}$ to twelfths. To sixteenths. To twentieths.

3. Reduce $\frac{5}{8}$ to twenty-fourths.

OPERATION.

$$\frac{5}{8}=\frac{5\times 3}{8\times 3}=\frac{15}{24}, \textit{ Ans.}$$

Since the proposed denominator, 24, is 3 times 8, the denominator of the given fraction, we multiply both terms of the fraction by 3, which will not change the value expressed (Art. 141), and have $\frac{15}{24}$. Therefore, $\frac{5}{8}$ is equal to $\frac{15}{24}$.

How do you reduce a whole number to an equivalent fraction? When have fractions a Common Denominator? What is a common denominator of several fractions? The Least Common Denominator? Which is expressed in larger terms, $\frac{2}{3}$ or $\frac{10}{15}$? *Ans.* $\frac{10}{15}$.

4. Reduce $\frac{3}{8}$ and $\frac{7}{16}$ to equivalent fractions having a common denominator.

OPERATION.

$$\left.\begin{array}{l} \frac{3}{8} = \frac{3 \times 2}{8 \times 2} = \frac{6}{16} \\ \qquad\qquad\quad \frac{7}{16} \end{array}\right\} Ans.$$

Since the denominator, 16, of the second fraction, is 2 times the denominator of the first fraction, we multiply both terms of the $\frac{3}{8}$ by 2, causing its denominator to become 16, and have $\frac{6}{16}$ and $\frac{7}{16}$. Therefore, $\frac{3}{8}$ and $\frac{7}{16}$ are equal to $\frac{6}{16}$ and $\frac{7}{16}$.

5. Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{1}{6}$ to equivalent fractions having the least common denominator.

OPERATION.

$$\begin{array}{ll} 2 = 2 & \frac{1}{2} = \frac{1 \times 6}{2 \times 6} = \frac{6}{12} \\ 4 = 2 \times 2 & \frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \\ 6 = 2 \times 3 & \frac{1}{6} = \frac{1 \times 2}{6 \times 2} = \frac{2}{12} \end{array} \left.\vphantom{\begin{array}{l}1\\1\\1\end{array}}\right\} Ans.$$

$$\text{Least com. mul.} = 2 \times 2 \times 3 = 12.$$

We find the least common multiple of the denominators to be 12, which must be the least common denominator (Art. 148). We then reduce the given fractions to equivalent fractions having this denominator, by multiplying both terms of each fraction by such a number as will make its denominator become 12, and have $\frac{6}{12}$, $\frac{9}{12}$, and $\frac{2}{12}$. Therefore, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{1}{6}$ are equal, respectively, to $\frac{6}{12}$, $\frac{9}{12}$, and $\frac{2}{12}$.

RULE. *Multiply both terms of each fraction by the denominators of the other fractions. Or,*

Find the least common multiple of the denominators for the least common denominator, and multiply the terms of each fraction by such a number as will reduce it to an equivalent fraction with that denominator.

All the fractions should be reduced to their smallest terms before finding the least common multiple of their denominators.

When the number by which to multiply the terms of any fraction is not apparent, it may be determined by dividing the common denominator by the denominator of the fraction under consideration.

What is the Rule? When the multiplier of the terms of any fraction is not apparent, how may it be determined?

Examples.

Reduce to equivalent fractions having a common denominator,

6. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{7}{12}$. *Ans.* $\frac{6}{12}$, $\frac{8}{12}$, $\frac{10}{12}$, $\frac{7}{12}$.
7. $\frac{4}{5}$, $\frac{3}{10}$, and $\frac{7}{20}$.
8. $\frac{3}{6}$, $\frac{11}{12}$, and $\frac{1}{18}$. *Ans.* $\frac{18}{36}$, $\frac{33}{36}$, $\frac{2}{36}$.

Reduce to equivalent fractions having the least common denominator,

9. $\frac{4}{9}$, $\frac{3}{5}$, and $\frac{8}{15}$. *Ans.* $\frac{20}{45}$, $\frac{27}{45}$, $\frac{24}{45}$.
10. $\frac{5}{6}$, $\frac{5}{8}$, $\frac{2}{9}$, and $\frac{13}{24}$. *Ans.* $\frac{60}{72}$, $\frac{45}{72}$, $\frac{16}{72}$, $\frac{39}{72}$.
11. $\frac{2}{3}$, $\frac{7}{8}$, $\frac{11}{12}$, and $\frac{19}{20}$. 80 105 110 114
12. $\frac{3}{15}$, $\frac{2}{4}$, $\frac{7}{10}$, and $\frac{13}{25}$. *Ans.* $\frac{60}{300}$, $\frac{150}{300}$, $\frac{210}{300}$, $\frac{156}{300}$.

150. If there are whole or mixed numbers with the given fractions, they must be reduced to improper fractions, before applying the rule.

13. Reduce $\frac{1}{2}$, $3\frac{1}{4}$, 6, and $\frac{5}{8}$ to equivalent fractions having the least common denominator. *Ans.* $\frac{4}{8}$, $\frac{26}{8}$, $\frac{48}{8}$, $\frac{5}{8}$.

14. Reduce $\frac{3}{7}$, $\frac{5}{21}$, $3\frac{1}{7}$, and $\frac{9}{14}$ to equivalent fractions having the least common denominator.

15. Reduce 1, $4\frac{1}{9}$, $8\frac{1}{7}$, and $12\frac{1}{2}$ to equivalent fractions having a common denominator. *Ans.* $\frac{126}{126}$, $\frac{518}{126}$, $\frac{1026}{126}$, $\frac{1575}{126}$.

ADDITION.

151. Addition of Fractions is the process of finding a number equal to two or more fractions.

152. Numbers to be added must be of like name or kind. (Art. 39. 1.) Hence,

Fractions can be added only when they express like fractional units, or have a common denominator.

1. What is the sum of $\frac{2}{7}$, $\frac{3}{7}$, and $\frac{5}{7}$?

What must be done when there are whole or mixed numbers with the given fractions? What is Addition of Fractions? When, only, can fractions be added?

Since the fractional units are of like name and kind, the sum of 2 *sevenths*, 3 *sevenths*, and 5 *sevenths*, is $2 + 3 + 5$, *sevenths*, or $\frac{10}{7}$, which reduced is $1\frac{3}{7}$.

2. What is the sum of $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$?
3. What is the sum of $\frac{6}{20}$, $\frac{11}{20}$, and $\frac{9}{20}$?
4. What is the sum of $\frac{3}{25}$, $\frac{7}{25}$, and $\frac{10}{25}$?
5. Let it be required to find the sum of $\frac{3}{4}$ and $\frac{4}{5}$?

OPERATION.

$$\frac{3}{4} + \frac{4}{5} = \frac{15}{20} + \frac{16}{20} = \frac{15+16}{20} = \frac{31}{20} = 1\tfrac{11}{20}, \textit{ Ans.}$$

Reducing the given fractions to equivalent fractions having a common denominator, so that they may express like fractional units (Art. 152), we have 15 *twentieths* and 16 *twentieths*, which added give 31 *twentieths*, $\frac{31}{20}$, or, by reduction, $1\frac{11}{20}$. Therefore, the sum of $\frac{3}{4}$ and $\frac{4}{5}$ is $1\frac{11}{20}$.

RULE. *Reduce the fractions, if necessary, to equivalent fractions having a common denominator, and write the sum of the numerators over the common denominator.*

Examples.

Find the sum of

6. $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{10}$. *Ans.* $1\frac{19}{20}$.
7. $\frac{2}{7}$, $\frac{6}{21}$, and $\frac{31}{42}$.
8. $\frac{7}{10}$, $\frac{11}{12}$, and $\frac{4}{9}$. *Ans.* $2\frac{11}{180}$.
9. $\frac{8}{11}$, $\frac{7}{9}$, $\frac{4}{5}$, and $\frac{3}{4}$. *Ans.* $3\frac{109}{1980}$.
10. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{9}$. *Ans.* $2\frac{31}{36}$.
11. $\frac{1}{19}$, $\frac{7}{19}$, $\frac{11}{19}$, and $\frac{21}{19}$.
12. $\frac{7}{8}$, $\frac{1}{12}$, $\frac{17}{18}$, $\frac{23}{24}$, and $\frac{20}{27}$. *Ans.* $3\frac{65}{108}$.
13. $\frac{1}{1}$, $\frac{23}{27}$, $\frac{2}{5}$, $\frac{7}{14}$, and $\frac{11}{15}$. *Ans.* $3\frac{131}{270}$.
14. What is the value of $\frac{31}{5} + \frac{7}{9} + \frac{15}{16} + \frac{61}{24}$? *Ans.* $10\frac{323}{720}$.
15. What is the value of $\frac{10}{11} + \frac{16}{7} + \frac{5}{14} + \frac{33}{22}$? *Ans.* $5\frac{4}{77}$.

153. When there are mixed numbers, we may

Add the fractional and integral parts separately, and then add their sums.

16. What is the sum of $7\frac{1}{2}$, $13\frac{1}{4}$, and $5\frac{3}{8}$?

Why, in the operation of example 5, are the fractions reduced to equivalent fractions having a common denominator? What is the Rule? How may we add when there are mixed numbers?

OPERATION.

$$\begin{array}{l} \frac{1}{2} + \frac{1}{4} + \frac{3}{8} = \frac{4}{8} + \frac{2}{8} + \frac{3}{8} = \frac{9}{8} = \ \ 1\frac{1}{8} \\ 7 + 13 + 5 \qquad\qquad\qquad\qquad\quad\ = 25 \\ \hline \qquad\qquad\qquad\qquad\qquad\qquad\quad 26\frac{1}{8}, \textit{Ans.} \end{array}$$

What is the sum of

17. $4\frac{2}{15}$, $2\frac{7}{20}$, $1\frac{1}{12}$, and $3\frac{5}{18}$? *Ans.* $10\frac{38}{45}$.
18. $\frac{6}{7}$, $\frac{6}{5}$, $3\frac{2}{35}$, and $\frac{41}{70}$?
19. $21\frac{4}{9}$, $18\frac{2}{3}$, 4, and $26\frac{5}{6}$? *Ans.* $70\frac{17}{18}$.
20. $3\frac{1}{42}$, $\frac{5}{21}$, $\frac{31}{63}$, and $1\frac{1}{14}$? *Ans.* $5\frac{34}{63}$.
21. $17\frac{2}{9}$, 10, $14\frac{7}{9}$, and $13\frac{2}{9}$?
22. $\frac{3}{8}$, $3\frac{14}{15} + 10\frac{2}{3}$, and $\frac{9}{12}$? *Ans.* $15\frac{11}{24}$.

SUBTRACTION.

154. Subtraction of Fractions is the process of subtracting one fraction from another.

155. One number can be taken from another only when of the same name or kind (Art. 44. 1). Hence,

A fraction can be subtracted from another, only when both express like fractional units, or have a common denominator.

1. What is the difference between $\frac{7}{9}$ and $\frac{4}{9}$?

Since the fractional units are of like name and kind, the difference between 7 *ninths* and 4 *ninths*, is 7 — 4, *ninths*, or $\frac{3}{9}$, which reduced is $\frac{1}{3}$.

2. What is the difference between $\frac{17}{20}$ and $\frac{13}{20}$?
3. What is the difference between $\frac{13}{15}$ and $\frac{4}{15}$?
4. What is the difference between $\frac{19}{21}$ and $\frac{11}{21}$?
5. What is the difference between $\frac{7}{8}$ and $\frac{3}{4}$?

OPERATION.

$$\frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{6}{8} = \frac{7-6}{8} = \frac{1}{8}, \textit{ Ans.}$$

Reducing the given fractions to equivalent fractions having a common denominator, so that they may express like fractional units, we have 7 *eighths* and 6 *eighths*, whose difference is 1 *eighth*, or $\frac{1}{8}$. Therefore the difference between $\frac{7}{8}$ and $\frac{3}{4}$ is $\frac{1}{8}$.

What is Subtraction of Fractions? When, only, can one fraction be subtracted from another? Why, in the operation, do we reduce the fractions *to equivalent fractions* having a common denominator?

RULE. *Reduce the fractions, if necessary, to equivalent fractions having a common denominator, and write the difference of the numerators over the common denominator.*

Examples.

What is the difference between,

6. $\frac{9}{10}$ and $\frac{7}{15}$?	*Ans.* $\frac{13}{30}$.	11. $\frac{3}{4}$ and $\frac{1}{3}$?	*Ans.* $\frac{5}{12}$.
7. $\frac{8}{9}$ and $\frac{8}{13}$?		12. $\frac{35}{37}$ and $\frac{17}{37}$?	
8. $\frac{15}{16}$ and $\frac{1}{7}$?	*Ans.* $\frac{89}{112}$.	13. $\frac{49}{50}$ and $\frac{5}{9}$?	*Ans.* $\frac{191}{450}$.
9. $\frac{1}{8}$ and $\frac{1}{9}$?		14. $\frac{9}{20}$ and $\frac{7}{15}$?	*Ans.* $\frac{1}{60}$.
10. $\frac{97}{100}$ and $\frac{3}{7}$?	*Ans.* $\frac{379}{700}$.	15. $\frac{16}{21}$ and $\frac{8}{15}$?	*Ans.* $\frac{8}{35}$.

156. When there are mixed numbers, we may either

Reduce them to improper fractions and apply the rule, or subtract the whole numbers and fractions separately and unite the results.

16. What is the difference between $5\frac{3}{4}$ and $3\frac{1}{8}$?

OPERATION.

$$5\tfrac{3}{4} - 3\tfrac{1}{8} = \tfrac{46}{8} - \tfrac{25}{8} = \tfrac{21}{8} = 2\tfrac{5}{8},\ \textit{Ans.}$$
$$\text{Or, } 5\tfrac{3}{4} - 3\tfrac{1}{8} = 5\tfrac{6}{8} - 3\tfrac{1}{8} = 2\tfrac{5}{8},\ \textit{Ans.}$$

17. What is the difference between $17\frac{9}{10}$ and $13\frac{9}{20}$? *Ans.* $4\frac{9}{20}$.

18. What is the difference between $18\frac{11}{12}$ and $18\frac{3}{4}$? *Ans.* $\frac{1}{6}$.

19. How much greater is $1\frac{15}{16}$ than $1\frac{11}{12}$? *Ans.* $\frac{1}{48}$.

20. Take $6\frac{1}{2}$ from $14\frac{5}{12}$.

OPERATION.

$$14\tfrac{5}{12} - 6\tfrac{1}{2} = \tfrac{173}{12} - \tfrac{78}{12} = \tfrac{95}{12} = 7\tfrac{11}{12},\ \textit{Ans.}$$
$$\text{Or, } 14\tfrac{5}{12} - 6\tfrac{1}{2} = 13\tfrac{17}{12} - 6\tfrac{6}{12} = 7\tfrac{11}{12},\ \textit{Ans.}$$

Here, in the second process, as we cannot take $\frac{6}{12}$ from $\frac{5}{12}$, we reduce 1 of the 14 units to *twelfths*, making $\frac{12}{12}$, and, adding the $\frac{5}{12}$, have $\frac{17}{12}$; taking, then, the 6 units from the remaining 13 units and the $\frac{6}{12}$ from $\frac{17}{12}$, and, uniting the results, we have $7\frac{11}{12}$.

What is the Rule? When there are mixed numbers how may we proceed?

Subtract,

21. $\frac{6}{7}$ from $6\frac{5}{7}$. *Ans.* $5\frac{6}{7}$.	23. $\frac{3}{16}$ from 3. *Ans.* $2\frac{13}{16}$.
22. $16\frac{7}{8}$ from 19. *Ans.* $2\frac{1}{8}$.	24. $13\frac{5}{6}$ from $42\frac{2}{3}$. *Ans.* $28\frac{5}{6}$.

APPLICATIONS.

1. A farmer sold $3\frac{1}{2}$ tons of hay to one person, $2\frac{3}{4}$ to another, and $1\frac{4}{5}$ to a third; what was the whole quantity sold?

SOLUTION. *The whole quantity sold must equal* $3\frac{1}{2} + 2\frac{3}{4} + 1\frac{4}{5}$, *tons, or, reducing the fractions to equivalent fractions having a common denominator,* $3\frac{10}{20} + 2\frac{15}{20} + 1\frac{16}{20}$, *tons, which are* $8\frac{1}{20}$ *tons. Therefore, etc.*

2. Joseph Kirk owns $\frac{1}{5}$ of a factory, his brother owns $\frac{3}{20}$ of it, and his father $\frac{9}{80}$ of it; how much of it do they all own?

Ans. $\frac{37}{80}$.

3. A man has 3 sheep; the first is worth $\$6\frac{3}{4}$, the second $\$8\frac{5}{8}$, and the third $\$9\frac{1}{2}$; how many dollars are the whole worth?

4. Two boys were talking of their ages; one said he was $9\frac{2}{7}$ years old, the other said he was $4\frac{5}{11}$ years older; what was the age of the oldest? *Ans.* $13\frac{57}{77}$ years.

5. If there is a pole standing so that $\frac{3}{8}$ of it is in the mud, $\frac{2}{5}$ of it in the water, and the rest above water, how much of it is above water?

SOLUTION. *If* $\frac{3}{8}$ *of the pole is in the mud and* $\frac{2}{5}$ *of it in the water, there must be in the mud and water* $\frac{3}{8} + \frac{2}{5}$ *of it or* $\frac{15}{40} + \frac{16}{40} = \frac{31}{40}$ *of it; and as the rest of it is above water, there must be above water* $\frac{40}{40} - \frac{31}{40} = \frac{9}{40}$ *of it. Therefore, etc.*

6. A lady, being asked her age, said her husband was $37\frac{5}{9}$ years old, and she was not so old as her husband by $8\frac{9}{13}$ years; what was her age? *Ans.* $28\frac{101}{117}$ years.

7. If I have $16\frac{1}{3}$ barrels of apples and sell $12\frac{1}{2}$ barrels, how many barrels shall I have left? *Ans.* $3\frac{5}{6}$ barrels.

8. A man having two tracts of land, one of $131\frac{7}{8}$ acres, and

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REVIEW QUESTIONS. What is a Proper Fraction? (135) What is an Improper Fraction? (136) What is a Mixed Number? (137) The terms *of a fraction? (138)*

the other of $160\frac{9}{16}$ acres, sold $150\frac{7}{20}$ acres; how many acres had he remaining? *Ans.* $142\frac{7}{80}$.

MULTIPLICATION.

157. Multiplication of Fractions is the process of multiplying, when one or both of the factors are fractions.

CASE I.

158. To multiply a fraction by a whole number.

1. What is the product of $\frac{4}{7}$ multiplied by 3?

The product of $\frac{4}{7}$ multiplied by 3, or 3 times 4 *sevenths*, is 12 *sevenths*, or $\frac{12}{7}$, which reduced is $1\frac{5}{7}$.

2. What is the product of $\frac{3}{8}$ multiplied by 4?

3. Let it be required to multiply $\frac{4}{15}$ by 3.

OPERATION.

$$\frac{4}{15}\times 3=\frac{4\times 3}{15}=\frac{12}{15}=\frac{4}{5},\ Ans.$$

$$\text{Or,}\quad \frac{4}{15}\times 3=\frac{4}{15\div 3}=\frac{4}{5},\ Ans.$$

3 times 4 *fifteenths*, or $\frac{4\times 3}{15}$, is $\frac{12}{15}$, which reduced is $\frac{4}{5}$. Or,

Since dividing the denominator multiplies the fraction (Art. 141. 2), 3 times $\frac{4}{15}$ is $\frac{4}{15\div 3}$, *or* $\frac{4}{5}$.

That is, the result is the same whether we take 3 times the *number* of parts by multiplying the numerator 4 by 3, or, the number of parts remaining unchanged, we make their *size* 3 times as large by dividing the denominator 15 by 3.

RULE. *Multiply the numerator by the whole number; or, Divide the denominator by the whole number, when it can be done without a remainder.*

Examples.

Multiply

4. $\frac{43}{48}$ by 8.	*Ans.* $\frac{43}{6}$.	9. $\frac{3}{4}$ by 3.	*Ans.* $2\frac{1}{4}$.
5. $\frac{5}{61}$ by 11.		10. $\frac{19}{20}$ by 20.	
6. $\frac{73}{125}$ by 25.	*Ans.* $\frac{73}{5}$.	11. $\frac{7}{15}$ by 45.	*Ans.* 21.
7. $\frac{98}{100}$ by 15.	*Ans.* $14\frac{7}{10}$	12. $\frac{25}{28}$ by 9.	*Ans.* $8\frac{1}{28}$.
8. $\frac{5}{12}$ by 120.	*Ans.* 50.	13. $\frac{487}{1000}$ by 100.	*Ans.* $48\frac{7}{10}$.

What is Multiplication of Fractions? Repeat the Rule.

159. When the multiplicand is a *mixed number,* we may

Multiply the fractional and integral parts separately, and add the products. Or,

Reduce the mixed number to an improper fraction, and then multiply.

14. Let it be required to multiply $7\frac{3}{4}$ by 5.

OPERATION.

$$\begin{array}{ll} \frac{3}{4} \times 5 = \frac{3 \times 5}{4} = \frac{15}{4} = & 3\frac{3}{4} \\ 7 \times 5 = & 35 \\ \hline 7\frac{3}{4} \times 5 = & 38\frac{3}{4}, \textit{Ans.} \end{array}$$

$$\text{Or, } 7\frac{3}{4} \times 5 = \frac{31 \times 5}{4} = \frac{155}{4} = 38\frac{3}{4}, \textit{Ans.}$$

Multiply

15. $5\frac{5}{8}$ by 8.	*Ans.* $45\frac{1}{3}$.	18. $25\frac{1}{4}$ by 15.	*Ans.* $378\frac{3}{4}$.
16. $3\frac{2}{15}$ by 45.		19. $19\frac{5}{7}$ by 10.	*Ans.* $198\frac{4}{7}$.
17. $31\frac{5}{10}$ by 60.	*Ans.* 1890.	20. $111\frac{7}{8}$ by 17.	*Ans.* $1901\frac{7}{8}$.

CASE II.

160. To multiply a whole number by a fraction.

Multiplying a number by a fraction is equivalent to taking the PART *of it denoted by the fraction.*

For, multiplying any number, as 6, by $\frac{1}{2}$, is taking 1 *half* of 6; by $\frac{1}{3}$ is taking 1 *third* of 6; by $\frac{2}{3}$ is taking 2 *thirds* of 6, etc.

1. How many are $\frac{2}{5}$ times ten, or $\frac{2}{5}$ of 10?

1 *fifth* of 10 is 2, and 2 *fifths* are 2 times 2, or 4.

2. How many are $\frac{3}{7}$ times 14? $\frac{5}{6}$ times 18? $\frac{7}{8}$ times 24?

3. Let it be required to multiply 16 by $\frac{3}{4}$.

OPERATION.

$$16 \times \frac{3}{4} = \frac{\overset{4}{\cancel{16}} \times 3}{\cancel{4}} = 12, \textit{Ans.}$$

1 *fourth* of 16 is 4, and 3 *fourths* of 16 are 3 times 4, or 12. Or,

3 times 16 is 16×3, and, as the multiplier is 3 *fourths,* the product must be only a *fourth* as large; and $\frac{1}{4}$ of 16×3 is $\frac{16 \times 3}{4}$, or 12.

When the multiplicand is a mixed number, how may we proceed? To what is multiplying a number by a fraction equivalent?

RULE. *Multiply the whole number by the numerator, and divide the product by the denominator.*

Examples.

Multiply

4. 124 by $\frac{3}{16}$.	*Ans.* $23\frac{1}{4}$.	8. 40 by $\frac{51}{120}$.	*Ans.* 17.
5. 63 by $\frac{3}{10}$.	*Ans.* $18\frac{9}{10}$.	9. 693 by $\frac{5}{9}$.	*Ans.* 385.
6. 72 by $\frac{7}{9}$.		10. 75 by $\frac{1}{12}$.	
7. 365 by $\frac{23}{100}$.	*Ans.* $83\frac{19}{20}$.	11. 1000 by $\frac{13}{100}$.	*Ans.* 130.

161. When the multiplier is a *mixed number*, we may

Multiply by the fractional and integral parts separately, and add the products. Or,

Reduce the mixed number to an improper fraction, and then multiply.

12. Let it be required to multiply 33 by $3\frac{2}{5}$.

OPERATION.

$$33 \times \frac{2}{5} = \frac{33 \times 2}{5} = \frac{66}{5} = 13\frac{1}{5}$$
$$33 \times 3 = \qquad 99$$
$$33 \times 3\frac{2}{5} = \qquad 112\frac{1}{5}, \textit{ Ans.}$$

Or, $33 \times 3\frac{2}{5} = 33 \times \frac{17}{5} = \frac{561}{5} = 112\frac{1}{5}$, *Ans.*

13. What is the value of $4 \times \frac{217}{684}$? *Ans.* $1\frac{46}{171}$.
14. What is the value of $106 \times 31\frac{3}{25}$?
15. Find the product of 536 multiplied by $67\frac{7}{8}$. *Ans.* 36381.

CASE III.

162. To multiply a fraction by a fraction.

1. How much is $\frac{3}{4}$ times $\frac{5}{7}$, or $\frac{3}{4}$ of $\frac{5}{7}$?

1 *fourth* of $\frac{5}{7}$ is 5 *twenty-eighths* (Art. 141), and 3 *fourths* are 3 times 5 *twenty-eighths*, which are 15 *twenty-eighths*, or $\frac{15}{28}$.

2. How much is $\frac{3}{5}$ times $\frac{3}{8}$? $\frac{4}{5}$ of $\frac{9}{10}$? $\frac{2}{3}$ of $\frac{7}{12}$?
3. Let it be required to multiply $\frac{4}{5}$ by $\frac{3}{8}$.

What is the Rule? How do you proceed when the multiplier is a mixed number?

OPERATION.

$$\frac{4}{5}\times\frac{3}{8}=\frac{\cancel{4}\times 3}{5\times \underset{2}{\cancel{8}}}=\frac{3}{10},\ \textit{Ans.}$$

1 *eighth* of $\frac{4}{5}$ is $\frac{4}{5\times 8}$, and 3 *eighths* of $\frac{4}{5}$ are 3 times $\frac{4}{5\times 8}$, which is $\frac{4\times 3}{5\times 8}$, or, by reducing, $\frac{3}{10}$. Or,

3 times $\frac{4}{5}$ is $\frac{4\times 3}{5}$, and, as the multiplier is 3 *eighths*, the product must be only 1 *eighth* as large, and $\frac{1}{8}$ of $\frac{4\times 3}{5}$ is $\frac{4\times 3}{5\times 8}$ (Art. 141), or $\frac{3}{10}$.

RULE. *Multiply by the numerator of the multiplier and divide the product by its denominator.* Or,

Multiply the numerators together for a new numerator, and the denominators for a new denominator.

This rule is *general*, and applies in the two preceding cases, since a whole number may be written in a fractional form (Art. 139).

Examples.

Multiply

4. $\frac{4}{8}$ by $\frac{7}{9}$. *Ans.* $\frac{7}{18}$.
5. $\frac{6}{7}$ by $\frac{19}{6}$.
6. $\frac{5}{10}$ by $\frac{5}{100}$. *Ans.* $\frac{25}{1000}$.
7. $\frac{1}{63}$ by $\frac{1}{12}$. *Ans.* $\frac{1}{756}$.
8. $\frac{5}{12}$ by $\frac{2}{11}$. *Ans.* $\frac{5}{66}$.
9. $\frac{487}{1000}$ by $\frac{10}{1}$. *Ans.* $4\frac{87}{100}$.

163. When fractions are connected by the word *of*, as $\frac{3}{4}$ of $\frac{4}{7}$, $\frac{3}{8}$ of $\frac{5}{6}$ of $\frac{7}{10}$, the expression is called a *Compound Fraction.*

A Compound Fraction may be treated as an expression of Multiplication (Art. 162).

10. What is the value of $\frac{3}{4}$ of $\frac{16}{21}$?

OPERATION.

$$\frac{3}{4}\text{ of }\frac{16}{21}=\frac{16}{21}\times\frac{3}{4}=\frac{\overset{4}{\cancel{16}}\times\cancel{3}}{\underset{7}{\cancel{21}}\times\cancel{4}}=\frac{4}{7},\ \textit{Ans.}$$

11. What is the value of $\frac{22}{23}$ of $\frac{36}{55}$? *Ans.* $\frac{72}{115}$.
12. What is the value of $\frac{2}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$? *Ans.* $\frac{2}{9}$.
13. What is the value of $\frac{7}{11}$ of $\frac{22}{23}$ of $\frac{46}{49}$? *Ans.* $\frac{4}{7}$.

164. When there are *mixed numbers* among the factors,

What is the Rule? What is said of the Rule? When fractions are connected by the word *of*, what is the expression called? How may a compound fraction be treated?

Reduce the mixed numbers to equivalent improper fractions, and then multiply.

Multiply

14. $14\frac{3}{7}$ by $\frac{5}{8}$.	*Ans.* $9\frac{1}{56}$.	17. $1\frac{9}{11}$ by $1\frac{37}{40}$.	*Ans.* $3\frac{1}{2}$.
15. $29\frac{4}{9}$ by $\frac{2}{3}$.		18. $2\frac{9}{28}$ by $1\frac{1}{65}$.	*Ans.* $2\frac{11}{28}$.
16. $19\frac{1}{8}$ by $1\frac{15}{17}$.	*Ans.* 36.	19. $27\frac{11}{12}$ by $8\frac{5}{6}$.	*Ans.* $246\frac{43}{72}$.

20. What is the value of $\frac{7}{8}$ of $\frac{5}{6}$ of $7\frac{1}{2}$? *Ans.* $5\frac{15}{32}$.

APPLICATIONS.

1. John has $\frac{19}{20}$ of a dollar, and his brother has 15 times as much; how much has his brother? *Ans.* $\$14\frac{1}{4}$.

2. What cost 196 bushels of corn, at $\frac{7}{8}$ of a dollar a bushel? 171 $\frac{1}{2}$

3. At $\frac{6}{10}$ of a dollar a day, how much can a boy earn in 26 days? 15 $\frac{3}{5}$

4. At $960 for a tract of land, what cost $\frac{17}{30}$ of it? *Ans.* $544.

5. At $7 a yard, what cost $\frac{15}{16}$ of a yard of cloth?

6. At $\$10\frac{5}{8}$ a barrel, what cost 24 barrels of flour?

SOLUTION. *At* $\$10\frac{5}{8}$ *a barrel,* 24 *barrels of flour will cost* 24 *times* $\$10\frac{5}{8}$, *or* 24 *times* $10 + 24 *times* $\$\frac{5}{8}$.

24 *times* $10 *is* $240; 24 *times* $\$\frac{5}{8}$, *or* $\$\frac{5\times 24}{8}$, *is* $15; *and* $240 + $15 *is* $255. *Therefore, etc.*

7. At the rate of $23\frac{21}{60}$ miles an hour, how far can a train of cars move in 24 hours? *Ans.* $560\frac{2}{5}$ miles.

8. If $2\frac{25}{36}$ of a yard of cloth be required to make a coat, how much will be required for 4 coats?

9. If a hat cost $\frac{1}{4}$ of $\$16\frac{3}{4}$, and a vest $\frac{8}{9}$ as much, what was the cost of the vest? *Ans.* $\$3\frac{13}{18}$.

10. William Benton has in his farm $\frac{5}{16}$ of 64 acres, and his son owns $\frac{4}{5}$ as many acres; how many does his son own? *Ans.* 16 acres.

When there are mixed numbers among the factors, how do you proceed?

REVIEW QUESTIONS. What is the Denominator of a Fraction? (132) The Numerator? (133) What does the numerator express? (138) The denominator? (138)

DIVISION.

165. Division of Fractions is the process of dividing, when the dividend or divisor, or both, are fractions.

CASE I.

166. To divide a fraction by a whole number.

1. What is the quotient of $\frac{10}{11}$ divided by 5?

The quotient of $\frac{10}{11}$ divided by 5, or $\frac{1}{5}$ of 10 *elevenths* is 2 *elevenths*, or $\frac{2}{11}$.

2. What is the quotient of $\frac{21}{25}$ divided by 7? Of $\frac{16}{17}$ divided by 4?

3. Let it be required to divide $\frac{12}{15}$ by 3.

OPERATION.

$$\frac{12}{15} \div 3 = \frac{12 \div 3}{15} = \frac{4}{15}, \textit{ Ans.}$$

$$\text{Or, } \frac{12}{15} \div 3 = \frac{\overset{4}{\cancel{12}}}{15 \times \cancel{3}} = \frac{4}{15}, \textit{ Ans.}$$

$\frac{1}{3}$ of 12 fifteenths, or $\frac{12 \div 3}{15} = \frac{4}{15}$. Or,

Since multiplying the denominator divides the fraction (Art. 141. 3), $\frac{1}{3}$ of $\frac{12}{15}$ is $\frac{12}{15 \times 3}$, which reduced is $\frac{4}{15}$.

That is, the result is the same whether we take $\frac{1}{3}$ of the number of parts by dividing the numerator 12 by 3, or, the number of parts remaining unchanged, we make their size $\frac{1}{3}$ as large by multiplying the denominator 15 by 3.

RULE. *Divide the numerator by the whole number, when it can be done without a remainder.* Or,

Multiply the denominator by the whole number.

Examples.

Divide

4. $\frac{14}{15}$ by 8. *Ans.* $\frac{7}{60}$.
5. $\frac{40}{67}$ by 20.
6. $\frac{35}{36}$ by 29. *Ans.* $\frac{35}{1044}$.
7. $\frac{31}{100}$ by 10. *Ans.* $\frac{31}{1000}$.
8. $\frac{3}{8}$ by 7. *Ans.* $\frac{3}{56}$.
9. $\frac{24}{109}$ by 48.
10. $\frac{4}{15}$ by 128. *Ans.* $\frac{1}{480}$.
11. $\frac{1771}{1232}$ by 23. *Ans.* $\frac{1}{16}$.
12. What is the value of $\frac{175}{400} \div 25$? *Ans.* $\frac{7}{400}$.

What is Division of Fractions? What effect has multiplying the denominator? What is the Rule?

167. When the dividend is a *mixed number*, we may either

Divide the integral part of the mixed number, and reducing the remainder to an improper fraction, divide it. Or,

Reduce the mixed number to an improper fraction, and then divide.

13. Let it be required to divide $26\frac{2}{9}$ by 4.

OPERATION.

$$4\,)\,26\tfrac{2}{9}\,(\,6$$
$$\underline{24}$$
$$2\tfrac{2}{9} = \tfrac{20}{9};\ \frac{\overset{5}{\cancel{20}}}{9 \times \cancel{4}} = \frac{5}{9};\ 6 + \tfrac{5}{9} = 6\tfrac{5}{9},\ \textit{Ans.}$$

Or, $26\frac{2}{9} = \frac{236}{9}$; $\frac{236}{9} \div 4 = \frac{236 \div 4}{9} = \frac{59}{9} = 6\frac{5}{9}$, *Ans.*

Divide

14. $75\frac{9}{12}$ by 9. *Ans.* $8\frac{5}{12}$.
15. $11\frac{39}{40}$ by 31. *Ans.* $\frac{479}{1240}$.
16. $203\frac{1}{15}$ by 60. *Ans.* $3\frac{173}{450}$.
17. $3\frac{1}{11}$ by 17. *Ans.* $\frac{2}{11}$.
18. $91\frac{2}{5}$ by 15. *Ans.* $6\frac{7}{75}$.
19. $813\frac{7}{10}$ by 100. *Ans.* $8\frac{137}{1000}$.

CASE II.

168. To divide a whole number by a fraction.

1. How many times is $\frac{1}{5}$ contained in 10 ?

1 is contained in 10, 10 times, and since 1 *fifth* must be contained in any number 5 times as many times as 1 is contained in it, 1 *fifth* is contained in 10, 5 times 10 times, or 50 times. Or,

10 is equal to $\frac{50}{5}$; and 1 *fifth* is contained in 50 *fifths*, 50 times.

2. How many times is $\frac{1}{7}$ contained in 6? $\frac{1}{6}$ in 16? $\frac{1}{9}$ in 16?

3. How many times is $\frac{2}{3}$ contained in 5?

5 is equal to $\frac{15}{3}$, and 2 *thirds* are contained in 15 *thirds*, $7\frac{1}{2}$ times.

4. How many times is $\frac{3}{8}$ contained in 7? $\frac{2}{9}$ in 3? $\frac{7}{10}$ in 6?

5. Let it be required to find the quotient of 12 divided by $\frac{1}{4}$.

When the dividend is a mixed number, how may we proceed ?

OPERATION.

$$12 \div \tfrac{1}{4} = \frac{12 \times 4}{1} = 48, \textit{ Ans.}$$

12 divided by 1 is $\frac{12}{1}$, or 12, and, since 1 *fourth* must be contained in any number 4 times as many times as 1 is contained in it, 12 divided by 1 *fourth* is 4 times $\frac{12}{1}$, or $\frac{12\times4}{1}$, which is 48.

6. Let it be required to find the quotient of 16 divided by $\frac{2}{5}$.

OPERATION.

$$16 \div \tfrac{2}{5} = \frac{\overset{8}{\cancel{16}} \times 5}{\cancel{2}} = 40, \textit{ Ans.}$$

$$\text{Or, } 16 \div \tfrac{2}{5} = \tfrac{80}{5} \div \tfrac{2}{5} = 40, \textit{ Ans.}$$

16 divided by 2 is $\frac{16}{2}$, and 16 divided by $\frac{2}{5}$ is 5 times $\frac{16}{2}$, or $\frac{16\times5}{2}$, which reduced is 40. Or,

Reducing the dividend 16 to an equivalent fraction of like name with the divisor, so that they may express like fractional units (Art. 60. 1), we have 80 *fifths* to be divided by 2 *fifths*, and 2 *fifths* are contained in 80 *fifths*, 40 times.

RULE. *Divide by the numerator of the divisor, and multiply by its denominator.* Or,

Reduce the given whole number to an equivalent fraction of the same denominator as the divisor, and divide the numerator of the dividend by the numerator of the divisor.

When the divisor is a *mixed number*, it must be reduced to an equivalent improper fraction before dividing.

Examples.

Divide

7. 5 by $\frac{7}{10}$.	*Ans.* $7\frac{1}{7}$.	11. 49 by $\frac{16}{29}$.	*Ans.* $88\frac{13}{16}$.
8. 29 by $\frac{3}{4}$.		12. 50 by $\frac{9}{11}$.	" $61\frac{1}{9}$
9. 10 by $\frac{1}{100}$.	*Ans.* 1000.	13. 176 by $\frac{11}{23}$.	*Ans.* 368.
10. 97 by $5\frac{4}{7}$.	*Ans.* $17\frac{16}{39}$.	14. 100 by $4\frac{7}{8}$.	*Ans.* $20\frac{20}{39}$.

CASE III.

169. To divide a fraction by a fraction.

1. How many times is $\frac{1}{3}$ contained in $\frac{3}{4}$?

Why, in the second operation of example 6, is the dividend 16 reduced to an equivalent fraction expressing fifths? What is the Rule? When the divisor is a mixed number, what must be done?

1 is contained in $\frac{3}{4}$, $\frac{3}{4}$ of a time, and, since 1 *third* must be contained in any number 3 times as many times as 1 is contained in it, 1 *third* is contained in $\frac{3}{4}$, 3 times $\frac{3}{4}$, or $\frac{9}{4}$ times, which is $2\frac{1}{4}$ times. Or,

$\frac{1}{3}$ is equal to $\frac{4}{12}$ and $\frac{3}{4}$ to $\frac{9}{12}$, and 4 *twelfths* are contained in 9 *twelfths*, $2\frac{1}{4}$ times.

2. How many times is $\frac{1}{5}$ contained in $\frac{2}{3}$? $\frac{1}{4}$ in $\frac{7}{8}$?

3. Let it be required to divide $\frac{4}{5}$ by $\frac{2}{3}$.

OPERATION.

$$\frac{4}{5} \div \frac{2}{3} = \frac{\overset{2}{\cancel{4}} \times 3}{5 \times \cancel{2}} = \frac{6}{5} = 1\tfrac{1}{5},\ \textit{Ans.}$$

Or, $\frac{4}{5} \div \frac{2}{3} = \frac{12}{15} \div \frac{10}{15} = 12 \div 10 = 1\frac{1}{5}$, *Ans.*

$\frac{4}{5}$ divided by 2 is $\frac{4}{5 \times 2}$, and, as the divisor is 2 *thirds*, the quotient must be 3 times as large, or $\frac{4 \times 3}{5 \times 2}$, which reduced is $1\frac{1}{5}$. Or,

$\frac{4}{5}$ is equal to $\frac{12}{15}$, and $\frac{2}{3}$ to $\frac{10}{15}$, and 10 *fifteenths* are contained in 12 *fifteenths*, $1\frac{1}{5}$ times.

RULE. *Divide by the numerator of the divisor, and multiply the quotient by the denominator.* Or,

Reduce the dividend and divisor, if necessary, to equivalent fractions having a common denominator, and divide the numerator of the dividend by the numerator of the divisor.

This rule is *general*, and applies in the preceding two cases, since a whole number may be written in a fractional form (Art. 139).

The process of the rule, in brief, consists *in inverting the terms of the divisor, and then finding the product of the dividend by the inverted divisor.*

Examples.

Divide

4. $\frac{9}{16}$ by $\frac{3}{4}$.	*Ans.* $\frac{3}{4}$.	9. $\frac{21}{25}$ by $\frac{14}{15}$.	*Ans.* $\frac{9}{10}$.
5. $\frac{13}{14}$ by $\frac{13}{15}$.	*Ans.* $1\frac{1}{14}$.	10. $\frac{7}{22}$ by $\frac{113}{355}$.	*Ans.* $\frac{2485}{2486}$.
6. $\frac{8}{15}$ by $\frac{9}{20}$.		11. $\frac{31}{60}$ by $\frac{23}{60}$.	
7. $\frac{7}{8}$ by $\frac{12}{19}$.	*Ans.* $1\frac{37}{96}$.	12. $\frac{155}{365}$ by $\frac{30}{365}$.	*Ans.* $5\frac{1}{6}$.
8. $\frac{5}{10}$ by $\frac{5}{100}$.	*Ans.* 10.	13. $\frac{15}{10000}$ by $\frac{3}{100}$.	*Ans.* $\frac{1}{20}$.

How many times is $\frac{1}{3}$ contained in 1? If 1 is contained once in any number, how many times is $\frac{1}{3}$ contained in the same number? Repeat the Rule. What is said of the Rule? The process of the rule?

170. When either the dividend or divisor, or both, are *mixed numbers*,

Reduce the mixed numbers to equivalent improper fractions, and then divide.

Divide

14. $56\frac{5}{9}$ by $\frac{3}{5}$. *Ans.* $94\frac{7}{27}$.
15. $\frac{3}{8}$ by $5\frac{1}{2}$. *Ans.* $\frac{3}{44}$.
16. $4\frac{3}{4}$ by $5\frac{1}{8}$. *Ans.* $\frac{38}{41}$.
17. $18\frac{53}{97}$ by $\frac{47}{85}$. *Ans.* $33\frac{2468}{4559}$.

171. When a fraction has one or both its terms fractional, as $\frac{\frac{3}{4}}{7}$, $\frac{5}{\frac{2}{3}}$, or $\frac{\frac{1}{2}}{\frac{4}{5}}$, the expression is called a *Complex Fraction.*

A Complex Fraction may be treated as a case in division (Art. 58).

18. What is the value of $\frac{\frac{6}{7}}{\frac{8}{11}}$?

OPERATION.

$$\frac{\frac{6}{7}}{\frac{3}{11}} = \frac{6}{7} \div \frac{3}{11} = \frac{\overset{2}{\cancel{6}} \times 11}{7 \times \cancel{3}} = \frac{22}{7} = 3\tfrac{1}{7}, \textit{Ans.}$$

Here, the given fraction is regarded as an expression of division; the numerator $\frac{6}{7}$ answering to the dividend, and the denominator $\frac{8}{11}$ to the divisor, and the value is found by performing the division indicated. (Art. 169).

19. $\frac{\frac{9}{19}}{6}$ = what? *Ans.* $\frac{3}{38}$.
20. $\frac{\frac{18}{35}}{12}$ = what?
21. $\frac{100}{\frac{17}{19}}$ = what? *Ans.* $111\frac{13}{17}$.
22. $\frac{\frac{4}{7}}{\frac{2}{3}}$ = what? *Ans.* $\frac{6}{7}$.
23. $\frac{3\frac{1}{6}}{9\frac{1}{2}}$ = what? *Ans.* $\frac{1}{3}$.
24. $\frac{2\frac{1}{2}}{\frac{1}{1728}}$ = what? *Ans.* 4320.

25. What is the quotient of $\frac{3\frac{1}{2}}{2\frac{1}{4}}$ divided by $\frac{6}{4\frac{1}{2}}$? *Ans.* $1\frac{1}{6}$.

26. What is the quotient of $\frac{7}{8}$ of $\frac{60\frac{5}{16}}{10\frac{3}{8}}$ divided by $\frac{5}{8}$?

Ans. $8\frac{68}{415}$.

When either dividend or divisor, or both, are mixed numbers, how do you *proceed?* When one or both terms of a fraction are fractional, what is the *expression called?* How may it be treated?

APPLICATIONS.

1. If 4 pounds of sugar cost $\frac{3}{4}$ of a dollar, how much will a pound cost? *Ans.* $\frac{3}{16}$ of a dollar.

2. If a horse consume $1\frac{7}{8}$ of a ton of hay in 5 months, how much will he consume in one month? *Ans.* $\frac{3}{8}$ of a ton.

3. If 10 bushels of wheat cost $\$13\frac{3}{4}$, how much is it a bushel? *Ans.* $\$1\frac{3}{8}$.

4. If $\$250\frac{3}{8}$ are divided equally among 19 men, how much will each man receive? *Ans.* $\$13\frac{27}{152}$.

5. If $\frac{3}{5}$ of a barrel of flour cost $9, how much will a whole barrel cost?

SOLUTION. *If $\frac{3}{5}$ of a barrel of flour cost $9, $\frac{1}{5}$ of a barrel must cost $\frac{1}{3}$ of $9, or $3; and $\frac{5}{5}$, or a whole barrel, must cost 5 times $3, which is $15. Therefore, etc.*

6. When $\$4\frac{3}{5}$ is paid for $\frac{2}{3}$ of a cord of wood, how much is it a cord? *Ans.* $\$6\frac{9}{10}$.

7. If $104\frac{2}{5}$ yards of cloth can be bought for $87, how much is the cloth a yard? *Ans.* $\frac{5}{6}$ of a dollar.

8. At $\frac{3}{8}$ of a dollar a pound, how many pounds of coffee can be bought for $\frac{15}{16}$ of a dollar?

SOLUTION. *At $\frac{3}{8}$ of a dollar a pound, as many pounds of coffee can be bought for $\frac{15}{16}$ of a dollar as $\frac{3}{8}=\frac{6}{16}$ of a dollar are contained times in $\frac{15}{16}$ of a dollar, which are $2\frac{1}{2}$. Therefore, etc.*

9. At $\$9\frac{1}{2}$ a barrel, how much flour can be bought for $\$3\frac{1}{6}$? *Ans.* $\frac{1}{3}$ of a barrel.

10. At $\frac{5}{6}$ of a dollar a bushel, how many bushels of corn can be bought for $87?

11. At the rate of $17\frac{1}{2}$ miles an hour, how long will it take a train of cars to run $1806\frac{7}{8}$ miles? *Ans.* $103\frac{1}{4}$ hours.

REVIEW QUESTIONS. What is Reduction of Fractions? (142) When is a fraction expressed in its smallest terms? (143) When have fractions a Common Denominator? (147) When, only, can fractions be added? (152) When, only, can fractions be subtracted? (155) What is multiplying a number by a fraction equivalent to? (160)

RELATIONS OF NUMBERS.

172. The **Relation of Numbers** is the number of times one number contains another with which it is compared, or the *part* the latter is of the former.

173. Numbers can only be compared, and their relations expressed, when they represent units of the same name or kind.

174. To find the relation of numbers, or the part that one number is of another.

1. What part of 5 is 3?

1 is $\frac{1}{5}$ of 5, and 3 is 3 times $\frac{1}{5}$ of 5, or $\frac{3}{5}$ of 5.

2. What part of 9 is 5? Of 12 is 4? Of 8 is 5?
3. What part of 6 is $\frac{5}{7}$?

OPERATION.

$\frac{5}{7\times6} = \frac{5}{42}$, *Ans.*

Or, $6 = \frac{42}{7}$; $\frac{5}{7} \div \frac{42}{7} = \frac{5}{42}$, *Ans.*

1 is $\frac{1}{6}$ of 6, and $\frac{5}{7}$ of 1 is $\frac{5}{7}$ of $\frac{1}{6}$, or $\frac{5}{42}$ of 6.

Or, 6 is $\frac{42}{7}$; and $\frac{5}{7}$ is the same part of $\frac{42}{7}$ as 5 is of 42, or $\frac{5}{42}$.

4. What part of 2 is $\frac{3}{5}$? Of 1 is $\frac{2}{3}$? Of 3 is $\frac{2}{5}$?
5. What part of $\frac{3}{4}$ is 5?

OPERATION.

$\frac{5\times4}{3} = \frac{20}{3} = 6\frac{2}{3}$, *Ans.*

Or, $5 = \frac{20}{4}$; $\frac{20}{4} \div \frac{3}{4} = \frac{20}{3} = 6\frac{2}{3}$, *Ans.*

$\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{3}{4}$, and $\frac{4}{4}$, or 1, is $\frac{4}{3}$ of $\frac{3}{4}$, and 5 is 5 times $\frac{4}{3}$ of $\frac{3}{4}$, or $\frac{20}{3}$ of $\frac{3}{4}$, which is $6\frac{2}{3}$ times $\frac{3}{4}$.

Or, 5 is $\frac{20}{4}$, and $\frac{20}{4}$ is the same part of $\frac{3}{4}$ as 20 is of 3, or $\frac{20}{3}$, which is $6\frac{2}{3}$ times $\frac{3}{4}$.

6. What part of $\frac{2}{3}$ is 1? Of $\frac{3}{4}$ is 2? Of $\frac{2}{9}$ is 3?
7. What part of $\frac{3}{4}$ is $\frac{2}{5}$?

OPERATION.

$\frac{2\times4}{5\times3} = \frac{8}{15}$, *Ans.*

Or, $\frac{2}{5} = \frac{8}{20}$, $\frac{3}{4} = \frac{15}{20}$; $\frac{8}{20} \div \frac{15}{20} = \frac{8}{15}$, *Ans.*

$\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{3}{4}$, and $\frac{4}{4}$, or 1, is $\frac{4}{3}$ of $\frac{3}{4}$, and $\frac{2}{5}$ is $\frac{2}{5}$ of $\frac{4}{3}$ of $\frac{3}{4}$, or $\frac{8}{15}$ of $\frac{3}{4}$.

What is the Relation of Numbers? When can numbers be compared and *their relations* expressed?

Or, $\frac{3}{4}$ is $\frac{15}{20}$ and $\frac{2}{5}$ is $\frac{8}{20}$, and $\frac{8}{20}$ is the same part of $\frac{15}{20}$, as 8 is of 15, or $\frac{8}{15}$; hence, $\frac{2}{5}$ is $\frac{8}{15}$ of $\frac{3}{4}$.

8. What part of $\frac{1}{2}$ is $\frac{1}{4}$? Of $\frac{5}{6}$ is $\frac{2}{3}$? Of $\frac{3}{8}$ is $\frac{3}{10}$?

RULE. *Divide the number denoting the part, by that with which it is compared.*

Examples.

What part of

9. 45 is 9?	*Ans.* $\frac{1}{5}$.	13. 13 is $\frac{5}{7}$?	*Ans.* $\frac{5}{91}$.
10. 16 is 56?	*Ans.* $3\frac{1}{2}$.	14. $\frac{24}{25}$ is 8?	*Ans.* $8\frac{1}{3}$.
11. 35 is 21?		15. $\frac{11}{12}$ is $\frac{13}{15}$?	*Ans.* $\frac{52}{55}$.
12. 9 is $\frac{14}{100}$?	*Ans.* $\frac{7}{450}$.	16. $30\frac{1}{4}$ is $5\frac{1}{2}$?	*Ans.* $\frac{2}{11}$.

17. What is the relation of $\frac{42}{54}$ to $\frac{28}{36}$? *Ans.* As 1 to 1.

18. What is the relation of 125 to 1000? *Ans.* $\frac{1}{8}$.

APPLICATIONS.

1. A liberty pole, whose hight was 75 feet, was broken off by the wind 45 feet from the ground; what part of it was left standing? *Ans.* $\frac{2}{5}$ of it.

2. If I perform a piece of work in 6 days, what part of it can I perform in $1\frac{1}{2}$ days?

3. When 24 pairs of boots cost \$156, what will 20 pairs cost?

OPERATION.

$$\frac{20}{24}=\frac{5}{6};\quad \frac{\$\overset{26}{\cancel{156}}\times 5}{\cancel{6}}=\$130,\ Ans.$$

When 24 pairs of boots cost \$156, 20 pairs, which are $\frac{20}{24}$, or $\frac{5}{6}$, of 24 pairs, will cost $\frac{5}{6}$ of \$156, or \$130. Therefore, etc.

4. A and B mowed a field for a certain sum of money, A mowing $12\frac{1}{2}$ acres, and B $18\frac{3}{4}$ acres. What should be A's share of the money, if B's is \$30? *Ans.* \$20.

5. If a boy has spent $\frac{3}{8}$ of $\frac{9}{10}$ of his money, what part of $\frac{9}{10}$ of his money has he left?

6. A farmer owns 320 acres of land, and his son $\frac{3}{8}$ as many.

What is the Rule?

Should the son add 40 acres to his land, how would his number of acres compare with the number his father has?

Ans. It would be $\frac{1}{2}$ as many.

REVIEW EXERCISES.

1. Reduce $\frac{3619}{6251}$ to its smallest terms. *Ans.* $\frac{11}{19}$.

2. Change $\frac{5}{61}$ to an equivalent fraction having 671 for its denominator. *Ans.* $\frac{55}{671}$.

3. Change $100\frac{100}{199}$ to an improper fraction. *Ans.* $\frac{20000}{199}$.

4. Reduce $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{3}{10}$ to equivalent fractions, having 30 for their denominator.

5. If I should receive from one person $\$19\frac{9}{10}$, and from another $\$51\frac{5}{6}$, and then pay away $\$63\frac{3}{4}$, how much should I have remaining? *Ans.* $\$7\frac{59}{60}$.

6. Joseph has $\$13\frac{3}{20}$, Andrew has $\$7\frac{3}{5}$ more than Joseph, and Henry has as much as both of them; how many dollars each have Andrew and Henry?

Ans. Andrew, $\$20\frac{3}{4}$; Henry, $\$33\frac{9}{10}$.

7. If $\frac{5}{8}$ be a minuend, and $\frac{2}{5}$ a remainder, what is the subtrahend?

8. A merchant owned $\frac{11}{16}$ of a ship, and sold $\frac{2}{3}$ of his share; what part of the whole ship did he sell? *Ans.* $\frac{11}{24}$.

9. If a product be $12\frac{1}{10}$, and the multiplicand $3\frac{2}{3}$, what is the multiplier? *Ans.* $3\frac{3}{10}$.

10. How many oranges can be bought for $2\frac{1}{3}$ times $\frac{1}{3}$ of 2 cents, when they are $\frac{1}{3}$ of $\frac{9}{11}$ of $2\frac{2}{3}$ cents apiece?

11. A has \$3240, and B has $\frac{4}{5}$ as much, lacking \$500; how much has B? *Ans.* \$2092.

12. If there leaks out from a cask containing $31\frac{1}{2}$ gallons of cider, $\frac{2}{3}$ of a gallon each week, in how many weeks will the whole have leaked out? *Ans.* $47\frac{1}{4}$ weeks.

REVIEW QUESTIONS. What is the unit of a fraction? (131) What is a fractional unit? (131)

13. If the divisor is $\frac{5}{6}$ and the quotient only $\frac{3}{4}$ as large, what must the dividend be? *Ans.* $\frac{25}{48}$.

14. What is the value of $\dfrac{\frac{3}{4}}{\frac{6}{5} \times \frac{5}{7}} + (3\frac{7}{8} \div \frac{9}{10}) - 1\frac{3}{16}$?

Ans. $3\frac{143}{144}$.

15. If 3 be added to each term of the fraction $\frac{2}{3}$, how much will the value expressed be increased or diminished?

Ans. Increased $\frac{1}{6}$.

16. If 3 be added to each term of the fraction $\frac{3}{2}$, how much will the value expressed be increased or diminished?

Ans. Diminished $\frac{3}{10}$.

17. What number must be multiplied by $\frac{17}{19}$, that the product may be 100? *Ans.* $111\frac{13}{17}$.

18. If I pay away $\frac{1}{2}$ of my money, then $\frac{1}{3}$ of what remains, and then $\frac{1}{4}$ of what still remains, what fraction of the whole will be left? *Ans.* $\frac{1}{4}$.

19. A farmer, having a flock of 120 sheep, sold 30, and the dogs killed $\frac{1}{9}$ of the remainder. What part of the original number then remained? *Ans.* $\frac{2}{3}$.

20. If 8 horses eat in a certain time $1\frac{12}{31}$ tons of hay, how much will one horse eat in the same time?

21. At $\$8\frac{3}{8}$ a ton, how many tons of coal can be purchased for $\$41\frac{7}{8}$? *Ans.* 5 tons.

22. Bought 4 casks of molasses, containing $31\frac{4}{5}$ gallons each. What part of the whole is 100 gallons? *Ans.* $\frac{125}{159}$.

23. A man owning $\frac{9}{15}$ of a steamboat, said that his part was 10 times as large as that of another man, who owned $\frac{3}{40}$; but not being a good arithmetician, he had made a wrong calculation. How many times as large, in fact, was his part?

Ans. 8 times.

Review Questions. How may a fractional expression be explained? (140) How may a fraction be regarded? (141) What is the value expressed by a fraction? (141) How is a fraction reduced to its smallest terms? (143) How is a mixed number reduced to an equivalent improper fraction? (145)

EXERCISES IN ANALYSIS.*

1. If 11 tons of coal cost \$99, what will $7\frac{5}{9}$ tons cost?

SOLUTION.—*If* 11 *tons cost* \$99, 1 *ton will cost* $\frac{1}{11}$ *of* \$99, *or* \$9. *If* 1 *ton cost* \$9, $7\frac{5}{9}$ *tons will cost* $7\frac{5}{9}$ *times* \$9, *or* \$68. *Therefore, etc.*

2. If 20 barrels of flour cost \$210, what will 27 barrels cost? *Ans.* $\$283\frac{1}{2}$.

3. If 27 barrels of flour cost $\$283\frac{1}{2}$, what will 20 barrels cost? 210.

4. If $\frac{4}{7}$ of a yard of cloth cost \$2.80, what will $5\frac{4}{5}$ yards cost?

SOLUTION. *If* $\frac{4}{7}$ *of a yard of cloth cost* \$2.80, $\frac{1}{7}$ *of a yard will cost* $\frac{1}{4}$ *of* \$2.80, *or* \$.70, *and* $\frac{7}{7}$, *or* 1 *yard, will cost* 7 *times* \$.70, *or* \$4.90.

If 1 *yard cost* \$4.90, $5\frac{4}{5}$ *yards will cost* $5\frac{4}{5}$ *times* \$4.90, *or* \$28.42. *Therefore, etc.*

5. If $\frac{3}{4}$ of a pound of tea cost \$.60, what will $553\frac{1}{4}$ pounds cost? *Ans.* \$442.60.

6. When $\frac{3}{8}$ of an acre of land cost \$75, what will $7\frac{4}{5}$ acres cost? *Ans.* \$1560.

7. If $7\frac{4}{5}$ acres of land cost \$1560, what will $\frac{3}{8}$ of an acre cost?

8. If \$2.80 will buy $\frac{4}{7}$ of a yard of cloth, how many yards will \$28.42 buy?

SOLUTION. *If* \$2.80 *will buy* $\frac{4}{7}$ *of a yard of cloth*, $\frac{1}{4}$ *of* \$2.80, *or* \$.70, *will buy* $\frac{1}{7}$ *of a yard, and* 7 *times* \$.70, *or* \$4.90, *will buy* $\frac{7}{7}$, *or* 1 *yard.*

If \$4.90 *will buy* 1 *yard of cloth, as many yards can be bought for* \$28.42 *as* \$4.90 *is contained times in* \$28.42, *or* $5\frac{4}{5}$ *yards. Therefore, etc.*

9. If \$7 will buy $5\frac{3}{8}$ bushels of rye, how many bushels will \$15 buy? *Ans.* $11\frac{29}{56}$ bushels.

10. If \$5.60 will pay for $\frac{7}{9}$ of a ton of coal, what part of a ton will \$5.40 purchase? *Ans.* $\frac{3}{4}$ of a ton.

REVIEW QUESTIONS. How are fractions reduced to equivalent fractions having a common denominator? (149) The Rule for Addition of Fractions? (152)

* Optional.

11. When $19\frac{1}{2}$ pounds of coffee cost $\$11\frac{1}{2}$, how many pounds can be obtained for $\$2\frac{7}{8}$? *Ans.* $4\frac{7}{8}$ pounds.

12. When $\$33\frac{11}{12}$ will pay for $4\frac{5}{8}$ barrels of flour, how much can be purchased with \$27.50? *Ans.* $3\frac{3}{4}$ barrels.

13. When $4\frac{2}{5}$ tons of hay will suffice for 11 horses for a certain time, for how many horses will $7\frac{1}{5}$ tons suffice for the same time? *Ans.* 18 horses.

14. If A can do a piece of work in 7 days, and B the same work in 5 days, in what time can both do it by working together?

SOLUTION. *If A can do a piece of work in 7 days, he can do $\frac{1}{7}$ of it in 1 day; and if B can do the same in 5 days, he can do $\frac{1}{5}$ of it in 1 day.*

If A can do $\frac{1}{7}$ of it in 1 day, and B $\frac{1}{5}$ in 1 day, they can, by working together, do $\frac{1}{7}+\frac{1}{5}$, or $\frac{12}{35}$, of it in one day.

If by working together they can do $\frac{12}{35}$ of the work in 1 day, they can do $\frac{1}{35}$ of it in $\frac{1}{12}$ of a day, and $\frac{35}{35}$, or the whole, in 35 times $\frac{1}{12}$ of a day, which is $\frac{35}{12}$ of a day, or $2\frac{11}{12}$ days. Therefore, etc.

15. A man can trench a garden in 13 days, and his son can do the same in 10 days; in what time can both working together do it? *Ans.* $5\frac{15}{23}$ days.

16. A can mow $\frac{1}{10}$ of a field in a day, and B $\frac{1}{15}$; in what time can both, by working together, mow it?

17. A cistern has 3 pipes; the first will fill it in 10 hours, the second in 15 hours, and the third in 16 hours. What time will it take them all to fill it? *Ans.* $4\frac{4}{11}$ hours.

18. In an orchard, $\frac{1}{3}$ of the trees bear apples, $\frac{1}{5}$ peaches, $\frac{2}{7}$ pears, and the remainder, which is 38, cherries. How many trees are there in the orchard?

SOLUTION. *Since $\frac{1}{3}$ of the trees bear apples, $\frac{1}{5}$ peaches, and $\frac{2}{7}$ pears, $\frac{1}{3}+\frac{1}{5}+\frac{2}{7}$, or $\frac{86}{105}$, bear apples, peaches, and pears, and $\frac{105}{105}$, or the whole orchard, less $\frac{86}{105}$, or $\frac{19}{105}$, must bear cherries.*

REVIEW QUESTIONS. The Rule for Subtraction of Fractions? (155) General Rule for Multiplication of Fractions? (162) Division of Fractions? (169)

* This page is optional.

If the 38 *trees bearing cherries are* $\frac{19}{105}$ *of the orchard,* $\frac{1}{105}$ *is* $\frac{1}{19}$ *of* 38 *trees, or* 2 *trees, and if* 2 *trees are* $\frac{1}{105}$, $\frac{105}{105}$, *or the whole orchard, must be* 105 *times* 2 *trees, or* 210 *trees. Therefore, etc.*

19. A person having bought a house for a certain sum, found that after he had paid one-half and one-third of the cost, he then owed $400. How much had he paid? 2000

20. A farmer had $\frac{3}{8}$ of his sheep in one field, $\frac{5}{12}$ in a second field, and the remainder, 75, in a yard. How many had he in each of the two fields?

Ans. In the first, 135; in the second, 150.

21. A, B, and C purchase a mill; A pays $\frac{2}{9}$, B $\frac{2}{7}$, and C $2000 of the cost. What are the sums paid by A and B?

Ans. By A, $903\frac{7}{31}$; by B, $1161\frac{9}{31}$.

22. A and B together have $136, and $\frac{2}{3}$ of A's money is equal to $\frac{3}{4}$ of B's. How much has each?

SOLUTION. *Since* $\frac{2}{3}$ *of A's money equals* $\frac{3}{4}$ *of B's,* $\frac{1}{3}$ *of A's equals* $\frac{1}{2}$ *of* $\frac{3}{4}$, *or* $\frac{3}{8}$, *of B's, and* $\frac{3}{3}$, *or the whole of A's, equals* 3 *times* $\frac{3}{8}$ *of B's, or* $\frac{9}{8}$ *of B's.*

If A's money equals $\frac{9}{8}$ *of B's, as B's must equal* $\frac{8}{8}$ *of itself, A and B together have* $\frac{9}{8}+\frac{8}{8}$, *or* $\frac{17}{8}$, *of B's money.*

If $\frac{17}{8}$ *of B's money is* $136, $\frac{1}{8}$ *is* $\frac{1}{17}$ *of* $136, *or* $8, *and* $\frac{9}{8}$, *or A's money, is* 9 *times* $8, *or* $72, *while* $\frac{8}{8}$, *or B's money, is* 8 *times* $8, *or* $64. *Therefore, etc.*

23. The sum of two numbers is 350, and $\frac{3}{8}$ of the larger number is $\frac{1}{2}$ of the smaller; what are the numbers?

Ans. 200 and 150.

24. My carriage is worth $2\frac{1}{2}$ times as much as my horse, and both together are worth $420. What is the value of each? 120 300

25. A gentleman bought a horse, chaise, and harness, for $300. The chaise cost $\frac{1}{2}$ as much as the horse, and the harness $\frac{1}{3}$ as much as the chaise and horse both. What was the cost of each? *Ans.* Horse, $150; chaise, $75; harness, $75.

REVIEW QUESTIONS. What is meant by the Relations of Numbers? (172) What is the Rule for finding what part one number is of another? (174)

* This page is optional.

DECIMAL FRACTIONS.

175. If a unit be divided into ten equal parts, each of these parts will be 1 *tenth.*

If each tenth be divided into ten equal parts, each part will be $\frac{1}{10}$ of $\frac{1}{10}$, or 1 *hundredth.*

If each hundredth be divided into ten equal parts, each part will be $\frac{1}{10}$ of $\frac{1}{100}$, or 1 *thousandth.*

In like manner, we may, by dividing by ten, continue to obtain fractions, each of whose values is *one tenth* of the fraction preceding it; such fractions are called *Decimal Fractions.* Hence,

176. A **Decimal Fraction** is a fraction whose unit is divided into *tenths, hundredths, thousandths,* etc.

A decimal fraction, for brevity, is usually called a *decimal.*

NOTATION AND NUMERATION.

177. Decimal fractions are commonly written without the denominator, and distinguished from whole numbers by having the decimal point (.) at the left.

The figures at the right of the point are called *decimal figures.*

The first order to the right of the decimal point expresses *tenths.* Thus,

$\frac{1}{10}$	may be written	.1,	and read	1 tenth;
$\frac{2}{10}$	"	"	.2,	"	2 tenths;
$\frac{3}{10}$	"	"	.3,	"	3 tenths;

and so on.

The second order to the right of the decimal point expresses *hundredths.* Thus,

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When a unit is divided into ten equal parts, what is each of the parts called? What is a Decimal Fraction? What is a decimal fraction usually called? How are decimal fractions usually written? What does the first order at the right of the point express? Second?

$\frac{1}{100}$ may be written .01, and read 1 hundredth;
$\frac{2}{100}$ " " .02, " 2 hundredths;
$\frac{3}{100}$ " " .03, " 3 hundredths;
and so on.

The third order to the right of the decimal point expresses *thousandths.* Thus,

$\frac{1}{1000}$ may be written .001, and read 1 thousandth;
$\frac{2}{1000}$ " " .002, " 2 thousandths;
$\frac{3}{1000}$ " " .003, " 3 thousandths;
and so on.

The fourth order to the right expresses *ten-thousandths;* the fifth *hundred-thousandths,* and so on. Hence, the following

GENERAL PRINCIPLES.

1. *The value expressed by decimal figures is determined by the place of each with reference to the decimal point.*

2. *The denominator of a decimal is understood to be* 1, *with as many ciphers annexed as there are orders in the decimal expression.*

3. *Ten of any lower order of decimals are always equal to one of the next higher.*

178. A *Mixed Number* may be a whole number and a decimal expressed together, with the decimal point between them. Thus,

5.34, read five *units* and thirty-four *hundredths,* is a mixed number.

179. A whole number may be regarded as a decimal by placing the decimal point on the right of the order of *units;* and the expression may be read, according to the decimal

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What does the third order at the right of the point express? Fourth? How is the value expressed by a decimal determined? What is the denominator of a decimal? How many of one decimal order make one of the next higher? How may a mixed number be expressed decimally? How may a whole number be regarded as a decimal?

places annexed, as a number of *tenths*, *hundredths*, etc. Thus,

16 may be written 16.0, 16.00, etc., and read 160 *tenths*, 1600 *hundredths*, etc.

180. The orders of decimals are named from the decimal point to the right,

Tenths, *hundredths*, *thousandths*, *ten-thousandths*, *hundred-thousandths*, *millionths*, etc., according to the following

Table.

Integers.							Decimals.					
Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
3	4	6	7	8	2	.	5	0	9	1	3	2,

where the mixed number is read, three hundred forty-six thousand seven hundred eighty-two *units*, and five hundred nine thousand one hundred thirty-two *millionths*.

CASE I.

181. To read decimals expressed by figures.

1. Let it be required to read the decimal .035.

SOLUTION. The figures express 0 *tenth*, 3 *hundredths*, and 5 *thousandths;* or, since 3 hundredths equal 30 thousandths, and 30 thousandths plus 5 thousandths, equal 35 thousandths, we have, as the value expressed, 35 thousandths.

Therefore, .035 is read *thirty-five thousandths.*

Name *the orders of* decimals. What number is expressed in the Table?

RULE. *Read the decimal as a whole number, giving it the name of the right-hand order.*

Examples.

Write and read the following decimals:

2. .65	6. .506	10. .55555
3. .175	7. .7571	11. .00067
4. .012	8. .9999	12. .063443
5. .003	9. .00006	13. .134567

182. When the decimal is part of a *mixed number,* we may *read the integral part as expressing units.* Thus,

16.45 may be read, sixteen *units,* and forty-five *hundredths.*

Write and read the following mixed numbers:—

14. 4.5	16. 16.0005	18. 56783.56
15. 30.03	17. 100.1586	19. 56.78356

CASE II.

183. To write decimals in figures.

1. Let it be required to express in figures thirteen thousandths.

SOLUTION. Since thirteen thousandths equal 1 *hundredth* and 3 *thousandths,* we write 3 in the order of thousandths, 1 in the order of hundredths, and, as there are no tenths, 0 in the order of tenths; and, prefixing the decimal point, we have .013.

Therefore, thirteen thousandths is expressed by .013.

RULE. *Write the decimal in the manner of a whole number, and place the decimal point so that each figure shall express its proper value.*

Write in figures the following:—

2. Four thousandths. *Ans.* .004.
3. One hundred sixty-three ten-thousandths.

What is the Rule for reading decimals? When the decimal is part of a *mixed number,* how may the integral part be read? What is the Rule for *writing decimals?*

4. Five hundred nine units, and five hundredths.
Ans. 509.05.

5. Nineteen units, and nineteen thousandths. *Ans.* 19.019.

6. One thousand eight hundred eight units, and one hundred sixty-one hundred-thousandths. *Ans.* 1808.00161.

7. Three units, and one hundred two millionths.
Ans. 3.000102.

8. Fifteen thousand nine hundred eighty-three ten-millionths. *Ans.* .0015983.

9. One million units, and eleven hundred-millionths.
Ans. 1000000.00000011.

Write in the decimal form,

10. $5\frac{73}{100}$. *Ans.* 5.73.	13. $1\frac{134}{1000}$.	16. $100\frac{301}{1000000}$.
11. $\frac{1623}{10000}$. *Ans.* .1623.	14. $\frac{1689101}{1000000}$.	17. $18651\frac{2}{100000}$.
12. $17\frac{44}{1000}$. *Ans.* 17.044.	15. $\frac{1001}{10000000}$.	18. $7861\frac{13}{1000000}$.

REDUCTION.

184. *Annexing a cipher to a decimal does not alter the value of the decimal.*

For, the order of the significant figures of the decimal is not changed. Thus, .3, or .30, is the same as $\frac{3}{10}$.

185. Hence, to change decimals having different denominators to equivalent decimals having a common denominator,

Make each decimal have the same number of decimal places, by annexing ciphers.

Exercises.

Change to equivalent fractions having a common denominator,

1. .8 and .0015.
Ans. .8000 and .0015.

2. .005 and .31.

3. .9, .06 and 1.634.
Ans. .900, .060, and 1.634.

4. 1.16 and 17.0016.
Ans. 1.1600, and 17.0016.

What effect upon the value expressed has the annexing a cipher to a decimal? How are decimals changed to equivalent decimals having a common denominator?

5. .13, .178, and .3367.

6. 1.5, 10.44, and $1.9\dot{5}656$.
Ans. 1.50000, 10.44000, 1.95656.

CASE I.

186. To reduce a decimal to a common fraction.

1. Let it be required to change .35 to a common fraction.

OPERATION.

$.35 = \frac{35}{100} = \frac{7}{20}$, *Ans.*

Removing the decimal point and writing the denominator (Art. 177), we have $\frac{35}{100}$, which reduced is $\frac{7}{20}$. Therefore, .35 is equal to $\frac{7}{20}$.

RULE. *Omit the decimal point, write the proper denominator, and reduce the common fraction to its smallest terms.*

Examples.

Reduce to common fractions:—

2. .495.	*Ans.* $\frac{99}{200}$.	5. 12.8.	*Ans.* $12\frac{4}{5}$.
3. .0075.	*Ans.* $\frac{3}{400}$.	6. 68.1875.	*Ans.* $68\frac{3}{16}$.
4. .375.	*Ans.* $\frac{3}{8}$.	7. 100.00125.	*Ans.* $100\frac{1}{800}$.

A decimal with a common fraction annexed is called a *Complex Decimal;* as,

$.66\frac{2}{3}$, read sixty-six and two-thirds hundredths.

8. Reduce $.43\frac{3}{4}$ to a common fraction.

OPERATION.

$$.43\tfrac{3}{4} = \frac{43\frac{3}{4}}{100} = \frac{\frac{175}{4}}{100} = \frac{175}{400} = \frac{7}{16}, \textit{ Ans.}$$

Reduce to a common fraction,

9. $.83\frac{1}{3}$. *Ans.* $\frac{5}{6}$.	11. $.09\frac{1}{3}$. *Ans.* $\frac{7}{75}$.	13. $.0078\frac{1}{8}$. *Ans.* $\frac{1}{128}$.
10. $.41\frac{2}{3}$. *Ans.* $\frac{5}{12}$.	12. $.022\frac{2}{9}$.	14. $.0156\frac{1}{4}$. *Ans.* $\frac{1}{64}$.

CASE II.

187. To reduce a common fraction to a decimal.

1. Let it be required to change $\frac{3}{4}$ to a decimal fraction.

What is the Rule for reducing a decimal to a common fraction? What is a decimal with a common fraction annexed called?

OPERATION.	
$4\,)\,3.00$ $\overline{.75}$, *Ans.*	$\frac{3}{4}$ equals $3 \div 4$. 3 equals 30 tenths, or 3.0; $\frac{1}{4}$ of 30 tenths is 7 tenths, or .7, with 2 tenths, equal 20 hundredths, remaining. $\frac{1}{4}$ of 20 hundredths is 5 hundredths, or .05. Therefore, $\frac{3}{4}$ is equal to .75.

RULE. *Annex ciphers to the numerator, divide by the denominator, and point off as many places for decimals as there were ciphers annexed.*

Examples.

Reduce to decimals: —

2. $\frac{3}{8}$.	*Ans.* .375.	7. $\frac{7}{800}$.	*Ans.* .00875.
3. $\frac{19}{20}$.	*Ans.* .95.	8. $\frac{1}{128}$.	*Ans.* .0078125.
4. $\frac{1}{400}$.	*Ans.* .0025.	9. $\frac{11}{40}$.	
5. $\frac{3}{250}$.	*Ans.* .012.	10. $\frac{1}{64}$.	*Ans.* .015625.
6. $\frac{5}{8}$.	*Ans.* .625.		

188. When the division does not terminate, it may be carried to a desirable degree of exactness, and the sign + annexed to the result to indicate its incompleteness, or the remainder annexed, as a part of a complex decimal.

11. Reduce $\frac{7}{12}$ to a decimal of four places. *Ans.* .5833+.
12. Reduce $\frac{4}{99}$ to a decimal of four places. *Ans.* .0404+.
13. Reduce $\frac{1}{41}$ to a decimal of six places. *Ans.* .024390+.
14. Reduce $\frac{1}{9}$ to a complex decimal of three decimal places. *Ans.* $.111\frac{1}{9}$.
15. Reduce $\frac{7}{75}$ to a complex decimal of four decimal places. *Ans.* $.0933\frac{1}{3}$.
16. Reduce $\frac{8}{11}$ to a complex decimal of six places. *Ans.* $.727272\frac{8}{11}$.

189. When decimals have figures which continually repeat, as in the answers to the last three examples, they are said to be *indeterminate*, and are called *Infinite* or *Circulating Decimals;*

What is the Rule for reducing a common fraction to an equivalent decimal? When the division does not terminate, how can you proceed? When are decimals said to be indeterminate? What are such decimals called?

and the repeating figures, as 1, 3, and 72, in the answers referred to, are called *Repetends*.

A repetend may be distinguished, when of only one figure, by placing a dot over it, and when of more than one figure, by placing a dot over the first and last. Thus,

$.111\frac{1}{9} = .\dot{1}$, read repetend one; $.0933\frac{1}{3} = .09\dot{3}$, read nine hundredths and repetend three; and $.727272\frac{8}{11} = .\dot{7}\dot{2}$, read repetend seventy-two.

Every repetend is equivalent to a common fraction of which the repetend is the numerator, and the denominator as many 9's as there are figures in the repetend. Thus,

$$.09\dot{3} = .09\tfrac{3}{9} = \frac{9\frac{3}{9}}{100} = \frac{7}{75}, \text{ and } .\dot{7}\dot{2} = \tfrac{72}{99} = \tfrac{8}{11}.$$

17. Reduce $\frac{31}{33}$ to an equivalent repetend. *Ans.* $.\dot{9}\dot{3}$.

18. Reduce $\frac{1}{41}$ to an equivalent repetend. *Ans.* $.\dot{0}243\dot{9}$.

19. Reduce $.\dot{0}243\dot{9}$ to an equivalent common fraction.

$$.\dot{0}243\dot{9} = \tfrac{2439}{99999} = \tfrac{1}{41}, \textit{ Ans.}$$

20. Reduce $.\dot{1}111\dot{5}$ to an equivalent common fraction.

Ans. $\frac{1235}{11111}$.

ADDITION.

190. Since ten of any order of decimals make one of the order next higher (Art. 177. 3), decimals may be added in the same manner as whole numbers. Hence, the

RULE. *Write the numbers so that figures of the same order shall stand in the same column; add as in whole numbers, observing to note the decimal in the amount by the decimal point.*

What are the repeating figures called? How may a repetend be distinguished? To what is every repetend equivalent? How may decimals be *added*? What is the Rule?

Examples.

(1.)	(2.)	(3.)
32.4056	11.275	3.73737
245.379	.34132	.873
12.0476	.00414	51.77778
9.38	.0001	108.2
459.2375	23.001	73.46313
758.4497	34.62156	238.05128

4. What is the sum of 450, 31.47, 376.004, 1.08, 456, .76, and .05? *Ans.* 1315.364.

5. What is the sum of 65.36, 8.125, 983, .465, 7.365, and 8.12345?

6. Add seventy-three units and twenty-nine hundredths, eighty-seven units and forty-seven thousandths, three thousand and five units and one hundred six ten-thousandths, twenty-eight units and three hundredths, twenty-nine thousand units and five thousandths. *Ans.* 32193.3826.

SUBTRACTION.

191. Since ten of any order of decimals make one of the order next higher, decimals may be subtracted in the same manner as whole numbers. Hence, the

Rule. *Write the less number under the greater, so that figures of the same order shall stand in the same column; subtract as in whole numbers, observing to note the decimal in the difference by the decimal point.*

Examples.

	(1)	(2)	(3)
From	6827.4681	2.4181	576.271
Take	6018.91	1.2234	89.7167
Ans.	808.5581	1.1947	486.5543

How may decimals be subtracted? What is the Rule?

Here, in Example 3, as there are no ten-thousandths in the minuend to subtract from, we consider that order in the minuend as filled by 0, since annexing a cipher to a decimal does not alter its value. (Art. 184.)

4. From 96.71 take 96.709. *Ans.* .001.

5. What is the difference between 107, and .0007?

6. Take eighty-five units and seven hundred thirty-seven thousandths from one hundred. *Ans.* 14.263.

7. Take one thousand four units and four millionths from two thousand units and sixteen hundredths. *Ans.* 996.159996.

MULTIPLICATION.

192. *Each removal of the decimal point one place toward the right multiplies by* 10.

For, each figure is made by the removal to denote units of an order next higher; hence the value expressed is made tenfold (Art. 30).

Thus, $.08 \times 10 = .8$; $.8 \times 10 = 8$; $.306 \times 100 = 30.6$.

193. *Each removal of the decimal point one place toward the left divides by* 10.

For, each figure is made by the removal to denote units of an order next lower; hence, the value expressed is made one tenth as much as it was.

Thus, $6.5 \div 10 = .65$; $.65 \div 10 = .065$; $73.4 \div 100 = .734$.

194. To multiply when one or both of the factors are decimals.

1. Let it be required to multiply .567 by 4.

OPERATION.

$$\begin{array}{r} .567 \\ 4 \\ \hline \end{array}$$

Ans. 2.268

4 times 567 thousandths is 2268 thousandths, which, reduced by dividing the numerator 2268 by the denominator 1000, by pointing off three places from the right, is 2.268.

Therefore, .567 multiplied by 4 is 2.268.

What effect has each removal of the decimal point one place toward the right? Why? What effect has each removal of the decimal point one place toward the left? Why?

2. Let it be required to multiply 3.16 by .4.

OPERATION.

$$\begin{array}{r} 3.16 \\ .4 \\ \hline 1.264 \end{array}$$

Ans. 1.264

4 times 3.16 is 12.64; but as the multiplier is .4, the product must be only a tenth as large, and 12.64 divided by 10, by removing the decimal point one place to the left (Art. 193), is 1.264.

Therefore, 3.16 multiplied by .4 is 1.264.

If we change the given decimals to the form of common fractions, and then multiply, we have

$$3.16 \times .4 = \tfrac{316}{100} \times \tfrac{4}{10} = \tfrac{1264}{1000} = 1.264.$$

In like manner, it may be shown, in every case, that

The number of decimal places in the product is equal to the number of decimal places in both of the factors.

RULE. *Multiply as in whole numbers, and point off as many decimal places in the product as there are decimal places in the multiplicand and multiplier, supplying the deficiency, if any, by prefixing ciphers.* Or,

If the multiplier is a decimal, multiply by its numerator, and divide by its denominator.

When the multiplier is 10, 100, 1000, etc., the multiplication may be performed by removing the point to the right as many places as there are ciphers on the right of the multiplier. (Art. 192.)

Examples.

Multiply

3. 12.375 by 1.25. *Ans.* 15.46875.
4. 8.5 by 83.7. *Ans.* 711.45.
5. .3785 by .003. *Ans.* .0011355.
6. .125 by .025. *Ans.* .003125.
7. 4.3125 by 100. *Ans.* 431.25.
8. 4.3125 by 1000.
9. 4.3125 by 10000. *Ans.* 43125.
10. Multiply one thousand by fifteen ten-thousandths.
11. Multiply one thousand by one thousandth. *Ans.* 1.

To what is the number of decimal places in the product equal? What is the Rule? How may the multiplication be performed when the multiplier is 10, 100, etc.?

12. Multiply two hundred thousand by three tenths.

13. Multiply three tenths by three hundredths. *Ans.* .009.

14. Multiply one by one hundred-thousandth. *Ans.* .00001.

15. Multiply two hundred forty-one ten-thousandths by one hundred sixty-five thousandths.

16. Multiply two thousand five hundred thirty-four millionths by three thousand two hundred fifty-six hundred-thousandths. *Ans.* .00008250704.

DIVISION.

195. To divide when the divisor, or dividend, or both, are decimals.

1. Let it be required to divide 2.268 by 4.

OPERATION.

$$4\,)\,2.268$$
$$.567$$

$\frac{1}{4}$ of 2268 thousandths is 567 thousandths, or .567.
Therefore, 2.268 divided by 4 is .567.

2. Let it be required to divide 1.264 by .4.

OPERATION.

$$.4\,)\,1.264$$
$$Ans.\ 3.16$$

1.264 divided by 4 is .316; but, as the divisor is .4, the quotient must be ten times as large, and .316 multiplied by 10, by removing the decimal point one place to the right (Art. 192), is 3.16.

Therefore, 1.264 divided by .4 is 3.16.

3. Let it be required to divide .00115 by .05.

OPERATION.

$$.05\,)\,.00115$$
$$Ans.\ .023$$

.00115 divided by 5 is .00023; but, as the divisor is .05, the quotient must be one hundred times as large, and .00023 multiplied by 100, by removing the point two places to the right (Art. 192), is .023.

If we change the given decimals to the form of common fractions, and then divide, we have

$$.00115 \div .05 = \frac{115}{100000} \div \frac{5}{100} = \frac{115 \times 100}{100000 \times 5} = \frac{23}{1000} = .023.$$

In like manner it may be shown in every case that

The number of decimal places in the quotient is equal to the excess of the number of decimal places in the dividend over that in the divisor.

Explain the operations.

RULE. *If the divisor is a whole number, divide as in whole numbers, and point off as many decimal places in the quotient as there are such places in the dividend.* Or,

If the divisor is not a whole number, divide by its numerator, and multiply by its denominator.

If the divisor is a decimal we may make it a whole number, by removing its decimal point a sufficient number of places to the right, and remove the decimal point in the dividend as many places to the right; then divide, and point off in the quotient as many decimal places as there are in the changed dividend.

For, multiplying both divisor and dividend by the same number will not change the value expressed by the quotient (Art. 73). Thus,

OPERATION.

05.) 00.115

Ans. .023

In working the third example, if we multiply the given decimals by 100, by removing the points two places to the right, the divisor becomes a whole number, 5, and the dividend 115 thousandths.

$\frac{1}{5}$ of 115 thousandths is 23 thousandths, or .023. Therefore, .00115 divided by .05 is .023.

When the divisor is 10, 100, 1000, etc., the division may be performed by removing the point to the left as many places as there are ciphers in the divisor (Art. 193).

Examples.

Divide

4. .7935 by 23. *Ans.* .0345.
5. .7935 by 2.3. *Ans.* .345.
6. .7935 by .23. *Ans.* 3.45.
7. 7.935 by .23.
8. 79.35 by .23.
9. 793.5 by 2.3. *Ans.* 345.
10. 711.45 by 8.5. *Ans.* 83.7.
11. .0011355 by .003. *Ans.* .3785.
12. .056875 by 6.5. *Ans.* .00875.
13. 987.5 by 100.
14. 987.5 by 1000.
15. 987.5 by 10000. *Ans.* .09875.
16. What is the quotient of 365.8 divided by .002?

To what is the number of decimal places in the quotient equal? What is the Rule when the divisor is a whole number? When it is not a whole number? How may it be made a whole number? How may we divide when the divisor is *10, 100, 1000*, etc.?

Here, by removing the decimal point three places to the right, in both the divisor and dividend, we have

$$365.8 \div .002 = 365800 \div 2 = 182900 \textit{ Ans.}$$

Divide

17. 8.05 by .0023. *Ans.* 3500.
18. 2.117 by .0073.
19. 17.28 by .0144. *Ans.* 1200.
20. 186.9 by 7.476.

196. When there is a remainder after dividing, the division can be continued by annexing decimal ciphers to the dividend.

If the division does not terminate, it may be carried to a desirable degree of exactness, and the sign + annexed to the result, to indicate its incompleteness.

Divide,

21. 2.5 by .32. *Ans.* 7.8125.
22. .97 by .8. *Ans.* 1.2125.
23. 37.4 by 4.5. *Ans.* 8.311+.
24. 7.7 by 128. *Ans.* .06015625.
25. 4 by .00255. *Ans.* 1568.627+.
26. 7.43 by .0079. *Ans.* 940.5063.

APPLICATIONS.

1. A ship sails in four days as follows:—the first day 197.025 miles, the second 211 miles, the third 163.175, and the fourth 150.65; how far did it sail in the four days? *Ans.* 721.85 miles.

2. The difference between A's money and B's is $7691.55, and A's money, which is the least of the two, is $1006.45; required, B's money. *Ans.* $8698.

3. Mr. Wade had in his farm 640 acres, but has sold off 221.125 acres. How many acres has he left? *Ans.* 418.875 acres.

4. What cost 17.75 tons of coal, at $4.54 a ton? 80.585

5. Such a quantity of bread was divided equally among 13 sailors, as allowed each sailor 1.236 pounds. How many pounds were divided? 16.068

When there is a remainder after dividing, how may the division be continued? How may you proceed when the division does not terminate?

6. What is the cost of 19.95 tons of hay at $20 a ton? *Ans.* $399.

7. What improper fraction is equivalent to the sum of 14.5 and .5, divided by their difference? *Ans.* $\frac{15}{14}$.

8. If the length of a year be taken at 365.25 days, instead of 365.242264, the true length, what will be the error in 400 years? *Ans.* 3.0944 days.

9. A man bought a farm, consisting of 75.8 acres, at $31.50 per acre, and sold it for $2274; how much did he lose per acre? *Ans.* $1.50.

10. According to the United States Coast Survey, a meter is 39.3685 inches, and allowing 63360 inches in a mile, how many meters are there in a mile? 16097+

11. A young man inherited a sum of money; after spending .375 of it in dissipation, and .25 of it in bad trades, he had $1500 left. How much did he inherit? *Ans.* $4000.

12. If a merchant purchases 650 barrels of flour for $4875, and sells it at $8.25 a barrel, how much does he gain on a barrel? *Ans.* $.75.

13. If a section of land is worth $6400, what is .875 of it worth?

14. What part of a section of land worth $6400 can be purchased for $5600? *Ans.* .875.

15. What is the value of 60.5 tons of coal, when .9 of a ton is worth $6.66? *Ans.* $447.70

16. If I expend $128.925 for corn at $.60 a bushel, and barley at $.75, in equal quantities, how many bushels of each do I get, and how much money is paid for each kind of the grain?

Review Questions. What is a Decimal Fraction? (176) How is a decimal usually written? (177) What is the denominator understood to be? (177) What is the Rule for reading decimals expressed by figures? (181) For writing decimals in figures? (183) What effect has the annexing of a cipher to a decimal? (184) How may decimals be changed to equivalent decimals *having a common* denominator? (185)

Exercises in Analysis.

1. At $4.50 per hundred, what cost 9634 pounds of fish?

SOLUTION. 9634 *reduced to hundreds by pointing off two places from the right is* 96.34 *hundreds. At* $4.50 *per hundred,* 96.34 *hundreds of fish must cost* 96.34 *times* $4.50, *or* $433.53. *Therefore, etc.*

2. At $25 per hundred, what cost 820 apple-trees?

3. What is the freight on 5670 pounds, at $1.20 per hundred?

4. What is the cost of 960 watermelons, at $12½ per hundred? *Ans.* $120.

5. At $22.40 per thousand, what cost 43750 feet of boards?

SOLUTION. 43750 *reduced to thousands by pointing off three places from the right is* 43.75 *thousands. At* $22.40 *per thousand,* 43.75 *thousand of boards must cost* 43.75 *times* $22.40 *or* $980. *Therefore, etc.*

6. At $6.50 per thousand, what cost 31684 bricks? *Ans.* $205.946.

7. How much will it cost to transport 53725 pounds of freight, at $1.14 per thousand?

8. What is the cost of 36500 feet of timber, at $40 a thousand; 5680 feet of plank, at $50 a thousand, and 16 thousands of shingles, at $5.25? *Ans.* $1828.

9. At $6.25 per ton of 2000 pounds, what cost 4480 pounds of coal?

SOLUTION. 4480 *pounds are* 4.48 *thousand pounds, and since* 2 *thousand pounds make a ton, there are half as many tons as there are thousand pounds, or* 2.24 *tons. At* $6.25 *per ton,* 2.24 *tons of coal will cost* 2.24 *times* $6.25, *or* $14. *Therefore, etc.*

10. At $21 per ton, what cost 2560 pounds of hay?

11. At $9.50 per ton, what cost 3248 pounds of plaster?

12. What is the freight on 96880 pounds of coal at $2.50 a ton? *Ans.* $121.10.

REVIEW QUESTIONS. What is the Rule for reducing a common fraction to an equivalent decimal? (187)

13.* A and B have together a certain sum of money, and A's share is to B's as 2 is to 3. How many hundredths is each man's share?

SOLUTION. *Since A's share is to B's as 2 to 3, if the money they have together be divided into 2 + 3, or 5, equal parts, 2 of these parts, or $\frac{2}{5}$ of the money, is A's share, and 3 of these parts, or $\frac{3}{5}$ of the money, is B's share. But $\frac{2}{5}$ is equal to .40, and $\frac{3}{5}$ to .60; hence, A's share is .40, and B's .60. Therefore, etc.*

14. The cost of one house is to that of another as 5 to 7; how many hundredths of the cost of the two is the cost of each?

Ans. .41$\frac{2}{3}$; .58$\frac{1}{3}$.

15. Three men own a ship together; their parts of it are to each other as 1, 2, and 5; what are their shares, expressed in hundredths? *Ans.* .12$\frac{1}{2}$, .25, and .62$\frac{1}{2}$.

16. Built a house and barn for $2476.10; the cost of the house was to that of the barn as $\frac{4}{5}$ to $\frac{2}{3}$; what was the cost of each?

SOLUTION. *Since the cost of the house was to that of the barn as $\frac{4}{5}$ to $\frac{2}{3}$, it was as $\frac{12}{15}$ to $\frac{10}{15}$, or as 12 to 10. If then the money they both cost be divided into 12 + 10, or 22, equal parts, 12 of these parts, or $\frac{12}{22}$ of $2476.10, which is $1350.60, must be the cost of house, and 10 of these parts, or $\frac{10}{22}$ of $2476.10, which is $1125.50, must be the cost of the barn. Therefore, etc.*

17. Divide 398.60 into two parts, which shall be to each other as .35 to .65. *Ans.* 139.51; 259.09.

18. In a certain union school having 475 pupils, the number of boys is to that of girls as 13 to 12; what is the number of each? *Ans.* 247 boys; 228 girls.

19. If gunpowder is composed of .76 parts of nitre, .14 of charcoal, and .10 of sulphur, how much of each of these will be required for 2000 pounds of powder?

Ans. nitre, 1520 pounds; charcoal, 280 pounds; sulphur, 200 pounds.

REVIEW QUESTIONS. What effect has the removal of the decimal point one *place to the right? (192)* One place to the left? (193)

* This page is optional.

WEIGHTS AND MEASURES.

197. Measure is that by which extension, capacity, force, duration, or value is estimated or determined.

198. Extension is that which has one or more of the dimensions of length, breadth, and hight or depth; as

199. A LINE, or that which has only length;

A SURFACE, or that which has only length and breadth;

A SOLID, Body, or Volume, or that which has length, breadth, and hight or depth.

The FACES of a solid or volume are its bounding surfaces.

200. Weight is the measure of the quantity of matter in a body, determined by the force by which it is naturally drawn toward the earth.

201. A **Unit of Measure** is some quantity used as a standard of comparison in measuring a quantity of the same kind.

COMMON WEIGHTS AND MEASURES.

202. Common Weights and Measures are those in general use.

TROY WEIGHT.

203. Troy Weight is used for weighing gold, silver, and jewels.

Table.

24 grains (gr.)	are	1 pennyweight,	pwt.
20 pennyweights,		1 ounce,	oz.
12 ounces,		1 pound,	lb.

What is Measure? A Line? A Surface? A Solid, Body or Volume? Weight? A Unit of Measure? The Common Weights and Measures? *For what is Troy* Weight used? Repeat the Table.

APOTHECARIES, in mixing medicines, use the *pound*, *ounce* (℥), and *grain*, of this weight; but divide the ounce into 8 *drams* (ʒ), each equal to three *scruples* (℈), each scruple being equal to 20 *grains*.

A *carat*, for gold-weight, is 4 grains; for diamond-weight, is 3.2 grains.

A pound Troy contains 240 pennyweights, or 5760 grains.

AVOIRDUPOIS WEIGHT.

204. Avoirdupois Weight is used for nearly all articles estimated by weight, except gold, silver, and jewels.

Table.

16 drams (dr.)	are	1 ounce,	oz.
16 ounces,		1 pound,	lb.
25 pounds,		1 quarter,	qr.
4 quarters, or 100 lb.,		1 hundred-weight,	cwt.
20 hundred-weight,		1 ton,	T.

Formerly, 112 pounds, or 4 quarters of 28 pounds each, were reckoned a hundred-weight, and 2240 pounds a ton, now called the long ton. This is now seldom employed in this country, except at the mines for coal, or at the United States Custom-houses for goods imported from Great Britain, in which country such weight continues to be used.

A pound Avoirdupois is equivalent to 7000 grains Troy, so that 144 pounds Avoirdupois are equal to 175 pounds Troy.

LINEAR MEASURE.

205. Linear or **Long Measure** is used in estimating distances and the length of articles.

Table.

12 inches (in.)	are	1 foot,	ft.
3 feet,		1 yard,	yd.
$5\frac{1}{2}$ yards,		1 rod,	rd.
40 rods,		1 furlong,	fur.
8 furlongs,		1 common mile,	m.

How do Apothecaries divide the ounce? What is a carat? What does a pound Troy contain? For what is Avoirdupois Weight used? Repeat the Table? How was a hundred-weight formerly reckoned? Where is the long ton now used? To what is a pound Avoirdupois equal? For what is Linear *Measure used? Repeat the* Table.

In measuring *cloth* and other woven fabrics, the linear yard is divided into *halves*, *quarters*, *eighths*, and *sixteenths*. Formerly a sixteenth of a yard, or $2\frac{1}{4}$ inches, was called a *nail*.

An *engineer's chain*, or *measuring tape*, is usually 100 feet in length, with each foot divided into tenths. Surveyors, however, make frequent use of *Gunter's chain*, which is 4 rods, or 66 feet, in length, and divided into 100 links of 7.92 inches each. Links are usually expressed as hundredths of a chain.

4 inches equal 1 *hand* in measuring the hight of horses directly over the fore feet; 6 feet equal 1 *fathom*, in measuring depths at sea; and 3 common miles equal 1 *league* on the land.

A *geographic* or *nautical mile* is $\frac{1}{60}$ of the length of a degree of latitude.

The United States coast survey employ 6086.34 feet, or 1.15+ common miles, as the average length of a nautical mile, and 69.16 common miles as the length of *a degree of longitude* on the equator.

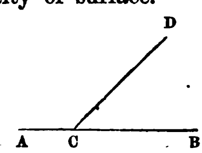
A mile is 320 rods = 1760 yards = 5280 feet = 63360 inches.

SURFACE MEASURE.

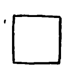
206. **Surface** or **Square Measure** is used in estimating surfaces.

207. The **Area** of a figure is its quantity of surface.

208. An **Angle** is the difference in direction of two straight lines, which meet at a point. Thus, A C D and D C B are angles.



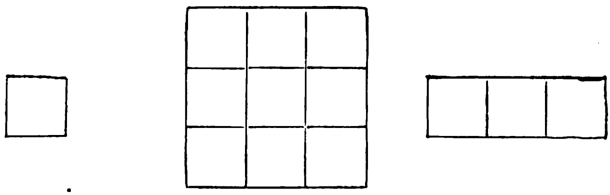
209. A **Square** is a surface having four equal sides and four equal angles.



The area of a square, each of whose sides is 1 foot, is 1 *square foot*.

How is the yard divided in measuring cloth? What is an engineer's chain, or measuring tape? Gunter's chain? A geographic or nautical mile? The length of a nautical mile? A degree of longitude on the equator? To what is a mile equal? For what is Surface Measure used? What is the Area of a figure? An Angle? A Square?

210. A **Rectangle** is any surface having four sides and four equal angles.



In the large square are represented 3 rows of small squares, of 3 squares each. If each of these small squares is supposed to be one square foot, then in the large square there evidently are in one of the equal rows, 3 times 1 square foot, or 3 square feet; and in the three rows, 3 times 3 square feet, or 9 square feet, or as many as the product of the number expressing the length by that expressing the breadth. Hence,

The area of a rectangle is equal to the product of the length by the breadth.

Table.

144 square inches (sq. in.)	are	1 square foot,	sq. ft.
9 square feet,		1 square yard,	sq. yd.
$30\frac{1}{4}$ square yards,		1 square rod or perch,	P.
160 square rods or perches,		1 acre,	A.
640 acres,		1 square mile,	M.

In surveying by Gunter's chain, 1 *square chain* is 16 square rods, and 10 square chains are 1 acre.

In measurement of government lands, 640 acres, or 1 square mile, make 1 *section of land.*

A *rood* equals 40 square rods or perches, but this denomination is not now much used.

An acre is 4840 square yards = 43560 square feet = 6272640 square inches.

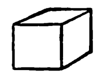
What is a Rectangle? To what is the area of a rectangle equal? Repeat the Table. How much is 1 square chain? 10 square chains? How many acres in one square mile or section of land? What does a rood equal? An acre?

SOLID MEASURE.

211. **Solid** or **Cubic Measure** is used in measuring bodies, or things having length, breadth, and hight or depth.

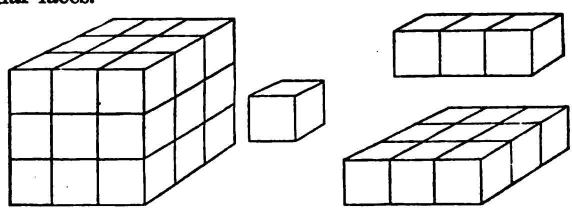
212. The **Contents** of a solid or volume **are the** number of times it contains a given unit of measure.

213. A **Cube** is a body bounded by six square and equal sides or faces.



If a cube has each of its equal faces 1 square foot, it is 1 solid or cubic foot.

214. A **Rectangular Solid** is a body bounded by rectangular faces.



In the large cube are represented 3 tiers of small cubes, of 3 rows each, and 3 in a row. If each of these small cubes is supposed to be one solid foot, then in the large cube there evidently are in one of the equal tiers 3 times three solid feet, or 9 solid feet, and in the 3 tiers, 3 times 9 solid feet, or 27 solid feet, or as many as the product of the numbers denoting the length, breadth, and hight or depth. Hence,

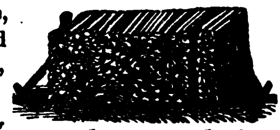
The contents of a rectangular solid are equal to the product of the length, breadth, and hight or depth.

Table.

1728 cubic inches (cu. in.)	are	1 cubic foot,	cu. ft.
27 cubic feet,		1 cubic yard,	cu. yd.
128 cubic feet,		1 cord,	C.

For what is Solid or Cubic Measure used? What is the Contents of a *solid or volume*? What is a Cube? A Rectangular Solid? To what is the *contents of a rectangular* solid equal? Repeat the Table.

A pile of wood 8 feet long, 4 feet wide, and 4 feet high, is a cord. A cord foot (c. f.) is 1 foot in length of this pile, or 16 cubic feet.



1 cord. 1 c. f.

A *perch* of masonry, or of building stone, is $24\frac{3}{4}$ cubic feet.

A ton of timber is commonly estimated at 40 solid feet, but as usually measured is 50 solid feet.

With transportation companies a ton of freight is quite variable, being for many articles estimated by the space occupied, and for others by weight.

A solid yard, or 27 solid feet, is equal to 46656 solid inches.

LIQUID MEASURE.

215. Liquid Measure is used in measuring all kinds of liquids.

Table.

4 gills (gi.)	are 1 pint,	pt.
2 pints,	1 quart,	qt.
4 quarts,	1 gallon,	gal.

A *barrel* (bar.), in some States is $31\frac{1}{2}$ gallons, and in others 28 to 32 gallons.

A *hogshead* (hhd.), when regarded as a measure, is 63 gallons; but the term is often applied to large casks of varying capacity.

Ale, beer, porter, and milk, were formerly sold by what was called *Beer Measure*, of which the gallon contained 282 cubic inches.

APOTHECARIES reckon one pint (O) equal to 16 fluid ounces (f. ℥); 1 fluid ounce equal to 8 fluid drams (f. ʒ).

The standard liquid gallon of the *United States* contains 231 cubic inches, and the Imperial gallon of *Great Britain*, 277.274 cubic inches.

A hogshead is 252 quarts, equal 504 pints, or 2016 gills.

What is a cord of wood? A perch of masonry? A ton of freight with transportation companies? A solid yard? For what is Liquid Measure used? Repeat the Table. What is a barrel? A hogshead? How many cubic inches in a gallon of Beer Measure? In the standard gallon of Liquid Measure? *What does* a hogshead equal?

DRY MEASURE.

216. Dry Measure is used in measuring such dry articles as grain, fruit, roots, coal, etc.

Table.

2 pints (pt.)	are	1 quart,	qt.
8 quarts,		1 peck,	pk.
4 pecks,		1 bushel,	bu.

The *chaldron*, a measure of 36 bushels, formerly employed with some kinds of coal, is now seldom used.

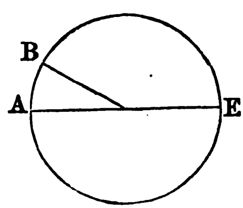
The standard bushel of the *United States* contains 2150.42 cubic inches; and the Imperial bushel of *Great Britain*, 2218.192 cubic inches.

CIRCULAR MEASURE.

217. Circular Measure is used for measuring angles, latitude and longitude, difference of direction, etc.

218. A **Circle** is a plane figure, bounded by a curved line, all the points of which are equally distant from a point within, called the *center*.

219. The **Circumference** of a circle is its entire bounding line.



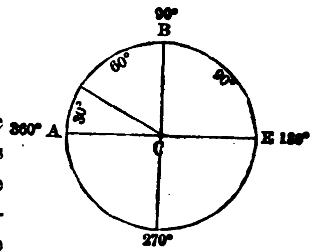
220. An **Arc** of a circle is any part of the circumference, as A B, or A B E.

221. A **Diameter** of a circle is any straight line drawn through the center, and terminated by the circumference, as A E.

222. The circumference of every circle, whatever, is supposed to be divided into 360 equal parts, called *degrees*.

For what is Dry Measure used? Repeat the Table? How many cubic inches does a standard bushel contain? An Imperial bushel? For what is Circular Measure used? What is a Circle? The Circumference of a circle? An arc? A diameter?

A Quadrant is a fourth of a circumference, or an arc of 90 degrees, as A B.



223. The ANGLE A C B, whose two lines meet at the center C, is measured by the arc of the circle drawn around that center, included between the opening of those lines. Thus, A C B is an angle of 90 degrees.

A *right angle* is an angle of 90 degrees.

Table.

60 seconds (″)	are	1 minute, ′
60 minutes,		1 degree, °
360 degrees,		1 circum., C.

A *sign*, used only in astronomy, is $\frac{1}{12}$ of the circumference of a circle, or 30°.

A *minute* of the earth's circumference is a geographic or nautical mile.

A degree is $\frac{1}{360}$ of the circumference of any circle, small or large.

A circumference of any circle has 21600 minutes, or 1296000 seconds.

TIME MEASURE.

224. Time is used in measuring portions of duration.

Table.

60 seconds (sec.) are	1 minute,	m.
60 minutes,	1 hour,	h.
24 hours,	1 day,	d.
365 days,	1 common year,	c. y.
366 days,	1 leap year,	l. y.

Also, 7 days make 1 *week*, 12 calendar months 1 *year*, and 100 years 1 *century*.

What is a Quadrant? A right angle? Repeat the Table. What is a sign? A minute? A degree of the earth's circumference? What does a circumference equal? What is time? Repeat the Table.

225. The Calendar Months, their names, order, and number of days, are as follows:

January,	1st mo.	31 days.	July,	7th mo.	31 days.
February,	2d "	28 or 29.	August,	8th "	31 "
March,	3d "	31 days.	September,	9th "	30 "
April,	4th "	30 "	October,	10th "	31 "
May,	5th "	31 "	November,	11th "	30 "
June,	6th "	30 "	December,	12th "	31 "

The number of days contained in each month may be remembered by recollecting that *the months are long and short alternately*, with the exception of August, which, as well as July, is *long*.

226. A *true year*, also called a *solar* or *tropical year*, is the exact time in which the earth makes a revolution around the sun, or 365 d. 5 h. 48 m. 49.7 sec.

227. The common year of 365 days comes short of the true year 5 h. 48 m. 49.7 sec., or 1 day, lacking only 44 m. 41.2 sec., in 4 years, so that an approximate correction of the calendar can be made by having every fourth year of 366 days.

But, by making every fourth year a leap year, there will be a gain in the calendar of 18 h. 37 m. 10 sec. in 100 years, or a little over 3 days in 400 years; hence, a second approximate correction can be made by having only every fourth of the centennial years a leap year. Hence,

Every year whose number can be divided by 4 without a remainder, except centennial years, and every centennial year whose number can be divided by 400 without a remainder, is a leap year and the others common years.

A common year has 8760 hours, equal 525600 minutes, or 31536000 seconds.

Give the Calendar Months, their names, order, and days. How may the number of days contained in each month be remembered? What is a true year? What years are leap years, and what years common years?

MISCELLANEOUS MEASURES.

228. Counting.

12 units	make	1 dozen.
12 dozen,		1 gross.
20 units,		1 score.
5 scores,		1 hundred.

229. Paper.

24 sheets	make	1 quire.
20 quires,		1 ream.
2 reams,		1 bundle.
5 bundles,		1 bale.

230. Capacity.

56 pounds of rye	are	1 bushel.
56 pounds of corn,		1 bushel.
60 pounds of wheat,		1 bushel.
60 pounds of beans,		1 bushel.
60 pounds of potatoes,		1 bushel.
60 pounds of clover-seed,		1 bushel.
100 pounds of fish,		1 quintal.
100 pounds of grain,		1 cental.
196 pounds of flour,		1 barrel.
200 pounds of beef or pork,		1 barrel.

Meal, either of *Indian corn* or *rye*, is usually estimated at 50 pounds to a bushel, and *wheat bran* at 20 pounds to a bushel.

A quarter of *grain*, in England, is equal to 8 Imperial bushels, or to 560 pounds.

Mental Exercises.

1. How many grains in 2 pennyweights? In 3 pennyweights? In 4 pennyweights?

2. In 40 pennyweights how many ounces?

3. How many ounces in 3 Troy pounds? In 12 Troy pounds?

4. How many ounces in 3 avoirdupois pounds?

5. In 60 hundred-weight how many tons? In 80 hundred-weight? In 100 hundred-weight?

6. How many yards in 2 rods? In 5 rods?

Repeat the Table of Counting. Of Paper. Of Capacity. How is meal estimated to a bushel? Wheat bran? Quarter of grain?

SOLUTION. *Since there are* $5\frac{1}{2}$ *yards in* 1 *rod, there are in* 2 *rods* 2 *times* $5\frac{1}{2}$ *yards;* 2 *times* 5 *yards are* 10 *yards, and* 2 *times* $\frac{1}{2}$ *a yard are* 1 *yard;* 10 *yards and* 1 *yard are* 11 *yards. Therefore, etc.*

7. In 30 feet how many yards? In 45 feet? In 48 feet?

8. How many square yards in 2 square rods? In 3 square rods?

9. In 54 cubic feet how many cubic yards?

10. In 10 gallons, how many quarts? In 15 gallons?

11. How many rods in 11 yards? In 22 yards?

SOLUTION. *Since there is in* $5\frac{1}{2}$ *yards, or in* 11 *half yards,* 1 *rod, there are as many rods in* 11 *yards, or* 22 *half yards, as* 11 *half yards are contained times in* 22 *half yards, which are* 2. *Therefore, etc.*

12. How many minutes in 3 degrees? In 5 degrees?

13. How many quarts in 7 pecks? In 10 pecks? In 12 pecks?

14. How many square rods in $\frac{3}{4}$ of an acre? In $\frac{3}{8}$ of an acre? In $\frac{5}{8}$ of an acre?

SOLUTION. *Since in* 1 *acre there are* 160 *square rods, in* $\frac{1}{4}$ *of an acre there must be* $\frac{1}{4}$ *of* 160 *square rods, or* 40 *square rods; and if* 1 *fourth is* 40 *square rods,* 3 *fourths must be* 3 *times* 40 *square rods, or* 120 *square rods. Therefore, etc.*

15. How many minutes in $\frac{3}{5}$ of an hour? In $\frac{3}{8}$ of an hour? In $\frac{7}{12}$ of an hour? In $\frac{3}{20}$ of an hour?

16. How many pounds in $\frac{3}{5}$ of a bushel of wheat? In $\frac{3}{4}$ of a bushel of corn? In $\frac{3}{10}$ of a barrel of beef?

17. What part of a furlong is 5 rods? Of a pound Troy is 8 ounces? Of a cubic yard is 18 cubic feet?

SOLUTION. *Since* 1 *furlong is* 40 *rods,* 5 *rods are* $\frac{5}{40} = \frac{1}{8}$ *of a furlong. Therefore, etc.*

18. What part of a hogshead are 15 gallons? Of a bushel are 24 quarts? Of a day is 18 hours?

19. What part of an acre is 120 rods? Of a bushel of wheat are 48 pounds? Of a barrel of beef are 60 pounds?

REVIEW QUESTIONS. What is Arithmetical Analysis? (75) What is a Rule? (11) What is a Formula? (69) What is a Solution? (10) An Operation? (8) An Answer? (9)

DECIMAL WEIGHTS AND MEASURES.

231. The **Metric System** of weights and measures, authorized by Congress, in 1866, to be used in the United States, is formed according to the decimal scale.

The HIGHER DENOMINATIONS of a weight or measure are expressed by prefixing to the name of its principal unit,

DEKA,	HECTO,	KILO,	MYRIA,
10,	100,	1000,	10000;

and the LOWER DENOMINATIONS by prefixing

DECI,	CENTI,	MILLI,
10th,	100th,	1000th.

MEASURES OF LENGTH.

232. The **Meter,** the principal unit for the measure of length, is very nearly one ten-millionth of the distance on the earth's surface from the equator to the pole.

Table.

10 *millimeters* (mm.) make	1 *centimeter* (cm.), equal to	.3937 inch.
10 centimeters,	1 decimeter,	3.937 inches.
10 decimeters,	1 METER (me.),	39.37 inches.
10 meters,	1 dekameter,	393.7 inches.
10 dekameters,	1 hectometer,	328 feet 1 inch.
10 hectometers,	1 *kilometer* (km.),	3280 feet 10 in.
10 kilometers,	1 myriameter,	6.2137 miles.

The *meter* is used as the unit of measure for all common lengths and distances. It is about 3 feet 3 inches, and 3 eighths of an inch in length.

The *kilometer* is taken as the unit in measuring long distances, as the length of roads, distances between cities, etc. It is about 200 rods, or ⅝ of a mile.

25 millimeters nearly replace the *inch*, 3 decimeters, the *foot*, 5 meters, the *rod*, and 1600 meters, the *mile*.

How is the Metric System formed? How are the Higher Denominations of each weight or measure expressed? The Lower Denominations? To what is the Meter equal? Repeat the Table. About how much is a Meter?

MEASURES OF SURFACE.

233. The **Square Meter,** the principal unit for the measure of surface, is the square whose side is one meter.

Table.

100 *sq. millimeters* ($mm.^2$),	are	1 *sq. centimeter* ($cm.^2$),	= .00155 sq. in.
100 sq. centimeters,		1 sq. decimeter,	.1076 sq. ft.
100 sq. decimeters,		1 SQ. METER ($m.^2$),	1.196 sq. yd.

Since the side of a square meter is 1 meter, or 10 decimeters, a square meter is equal to $10 \times 10 = 100$ square decimeters; since the side of a square decimeter is 1 decimeter, or 10 centimeters, a square decimeter is equal to $10 \times 10 = 100$ square centimeters, etc. Hence,

The scale is 100, and two orders of figures must be allowed to each denomination.

234. The **Are,** the principal unit in measuring land, is a square whose side is ten meters.

Table.

100 centiares are	1 ARE (ar.), equal to	119.6 sq. yd.
100 ares,	1 *hectare* (ha.),	2.471 acres.

A *centiare*, or square meter, is about $1\frac{1}{5}$ square yards, and a *hectare* about $2\frac{1}{2}$ acres.

40 ares nearly replace an *acre* of common surface measure.

MEASURES OF VOLUME.

235. The **Cubic Meter,** the principal unit for the measure of volume, is the cube, whose edge is one meter.

Table.

1000 *cu. millimeters* ($mm.^3$),	are	1 *cu. centimeter* ($cm.^3$),	= .061 cu. in.
1000 cu. centimeters,		1 cu. decimeter,	61.022 cu. in.
1000 cu. decimeters,		1 CU. METER ($m.^3$),	1.308 cu. yd.

To what is the square Meter equal? Repeat the Table. What is the *Are*? Repeat the Table. How much is a centiare? A hectare? What *nearly replaces the acre*? What is the cubic Meter? Repeat the Table.

Since the edge of a cubic meter is 1 meter, or 10 decimeters, a cubic meter is equal to $10 \times 10 \times 10 = 1000$ cubic decimeters; since the edge of a cubic decimeter is one decimeter, or 10 centimeters, a cubic decimeter is equal to $10 \times 10 \times 10 = 1000$ cubic centimeters, etc. Hence,

The scale is 1000, and three orders of figures must be allowed to each denomination.

236. The **Liter,** the principal unit for liquid or dry measure, is a cubic decimeter.

Table.

10 milliliters	are 1 *centiliter* (cl.), equal to	.338 fluid oz.
10 centiliters,	1 deciliter,	.845 gill.
10 deciliters,	1 LITER (lt.),	1.0567 quarts.
10 liters,	1 dekaliter,	2.6417 gallons.
10 dekaliters,	1 *hectoliter* (hl.)	26.417 gallons.
10 hectoliters,	1 kiloliter,	264.17 gallons.

The *liter* is used in measuring liquids, and is about $1\frac{1}{18}$ liquid quart.

The *hectoliter* is used in measuring grains and like articles, and is 2.837 bushels, or about $2\frac{5}{6}$ bushels, or $\frac{5}{6}$ of a barrel; a liter is very nearly .908 of a dry quart.

4 liters a little more than replace the *liquid gallon*, and 35 liters very nearly the *common bushel.*

A *milliliter* is equal to 1 cubic centimeter; a *centiliter* is equal to 10 cubic centimeters.

237. The **Stere,** the principal unit for measuring wood, is a cubic meter, or 1000 liters.

Table.

10 decisteres	are 1 STERE (st.), equal to	1.308 cubic yards.
10 steres,	1 dekastere,	13.08 cubic yards.

36 decisteres, or 3.6 steres, very nearly replace the common *cord.*

What is the Liter? Repeat the Table. For what is the liter used? About how much is a liter? For what is a hectoliter used? About how much is a hectoliter? What nearly replaces the liquid gallon? The common bushel? What is the Stere? Repeat the Table. What very nearly replaces the common cord?

WEIGHTS.

238. The **Gram,** the principal unit of weights, is the weight, in a vacuum, of a cubic centimeter of distilled water, at its greatest density.

Table.

10 *milligrams*	are 1 centigram,	equal to	.1543	grains.
10 centigrams,	1 decigram,		1.543	"
10 decigrams,	1 GRAM (gm.),		15.432	"
10 grams,	1 dekagram,		.3527	av. oz.
10 dekagrams,	1 hectogram,		3.5274	"
10 hectograms,	1 *kilogram* (k.),		2.2046	av. lb.
10 kilograms,	1 myriagram,		22.046	"
10 myriagrams,	1 quintal,		220.46	"
10 quintals,	1 millier, or *tonneau* (t.),		2204.6	"

The *kilogram*, or, for brevity, *kilo*, is the ordinary weight of commerce. It is about $2\frac{1}{5}$ pounds.

The *tonneau* (pronounced *tonno*), or *metric ton*, is used in weighing heavy articles, and is about 2200 pounds.

The *gram* is used in mixing medicines, weighing letters, gold, jewels, etc. 28 grams nearly replace an avoirdupois ounce; and $\frac{1}{2}$ kilo, a little more than a pound.

239. In expressing Metric Weights and Measures, by figures, the decimal point, as in United States Money, is placed between the unit, and its subdivisions written as decimal orders.

One, two, or three orders of figures must be allowed to each denomination lower than the unit, according as the scale is 10, 100, or 1000. Thus,

3 kiloliters, 7 hectoliters, 2 dekaliters, 5 liters, 6 centiliters, is written, as liters, 3725.06 lt.

4 cubic meters, 630 cubic centimeters, as cubic meters, 4.00063 m^3.

240. The *integer* of a metrical expression may be read as a number of its primary unit; and the *decimal part*, if any, as a number of the lowest denomination denoted. Thus,

---

What is the Gram? Repeat the Table. What is the kilogram called for brevity? About how much is a kilo? A tonneau? What very nearly replaces the avoirdupois ounce? The pound? How is the decimal point placed?

360.075 kilos may be read as three hundred sixty kilos, and seventy-five grams.

36.15 meters, as thirty-six meters, and fifteen centimeters.

Comparative Table.

A meter	=	39.37 inches.	An inch	=	.0254 meter.
A meter	=	3.28 feet.	A foot	=	.3048 meter.
A meter	=	1.0936 yards.	A yard	=	.9144 meter.
A kilometer	=	.62137 mile.	A mile	=	1.6093 kilometers.
A sq. meter	=	1550 sq. inches.	A sq. inch	=	.0006452 sq. meter.
A sq. meter	=	10.76 sq. feet.	A sq. foot	=	.0929 sq. meter.
A sq. meter	=	1.196 sq. yards.	A sq. yard	=	.8361 sq. meter.
An are	=	3.953 sq. rods.	A sq. rod	=	.2529 are.
A hectare	=	2.471 acres.	An acre	=	.4047 hectare.
A hectare	=	.00386 sq. mile.	A sq. mile	=	259 hectares.
A liter	=	33.81 fluid oz.	A fluid oz.	=	.02958 liter.
A liter	=	1.0567 quarts.	A quart	=	.9465 liter.
A liter	=	.26417 gallon.	A gallon	=	3.786 liters.
A hectoliter	=	2.837 bushels.	A bushel	=	.3524 hectoliter.
A liter	=	61.022 cu. inches.	A cu. inch	=	.01639 liter.
A hectoliter	=	3.531 cu. feet.	A cu. foot	=	.2832 hectoliter.
A stere	=	1.308 cu. yards.	A cu. yard	=	.7646 stere.
A stere	=	.2759 cord.	A cord	=	3.625 steres.
A gram	=	15.432 grains.	A grain	=	.0648 gram.
A kilogram	=	35.27 av. ounces.	An av. oz.	=	.0283 kilogram.
A kilogram	=	2.68 Tr. pounds.	A Troy lb.	=	.373 kilogram.
A kilogram	=	2.2046 av. pounds.	An av. lb.	=	.4536 kilogram.
A tonneau	=	1.1023 tons.	A ton	=	.9071 tonneau.

These equivalents are only determined approximately; but are sufficiently exact for all ordinary business.

How many inches make a meter? How many miles a kilometer? Square inches a square meter? Square rods an are? Acres a hectare? Quarts a liter? Bushels a hectoliter? Cords a stere? Grains a gram? Pounds Troy a kilo? Avoirdupois pounds a kilo? Tons a tonneau? Meters an inch? Kilometers a mile? Square meters a square yard? Ares a square rod? Hectares an acre? Liters a quart? Hectoliters a bushel? Steres a cord? Grams a grain? Kilograms a pound Troy? Kilograms an avoirdupois pound? Tonneaux a ton?

Exercises.

Read

1. 1686.45 me.
2. 537.125 k.
3. 634.56 t.
4. 76.7 st.
5. 100.56 ar.
6. 2.006 m^2.
7. 517.5 hl.
8. 3.1605 ha.
9. 15.1005 m^3.
10. 678.06 lt.

11. Write in figures thirty-one hectares and fifteen ares, as hectares.

12. Write in figures seventeen and five tenths steres.

13. Write in figures sixteen hundred meters and twenty-five centimeters.

14. Express by figures one hundred thirty-two cubic meters, and seven thousand three cubic centimeters.

15. Reduce 8.07018 kilometers to meters.

SOLUTION. *Since in* 1 *kilometer there are* 1000 *meters, in* 8.07018 *kilometers there must be* 8.07018 *times* 1000 *meters, or* 8070.18 *meters.*

16. Reduce 16.85 grams to kilograms.

SOLUTION. *Since in* 1000 *grams there is* 1 *kilogram, there are as many kilograms in* 16.85 *grams as* 1000 *grams are contained times in* 16.85 *grams, or* .01685 *kilogram.*

That is, to change from a number of one denomination to an equivalent number of another denomination of the same weight or measure, we may

Remove the decimal point one or more places to the right or left, as the case may require, supplying ciphers if necessary.

17. In 560.5 hectares, how many square meters?

18. In 5605000 square meters, how many hectares?

19. In .0605 liters, how many centiliters?

20. In 6.05 centiliters, how many liters?

21. Express .01531 tonneau as kilograms.

22. Express as millimeters and add, 31.55 meters, 9.005 meters, and 75 millimeters. *Ans.* 40.63mm.

23. From 56 hectares take 156 ares. *Ans.* 54.44 ha

How may we change from one denomination to another of the same *weight or measure?*

COMPARISON OF UNITS.

241. The units of the metric and common systems may be readily compared by means of the Comparative Table (Art. 240).

1. In 125 meters how many feet?

SOLUTION. *Since in* 1 *meter there are* 3.28 *feet, there will be in* 125 *meters* 125 *times* 3.28 *feet, or* 410 *feet.*

Or, *since in* 1 *foot there are* .3048 *meter, there will be as many feet in* 125 *meters, as* .3048 *meter are contained times in* 125 *meters, or* 410 *feet.*

2. In 150 acres how many hectares?
3. In 2.5 hectoliters how many bushels? *Ans.* 7.09+ bu.
4. In 15 cords how many steres? *Ans.* 54.375 steres.
5. Reduce 16.25 kilos to pounds avoirdupois.
6. How many tonneaux in 100 tons? *Ans.* 90.71 tonneaux.

CONVERSION OF PRICES OR RATES.

242. When the price or rate per unit is given either in the common or in the metric system, it may be readily found for a corresponding unit in the other system, since

The prices or rates must have the same relation to each other as have the magnitudes of the given units.

1. If the price per kilogram is $.80 cents, what is it per pound?

SOLUTION. *If the price per kilogram is* $.80, *it must be per pound, which is* .4536 *times a kilogram,* .4536 *times* $.80, *or* $.36+.

2. The price per liter being $1, what is it per quart? *Ans.* $.946+.

3. The produce per hectare being 10 hectoliters, what is it per acre? *Ans.* 4.047 hectoliters.

4. The produce per acre being 45 bushels, what is it per hectare? *Ans.* 111.195 bushels.

How may the units of the metric and common systems be compared? To what does the *price or rate of* units in the different systems correspond?

DENOMINATE NUMBERS.

243. A **Denominate Number** is a concrete number expressing weight, measure, or currency. Thus,

5 rods, 3 pounds, 2 ounces, 9 dollars, are each denominate numbers.

A SIMPLE DENOMINATE NUMBER expresses only units of a single denomination of a weight, measure, etc.

A COMPOUND DENOMINATE NUMBER expresses units of two or more denominations of the same kind of weight, measure, etc. Thus,

5 rods 3 yards, 8 pounds 6 ounces, 7 dollars 3 dimes 2 cents, are each compound denominate numbers.

REDUCTION.

244. Reduction of Denominate Numbers is the process of changing their denomination, without changing their value.

CASE I.

245. To reduce from a higher to a lower denomination.

1. Let it be required to reduce 63 lb. 0 oz. 10 pwt., to pennyweights.

OPERATION.

```
  63 lb. 0 oz. 10 pwt.
    12
  ----
  126
  63
  ----
  756 oz.
    20
  -----
  15120
     10
  -----
  15130 pwt., Ans.
```

Since in 1 pound there are 12 ounces, in 63 pounds there are 63 times 12 ounces, or 756 ounces.

Since in 1 ounce there are 20 pennyweights, in 756 ounces there are 756 times 20 pennyweights; and 10 pennyweights added, make 15130 pennyweights. Therefore, etc.

What is a Denominate Number? A Simple Denominate Number? A *Compound* Denominate Number? Reduction of Denominate Numbers?

RULE. *Multiply the number of the highest denomination given by the number required of the next lower denomination to make one of that higher, and to the product add the number, if any, of the lower denomination.*

Proceed in like manner till the whole is reduced to the required denomination.

Examples.

Reduce

2. 3 T. 15 cwt. to pounds.*
3. 2 lb. 8 oz. to drams.
4. 9 miles to rods.
5. 98 gal. 1 pt. to pints.
6. 583 A. 130 P. to square rods.
7. 17 cords to cubic inches.
8. 27 bu. 3 pk. to pints.
9. $5° \; 6' \; 15''$ to seconds.
10. In 5 fur. 12 rd. 4 yd. how many feet?
11. In 365 d. 5 h. 48 m. 50 sec. how many seconds?
12. In 24 hhd. 18 gal. 2 qt. how many pints?
13. In 17 m. 6 fur. 22 rd. 4 yd. 2 ft. 7 in. how many inches?
14. In 75 bu. 3 pk. 5 qt. how many quarts?
15. In 4 cwt. 99 lb. 10 oz. 12 dr. how many drams?
16. In 180 degrees how many seconds?

246. The rule also applies when the denominate number is a *common fraction* or *decimal.*

17. Reduce $\frac{3}{7}$ of a gallon to pints.

OPERATION.

$$\tfrac{3}{7} \times 4 = \tfrac{12}{7} \text{ qt.}; \quad \tfrac{12}{7} \times 2 = \tfrac{24}{7} = 3\tfrac{3}{7} \text{ pt. } Ans.$$

Since in 1 gallon there are 4 quarts, in $\frac{3}{7}$ of a gallon there are $\frac{3}{7}$ of 4 quarts, or $\frac{12}{7}$ quarts.

Since in 1 quart there are 2 pints, in $\frac{12}{7}$ quarts there are $\frac{12}{7}$ of 2 pints, or $\frac{24}{7} = 3\frac{3}{7}$ pints.

18. Reduce $\frac{7}{16}$ of an acre to square rods?
19. In $\frac{3}{8}$ of a yard how many quarters?
20. In $9\frac{3}{8}$ days how many minutes?
21. What part of a pennyweight is $\frac{7}{1920}$ of a pound?

What is the Rule?

* For answers, see corresponding examples in Case II.

22. Express $\frac{3}{350}$ of a cwt. in terms of a pound.

23. Reduce .145 cwt. to ounces.

OPERATION.

$$.145 \times 100 = 14.5 \text{ lb.};\ 14.5 \times 16 = 232 \text{ oz. } Ans.$$

24. In .0003 weeks how many minutes?

25. In 6.35 miles how many feet?

26. In .1756 kilometers how many meters?

27. What part of a quart is .0015 of a hogshead?

28. How many mills in $15.69?

29. How many kilograms in 3.675 tonneaux?

30. How many square rods in .94375 of an acre?

CASE II.

247. To reduce from a lower to a higher denomination.

1. Let it be required to reduce 15130 pennyweights to pounds.

OPERATION.

$$\begin{array}{r} 2|0\,)\,1513|0 \text{ pwt.} \\ \hline 12\,)\,756\,..\,10 \text{ pwt.} \\ \hline 63 \text{ lb.} \end{array}$$

Ans. 63 lb. 0 oz. 10 pwt.

Since in 20 pennyweights there is 1 ounce, in 15130 pennyweights there are as many ounces as 20 pennyweights are contained times in 15130 pennyweights, which are 756 times, and a remainder of 10 pennyweights.

Since in 12 ounces there is 1 pound, in 756 ounces there are as many pounds as 12 ounces are contained times in 756 ounces, or 63 times.

RULE. *Divide the given number by the number of its denomination required to make one of the next higher, and reserve the remainder, if any.*

Proceed in like manner with the quotient, and so continue till the whole is reduced to the required denomination.

The number of the required denomination, with the several remainders, if any, will be the answer.

REVIEW QUESTIONS. What is a Denominate Number? (243) A Simple Denominate Number? (243) A Compound Denominate Number? (243) A Measure? (197) A Weight? (200) Explain the operation. Repeat the Rule.

Examples.

Reduce

2. 7500 pounds to tons.	6. 93410 square rods to acres.
3. 640 drams to pounds.	7. 3760128 cubic in. to cords.
4. 2880 rods to miles.	8. 1776 pints to bushels.
5. 785 pints to gallons.	9. 18375 seconds to degrees.

10. In 3510 feet how many furlongs?

OPERATION.

$$\begin{array}{l} 3\,)\,3510,\ \text{ft.} \\ 5\tfrac{1}{2}\,)\,\overline{1170},\ \text{yd.} \\ \underline{\ 2\ }\quad\ \ \underline{\ 2\ } \\ 11\quad 2340,\ \ \text{hlf. yd.} \\ \quad 40\,)\,\overline{212}\ ..\ 8\ \text{hlf. yd.} = 4\ \text{yd.} \\ \qquad\quad 5\ ..\ 12\ \text{rd.} \end{array}$$

Ans. 5 fur. 12 rd. 4 yd.

Here, to divide 1170 by $5\frac{1}{2}$, we first reduce both numbers to halves, by multiplying by 2, and have 2340 *halves* to be divided by 11 halves, which gives 212, and a remainder of 8 halves = 4.

Reduce

11. 31556930 seconds to days.	14. 2429 quarts to bushels.
12. 12244 pints to hogsheads.	15. 127916 drams to cwt.
13. 1129171 inches to miles.	16. 648000 seconds to degrees.

17. Reduce $3\frac{3}{7}$ pints to the fraction of a gallon.

OPERATION.

$$3\tfrac{3}{7} = \tfrac{24}{7};\ \tfrac{24}{7} \div 2 = \tfrac{12}{7}\ \text{qt.};\ \tfrac{12}{7} \div 4 = \tfrac{3}{7}\ \text{gal. } Ans.$$

Since in 1 quart there are 2 pints, in $3\frac{3}{7} = \frac{24}{7}$ pints there are as many quarts as 2 pints are contained times in $\frac{24}{7}$ pints, which are $\frac{12}{7}$ times.

Since, in 1 gallon there are 4 quarts, in $\frac{12}{7}$ quarts there are as many gallons as 4 quarts are contained times in $\frac{12}{7}$, which is $\frac{3}{7}$ of a time.

18. Reduce 70 square rods to a fraction of an acre.

19. In $1\frac{1}{3}$ quarters how many yards?

REVIEW QUESTIONS. Repeat the Table of Troy Weight. (203) The Table of Avoirdupois Weight. (204) Of Linear Measure. (205) Of Surface Measure. (210) What is a Surface? (206) The area of a figure? (207) A Square? (209) A Rectangle? (210)

20. In 13920 minutes how many days?
21. What part of a pound is $\frac{7}{8}$ of a pennyweight?
22. Express $\frac{6}{7}$ of a pound in terms of a hundred-weight.
23. Reduce 232 ounces to a decimal of a hundred-weight.

OPERATION.

$232 \div 16 = 14.5$ lb.; $14.5 \div 100 = .145$ cwt. *Ans*

24. In 3.024 minutes how many weeks?
25. In 33528 feet how many miles?
26. In 175.6 meters how many kilometers?
27. What part of a hogshead is .378 of a quart?
28. How many dollars in 15690 mills?
29. How many tonneaux in 3675 kilograms?
30. How many square acres in 151 square rods.

CASE III.

248. To reduce a fraction of a given denomination to integers of lower denominations.

1. Let it be required to reduce $\frac{8}{9}$ of a pound Troy to equivalent integers.

OPERATION.

$\frac{8}{9} \times 12 = \frac{96}{9} = 10\frac{2}{3}$ oz.
$\frac{2}{3} \times 20 = \frac{40}{3} = 13\frac{1}{3}$ pwt.
$\frac{1}{3} \times 24 = \frac{24}{3} = 8$ gr.
Ans. 10 oz. 13 pwt. 8 gr.

Since 1 pound equals 12 ounces, $\frac{8}{9}$ of a pound equals $\frac{8}{9}$ of 12 ounces, or $10\frac{2}{3}$ ounces.

Since 1 ounce equals 20 pennyweights, $\frac{2}{3}$ of an ounce equals $\frac{2}{3}$ of 20 pennyweights, or $13\frac{1}{3}$ pennyweights.

Since 1 pennyweight equals 24 grains, $\frac{1}{3}$ of a pennyweight equals $\frac{1}{3}$ of 24 grains, or 8 grains.

Therefore, $\frac{8}{9}$ of a pound Troy is equal to 10 oz. 13 pwt. 8 gr.

RULE. *Multiply the fraction by the number required of the next lower denomination to make one of its denomination, and reduce the result, if possible, to a whole or mixed number.*

Proceed in like manner with the fractional parts, if any, and so continue till reduced as required.

Explain the operation. Repeat the Rule.

Examples.

2. Reduce $\frac{8}{9}$ of a furlong to equivalent integers.*
3. Reduce $\frac{1}{96}$ of a hogshead to equivalent integers.
4. Express in integers $\frac{19}{42}$ of a week.
5. Find the value of $\frac{5}{14}$ of a degree.
6. Find the value of $\frac{17}{24}$ of a hundred-weight.
7. Reduce .282 of a ton to equivalent integers.

FIRST OPERATION.

$.282 \times 20 = 5.64$ cwt.

$.64 \times 4 = 2.56$ qr.

$.56 \times 25 = 14$ lb.

Ans. 5 cwt. 2 qr. 14 lb.

SECOND OPERATION.

$$\begin{array}{rr} & .282 \\ & 20 \\ \hline \text{cwt.} & 5.640 \\ & 4 \\ \hline \text{qr.} & 2.560 \\ & 25 \\ \hline & 2.800 \\ & 11.20 \\ \hline \text{lb.} & 14.000 \end{array}$$

It is often most convenient to multiply the decimal part, without re-writing it, as in the second operation.

8. Reduce .875 of a hogshead to equivalent integers.
9. Reduce .4765625 of a mile to equivalent integers.
10. What is the value of .09375 of an acre?
11. Reduce 5.141 tons to equivalent integers.
12. What is the value of .761 of a day?

CASE IV.

249. To reduce integers of lower denominations to a fraction of a higher denomination.

1. Let it be required to reduce 10 oz. 13 pwt. 8 gr. to an equivalent fraction of a pound.

OPERATION.

8 gr. $= \frac{8}{24}$ pwt. $= \frac{1}{3}$ pwt.

$13\frac{1}{3}$ pwt. $= \frac{40}{3}$ pwt. $= \frac{40}{3} \div 20 = \frac{2}{3}$ oz.

$10\frac{2}{3}$ oz. $= \frac{32}{3}$ oz. $= \frac{32}{3} \div 12 = \frac{8}{9}$ lb.

Since 24 grains equal 1 pennyweight, 8 grains equal $\frac{8}{24}$, or $\frac{1}{3}$, of a pennyweight. Since 20 penny-

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Explain the operation of example 1.

* For answers, see corresponding examples in Case IV.

weights equal 1 ounce, $\frac{40}{3}$ pennyweights equal $\frac{1}{20}$ of $\frac{40}{3}$, or $\frac{2}{3}$ of an ounce.

Since 12 ounces equal 1 pound, $\frac{32}{3}$ ounces equal $\frac{1}{12}$ of $\frac{32}{3}$, or $\frac{8}{9}$ of a pound.

RULE. *Reduce the number of the lowest denomination to a fraction of the denomination next higher, and write it as a fractional part of the number of that higher denomination. Reduce the number thus formed, in like manner as before, and so continue till all the numbers are reduced as required.*

Examples.

2. Reduce 35 rd. 3 yd. 2 in. to an equivalent fraction of a furlong.

3. Reduce 2 qt. 1 pt. 1 gi. to an equivalent fraction of a hogshead.

4. Reduce 3 d. 4 h. to an equivalent fraction of a week.

5. Reduce 21′ $25\frac{5}{7}$″ to an equivalent fraction of a degree.

6. Reduce 2 qr. 20 lb. 13 oz. $5\frac{1}{3}$ dr. to an equivalent fraction of a hundred-weight.

7. Reduce 5 cwt. 2 qr. 14 lb. to an equivalent decimal of a ton.

FIRST OPERATION.	SECOND OPERATION.
14 lb. $= 14 \div 25 = .56$ qr.	25) 14.00 lb.
2.56 qr. $= 2.56 \div 4 = .64$ cwt.	4) 2.56 qr.
5.64 cwt. $= 5.64 \div 20 = .282$ T. *Ans.*	20) 5.640 cwt.
	.282 T. *Ans.*

The second operation, which is an abridged form of the first, is generally preferred, on account of its conciseness.

8. Reduce 55 gal. 0 qt. 1 pt. to an equivalent decimal of a hogshead.

9. Reduce 3 fur. 32 rd. 8 ft. 3 in. to an equivalent decimal of a mile.

What is the Rule? Explain the operation.

10. Reduce 15 square rods to an equivalent decimal of an acre.

11. Express 5 T. 2 cwt. 3 qr. 7 lb. as a mixed decimal of a ton.

12. What is the value of 18 h. 15 m. 50.4 sec. in the decimal of a day?

250. When it is required to find the part that one compound denominate number is of another (Art. 174),

Reduce the numbers to the same denomination, and divide the number denoting the part by that with which it is compared.

13. What part of 1 lb. 4 oz. 12 pwt. 12 gr. is 2 oz. 15 pwt. 10 gr.? *Ans.* $\frac{1}{6}$.

14. What part of 7 bu. 1 pk. is 2 qt. 1 pt.? *Ans.* $\frac{5}{464}$.

15. What part of 3 acres is 1 A. 26 P.? *Ans.* $\frac{31}{80}$.

16. What part of 1 T. 6 cwt. 15 lb. 10 oz. is 10 cwt. 46 lb. 4 oz.? *Ans.* $\frac{2}{5}$.

17. What decimal of 148 m. 4 fur. is 18 m. 4 fur. 20 rd.? *Ans.* .125.

18. What decimal of 7 w. 4 d. is 2 d. 17 m.? *Ans.* .0379585+.

19. What decimal of 45 T. 15 cwt. 25 lb. is 6 T. 10 cwt. 75 lb.? *Ans.* .142857+.

APPLICATIONS.

1. If a silver pitcher weighs 2 lb. 3 oz. 6 pwt., how many pennyweights is its weight? *Ans.* 546 pwt.

2. What is the cost of 3 cwt. 63 lb. of flour, at 5 cents a pound? *Ans.* $18.15.

3. How many acres in a rectangular field 80 rods long by 65 rods wide? *Ans.* 32 A. 80 P.

How do you find the part that one compound denominate number is of another?

4. How many bottles, holding 1 qt. 1 pt. each, will be required to bottle a hogshead of wine? *Ans.* 168 bottles.

5. What will be the cost of excavating a mass of earth 100.14 feet long, 12.45 feet wide, and 10 feet deep, at 20 cents a cubic yard? *Ans.* $92.351+.

6. What sum can be saved in 40 years by rising 45 minutes earlier, 300 days of each year, if an hour is worth 15 cents?
Ans. $1350.

7. A farm consists of exactly 788436 square feet; what is its value, at $\frac{5}{8}$ of a dollar per square rod? *Ans.* $1810.

8. How many tons of iron, at 3 cents a pound, can be bought for $353.79? *Ans.* 5 T. 17 cwt. 3 qr. 18 lb.

9. What is the value of a range of wood, 100 feet long, 4 feet wide, and 12 feet high, at $5 per cord?

10. Which are the leap years between 1866 and 1873?

11. How many square yards of carpeting will be required to carpet a room 24 feet long and 18 feet wide? *Ans.* 48 yards.

12. What is the cost of 2 T. 5 cwt. 2 qr. 20 lb. of beef, at $9 per hundred-weight? *Ans.* $411.30.

13. If you are 16 y. 3 w. 18 h. 30 m. old, how many minutes have you lived, allowing that every fourth year was leap year?
Ans. 8446710 minutes.

14. At $3.50 a gallon, how much wine can be bought for $264.25? *Ans.* 1 hhd. 12 gal. 2 qt.

15. At $9 a hundred-weight, how much beef can be bought for $229.05? *Ans.* 1 T. 5 cwt. 1 qr. 20 lb.

16. How many more minutes in March than in February of 1868? *Ans.* 2880 minutes.

17. When the distance between two places is 31.5 kilometres, how many miles are they apart?
Ans. 19 m. 4 fur. 23 rd. 6.7584 ft.

REVIEW QUESTIONS. What is a Cube? (213) A Rectangular Solid? *(214) Table of Solid* Measure? (214) A cord of wood? (214) A perch of *masonry? Table of* Liquid Measure? (215) Table of Dry Measure? (216)

18. How many cords of wood in a load piled in two tiers, each 4 feet wide and $6\frac{1}{2}$ feet high, the length of the wood being 4 feet? *Ans.* 1 C. 5 c. ft.

19. What is the value of .9628 of a common year?
Ans. 351 d. 10 h. 7 m. 40.8 sec.

20. How many hectoliters are 80 bu. 2 pk.?
Ans. 28.3682 hectolitres.

21. If a pound Avoirdupois of opium be divided into doses of 15 Troy grains each, and sold at 20 cents a dose, how much will it amount to? *Ans.* $93.33\frac{1}{3}$.

22. What is the cost of .695 ton of nails, at 8 cents a pound? *Ans.* $111.20.

23. What decimal of 4 d. 3 h. is 2 w. $6\frac{1}{4}$ d.? *Ans.* 4.909+.

24. If the length of a degree of longitude on the equator be taken at $69\frac{1}{2}$ miles, instead of 69.16, the true length, how much too large will it make the equatorial circumference of the earth? *Ans.* 122 m. 3 fur. 8 rd.

25. If a comet move at the rate of 40′ 30″ per minute, how long will it be in moving through 60° 45′? *Ans.* 1 h. 30 m.

26. There is a house 112 feet long, and each of the two sides of the roof is 25 feet wide; how many shingles will it take to cover it, if it require 6 shingles to cover a square foot?
Ans. 33600 shingles.

ADDITION.

251. 1. Let it be required to find the sum of 5 cwt. 2 qr. 10 lb., 2 cwt. 2 qr. 17 lb., and 10 cwt. 3 qr. 21 lb.

OPERATION.

	cwt.	qr.	lb.
	5	2	10
	2	2	17
	10	3	21
Ans.	19	0	23

For convenience, we write the numbers so that units of the same name stand in the same column, and begin at the right to add.

21, 17, 10, are 48; 48 lb. are equal to 1 qr. 23 lb.; we write the 23 lb., and add the 1 qr. in with quarters.

1, 3, 2, 2, are 8; 8 qr. are 2 cwt. 0 qr.; we write the 0 qr., and add the 2 cwt. in with hundred-weights.

REVIEW QUESTIONS. For what is Circular Measure used? (217) What is a *Circle?* (218) The circumference of a circle? (219) An Arc? (220) *A Diameter?* (221) Repeat the Table of Circular Measure. (223)

2, 10, 2, 5, are 19; 19 cwt. we write.
Therefore, the sum required is 19 cwt. 0 qr. 23 lb.

RULE. *Write the numbers so that units of the same name shall stand in the same column.*

Begin at the right, add each column, and reduce the sum to the next higher denomination; write the remainder, if any, and add the quotient with the next column.

PROOF. The same as in simple numbers.

Examples.

(2.)

	lb.	oz.	pwt.	gr.
	7	0	5	9
	5	6	6	7
	9	5	6	8
Sum	21	11	18	0
Proof	21	11	18	0

(3.)

	M.	fur.	rd.	yd.	ft.	in.
	78	5	30	4	1	7
	68	3	10	3	5	1
	60	3	14	0	2	6
	207	4	15	$4\frac{1}{2}$	0	2
or	207	4	15	4	1	8

In example 3, the $\frac{1}{2}$ yard in the sum is equal to 1 ft. 6 in.; by so considering it, and adding, a form of answer is obtained without a fraction.

(4.)

	A.	P.	sq. yd.	sq. ft.	sq. in.
	120	131	25	6	
	243	10	3	5	
	6	116	6	2	
	370	98	$4\frac{3}{4}$	4	
Or	370	98	5	1	108

(5.)

d.	h.	m.	sec.
121	16	45	15
	20	35	5
17	6	10	0
8	0	0	12
147	19	30	32

The second form of the answer to example 4 is obtained by regarding the $\frac{3}{4}$ yd. as equal to 6 sq. ft. 108 sq. in., and adding it to the 4 sq. yd. 4 sq. ft.

Explain the operation. Repeat the Rule. What is the Proof? How *may the fraction in the answer to example 3 be considered?*

6. Add 340 cu. yd. 18 cu. ft. 1312 cu. in., 748 cu. yd. 9 cu. ft. 716 cu. in., 130 cu. yd. 6 cu. ft. 310 cu. in., and 7 cu. yd. 20 cu. ft. 1642 cu. in.

7. Add 120 gal. 2 qt. 1 pt., 258 gal., 136 gal. 1 qt. 1 pt., and 118 gal. 0 qt. 1 pt. *Ans.* 10 hhd. 3 gal. 0 qt. 1 pt.

8. Add 14 bu. 2 pk. 5 qt., 23 bu. 3 pk., 8 bu. 7 qt., 19 bu. 1 pt., and 59 bu. 4 qt.

9. Add 6° 27′ 48″, 3° 32′ 12″, 8° 20′ 30″, 1° 39′ 31″, 9° 59′ 48″, and 7° 46′ 41″. *Ans.* 37° 46′ 30″.

10. What is the sum of 31 m. 5 fur. 30 rd. 2 yd., 42 m. 6 fur. 25 rd. 4 yd., 63 m. 0 fur. 10 rd. 3 yd., 7 fur. 17 rd. 5 yd., and 16 m. 16 rd. 4 yd.? *Ans.* 154 m. 4 fur. 21 rd. 1 yd. 1 ft. 6 in.

11. What is the sum of 4 T. 11 cwt. 2 qr. 22 lb. 12 oz., 16 cwt. 1 qr. 24 lb. 7 oz., 9 cwt. 11 lb. 15 oz., 3 T. 6 cwt. 1 qr. 2 lb. 7 oz., 2 T. 9 cwt. 1 lb. 6 oz.?

Ans. 11 T. 12 cwt. 2 qr. 12 lb. 15 oz.

12. What is the sum of 60 y. 90 d. 50 m., 6 y. 76 d. 1 h. 57 m., 3 h. 58 m., 6 y. 1 d. 2 h.? *Ans.* 72 y. 167 d. 8 h. 45 m.

252. When one or more of the denominate numbers to be added are fractional, before adding,

Reduce the fractional numbers to integers of lower denominations (Art. 248).

13. Add .282 of a ton, and $\frac{4}{5}$ of a hundred-weight.

OPERATION.

	cwt.	qr.	lb.
.282 T. =	5	2	14
$\frac{4}{5}$ cwt. =		3	5
Ans.	6	1	19

The value of .282 T. is 5 cwt. 2 qr. 14 lb., and of $\frac{4}{5}$ cwt. is 3 qr. 5 lb.; and adding, we have 6 cwt. 1 qr. 19 lb.

14. Add $\frac{3}{4}$ of a mile to $\frac{7}{10}$ of a furlong. *Ans.* 6 fur. 28 rd.

In the answer to example 4, how is the second form of answer obtained? How do you proceed when one or more of the numbers to be added are fractional?

15. Add .6 of an acre, .85 of an acre, and 17 A. 32 P.

16. Add $\frac{1}{3}$ of a week, $\frac{9}{13}$ of a day, and $\frac{1}{3}$ of an hour.
Ans. 3 d. 1 h. 6 m. $55\frac{5}{13}$ sec.

APPLICATIONS.

1. Bought three loads of hay; the first weighed 1 T. 14 cwt. 1 qr. 17 lb., the second 1 T. 2 qr. 17 lb., and the third 1 T. 2 qr. 10 lb.; what was the weight of the whole?
Ans. 3 T. 15 cwt. 2 qr. 19 lb.

2. A miner has an ingot of silver weighing 11 lb. 4 oz. 16 pwt. 11 gr., another weighing 2 lb. 5 oz. 6 pwt. 14 gr., and a third weighing 6 lb. 7 oz. 14 pwt. 17 gr.; required the entire weight. *Ans.* 20 lb. 5 oz. 17 pwt. 18 gr.

3. I have four pieces of rope; the first is 2 yd. 2 ft. 7 in. long, the second $\frac{6}{85}$ of a mile, the third $\frac{3}{88}$ of a furlong, and the fourth 3 yd. 1 ft. 11 in.; what is the length of the whole?
Ans. 37 rd. 2 yd. 1 ft. 6 in.

4. In one range of wood are 50 c. 104 cu. ft. 172 cu. in., in a second, 30 c. 110 cu. ft. 100 cu. in., in a third, 45 c. 3 cord ft., and in a fourth 9 c. 56 cu. ft. 678 cu. in.; how much wood in the four ranges?

5. What day of the year is the 16th of May in common years? *Ans.* 136th.

6. A ship sailed east from Boston, the first day 4° 45′, the second 3° 0′ 45″, the third 2° 25′ 5″, the fourth, 3° 10′ 15″; how far then was she from the place of departure?
Ans. 13° 21′ 5″.

7. Through how many degrees of latitude does a ship pass in going from Philadelphia, latitude 39° 58′ 24″ north, to Cape of Good Hope, latitude 32° 24′ 3″ south? *Ans.* 72° 22′ 27″.

REVIEW QUESTIONS. What is Time? (224) Repeat the Table of Time. *(224)* Give the names of the Calendar Months. (225) What is the length *of a true year? (226)*

8. My farm consists of three fields; the first contains 5.88125 acres, the second of $19\frac{5}{8}$ acres, and the third of 41 A. 17 P.; how many acres does it contain? *Ans.* 66 A. 98 P.

SUBTRACTION.

253. 1. Let it be required to find the difference between 19 cwt. 2 qr. 17 lb., and 15 cwt. 3 qr. 15 lb.

OPERATION.

	cwt.	qr.	lb.
	19	2	17
	15	3	15
Ans.	3	3	2

For convenience, we write the subtrahend under the minuend, so that units of the same name stand in the same column, and begin at the right to subtract.

15 lb. from 17 lb. leaves 2 lb., which we write.

We can not take 3 qr. from 2 qr., but we can take 1 cwt. from the 19 cwt., leaving 18 cwt., and the 1 cwt. taken is 4 qr., which added to the 2 qr. makes 6 qr.; 3 qr. from 6 qr. leaves 3 qr., which we write.

15 cwt. from 18 cwt. leaves 3 cwt., which we write.

Therefore, the difference required is 3 cwt. 3 qr. 2 lb.

Instead of making the upper number, 19 cwt., less by 1 cwt., the result would have been the same, if we had increased the corresponding lower number, 15 cwt., by 1 cwt. (Art. 45.)

RULE. *Write the less number under the greater, so that units of the same name shall stand in the same column.*

Beginning at the right, subtract each denomination of the subtrahend from the corresponding denomination of the minuend.

If the number of any denomination in the subtrahend is greater than that of the same denomination in the minuend, increase the upper number by as many units of that denomination as make one of the next higher, before subtracting; and consider the number of the next higher denomination of the minuend diminished by one.

PROOF. The same as in subtraction of simple numbers.

Explain the operation. Repeat the Rule.

Examples.

	(2.)					(3.)				
	lb.	oz.	pwt.	gr.	m.	fur.	rd.	yd.	ft.	
	10	6	15	16	16	7	8	3	2	
	8	3	19	11	11	6	29	4	1	
Ans.	2	2	16	5	5	0	18	$4\frac{1}{2}$	1	in.
Proof.	10	6	15	16	or 5	0	18	4	2	6

4. From 780 A. 40 P., subtract 396 A. 135 P.

Ans. 383 A. 65 P.

5. Take 13 gal. 3 qt. 1 pt. from 1 hhd. 2 gal. 1 qt. 0 pt. 3 gi.
6. Take 5 bu. 3 pk. 1 pt. from 60 bu. 1 pk. 7 qt.

Ans. 54 bu. 2 pk. 6 qt. 1 pt.

7. Take 310 d. 5 h. 45 m. from one common year.
8. Subtract 10 cu. yd. 25 cu. ft. 1014 cu. in. from 23 cu. yd. 13 cu. ft. 357 cu. in. *Ans.* 12 cu. yd. 14 cu. ft. 1071 cu. in.

254. When one or both of the given denominate numbers are fractional, before subtracting,

Reduce the fractional number, or numbers, to integers of a lower denomination.

9. From $\frac{11}{18}$ lb. Troy, subtract .17 lb. Troy.

OPERATION.

	oz.	pwt.	gr.
$\frac{11}{18}$ lb. =	7	6	16
.17 lb. =	2	0	19.2
Ans.	5	5	20.8

The value of $\frac{11}{18}$ lb. Troy is 7 oz. 6 pwt. 16 gr., and of .17 lb. Troy is 2 oz. 0 pwt. 19.2 gr.; and, subtracting, we have 5 oz. 5 pwt. 20.8 gr.

10. From $\frac{3}{8}$ of a quarter, take $\frac{5}{9}$ of a pound.

Ans. 16 lb. 1 oz. $12\frac{4}{9}$ dr.

11. From $\frac{3}{8}$ of an acre take $\frac{7}{16}$ of an acre.
12. From $\frac{1}{4}$ of a square yard take $\frac{1}{6}$ of a yard square.
13. Subtract $\frac{5}{14}$ of a degree from .37 of a degree.

Ans. $46\frac{2}{7}$ seconds.

How is obtained the second form of answer to example 3? When one or both of the given denominate numbers are fractional, how do you proceed?

255. The rule applies to finding the difference between two dates.

14. What time elapsed between May 16, 1819, and March 9, 1865?

OPERATION.

	y.	mo.	d.
	1865	2	9
	1819	4	16
Ans.	45	9	21

We place the *numbers* of the earlier date under those of the later, writing the number of the year, then in order the number of months and days, which have elapsed of the year, and, in subtracting, *count as many days to a month as are in the month next earlier than that named in the later date.*

Here that month has 28 days.

It is quite common to count every month as containing 30 days, which, in this case, would have given the difference 2 days more, and to that extent wanting in exactness.

15. What time elapsed between Feb. 14, 1807, and Sept. 5, 1866? *Ans.* 59 y. 6 m. 22 d.

16. What time elapsed between July 4, 1 o'clock, P. M., 1776, and January 8, 6 o'clock, A. M., 1865?
Ans. 88 y. 6 mo. 3 d. 17 h.

Since the hours of the day begin to count from midnight, 6 o'clock, A. M., is 6 hours from that time, and 1 o'clock, P. M., 13 hours.

17. What time elapsed between Oct. 14, 1492, and April 3, 1865?

18. What time elapsed between the termination of the American Revolution, January 20, 1783, and the evacuation of Fort Sumter, April 14, 1861. *Ans.* 78 y. 2 mo. 25 d.

APPLICATIONS.

1. A forwarding house having received 20 T. 2 qr. 14 lb. of freight has shipped 10 T. 13 cwt. 2 qr. 14 lb. by steamer, and the balance by sail vessel; what quantity was sent by the latter conveyance? *Ans.* 9 T. 7 cwt.

2. Bought 7 cords of wood, and 2 c. 78 cu. ft. having been stolen, how much remains?

In subtracting dates, how many days are counted to a month? From what time do the hours of the day begin to count?

3. There are two cities, 98 m. 5 fur. 3 rd. apart; how far is a man traveling between these cities from one of them, if he is 12 miles $6\frac{1}{10}$ furlongs from the other?

Ans. 85 m. 6 fur. 39 rd.

4. The longitude of Boston is 71° 3′ 30″ west of Greenwich, and that of San Francisco 122° 26′ 48″; what is the difference? *Ans.* 51° 23′ 18″.

5. The latitude of New Orleans is 29° 57′ 30″ north, and that of Chicago 42°; what is the difference?

6. How many days are there from the 5th of January till the 25th of the next June of a common year?

SOLUTION. 31 *days in January, less* 5 *days* = 26 *days of January;* 26 *days of January* + 28 *days of February* + 31 *days of March* + 30 *days of April* + 31 *days of May* + 25 *days of June* = 171 *days, Ans.*

7. How many days from Feb. 22, to the next July 4th, of a leap year? *Ans.* 133 days.

8. If a man was born February 29, 1820, how many anniversaries of his birth-day will he have had on February 29, 1880? *Ans.* 15.

9. If September 21st is Monday, on what day will the 25th of the next December be?

SOLUTION. *The number of days between Sept.* 21 *and Dec.* 25 *is* 95 *days, or* 13 *weeks* 4 *days; counting four days after Monday, the* 25*th of December must be Friday.*

10. A farmer has in one crib $325\frac{15}{16}$ bushels of corn, in a second $43\frac{5}{8}$ bushels, and in a third 587 bu. 3 pk. 7 qt. Of this he will have to pay for rent 367 bu. 2 pk. 4 qt.; to use for fatting cattle, 56 bu. 2 pk. 3 qt.; for fatting hogs, 35 bu. 3 pk. 2 qt., and will require for his own use 298 bushels; how much will he have left to dispose of? *Ans.* 199 bu. 2 pk.

REVIEW QUESTIONS. For what purposes are decimal weights and measures generally adopted? (232) Upon what scale is the Metrical System formed? (239)

MULTIPLICATION.

256. 1. Let it be required to find the product of 4 bu. 3 pk. 4 qt. 1 pt. multiplied by 7.

OPERATION.

	bu.	pk.	qt.	pt.
	4	3	4	1
				7
Ans.	34	0	7	1

For convenience, we begin with the lowest denomination to multiply.

7 times 1 pt. are 7 pt., which equal 3 qt. 1 pt.; we write the 1 pt., and reserve the 3 qt. to add to the product of the quarts.

7 times 4 qt. are 28 qt., which, with the 3 qt. added, are 31 qt., or 3 pk. 7 qt.; we write the 7 qt. and reserve the 3 pk. to add to the product of the pecks.

7 times 3 pk. are 21 pk., which, with the 3 pk. added, are 24 pk., or 6 bu. 0 pk.; we write the 0 pk., and reserve the 6 bu. to add to the product of bushels.

7 times 4 bu. are 28 bu., which, with the 6 bu. added, are 34 bu., which we write.

Therefore, the product required is 34 bu. 0 pk. 7 qt. 1 pt.

RULE. *Beginning at the right, multiply the number of each denomination in its order, and reduce each product to the next higher denomination; write the remainder, if any, and add the quotient to the next product.*

PROOF. The same as in multiplication of simple numbers.

Examples.

(2.)

cwt.	qr.	lb.	oz.	dr.
	3	6	13	15
				8
6	2	4	15	8

(3.)

m.	fur.	rd.	yd.	ft.
1	2	15	2	1
				9

4. Multiply 3 hhd. 57 gal. 3 qt. 1 pt. by 11.
Ans. 43 hhd. 6 gal. 2 qt. 1 pt.
5. Multiply 36 d. 21 h. 48 m. 56 sec. by 6.
6. Multiply 111 C. 7 c. ft. 7 cu. ft. by 12.
Ans. 1343 C. 1 c. ft. 4 cu. ft.
7. Multiply 4 bu. 1 pk. 5 qt. 1 pt. by 7.
Ans. 30 bu. 3 pk. 6 qt. 1 pt.

Explain the operation. Repeat the Rule. What is the Proof?

8. Multiply 17 A. 71 P. by 72. *Ans.* 1255 A. 152 P.

9. Multiply 2 lb. 7 oz. 9 pwt. 22 gr. by 50.
Ans. 131 lb. 2 oz. 15 pwt. 20 gr.

APPLICATIONS.

1. What is the weight of 5 hogsheads of sugar, if each weighs 12 cwt. 1 qr. 23 lb.? *Ans.* 3 T. 2 cwt. 1 qr. 15 lb.

2. What is the weight of 12 spoons, if each weighs 1 oz. 12 pwt. 20 gr.?

3. If a car will take on 4 C. 56 cu. ft. of wood, how much will 8 cars take on? *Ans.* 35 C. 64 cu. ft.

4. If the daily motion of the moon is 13° 10′ 35″, how much is it in 15 days? *Ans.* 197° 38′ 45″.

5. If 3 yd. $1\frac{1}{4}$ qr. of cloth are required for one garment, how much is required for 14?

6. How much molasses in 25 casks, if each contains 61 gal. 1 qt. 1 pt.? *Ans.* 1534 gal. 1 qt. 1 pt.

7. If a man travel at the rate of 22 m. 7 fur. 32 rd. 4 yd. per day, how far can he travel in 56 days?
Ans. 1286 m. 5 fur. 32 rd. 4 yd.

8. How much land in 9 farms, each containing 74 A. 87 P. 4 sq. yd.? *Ans.* 670 A. 144 P. 5 sq. yd. 6 sq. ft. 108 sq. in.

DIVISION.

257. 1. Let it be required to find the quotient of 34 bu. 0 pk. 7 qt. 1 pt. divided by 7.

OPERATION.

	bu.	pk.	qt.	pt.
7)	34	0	7	1
	4	3	4	1

For convenience, we begin with the highest denomination to divide.

One seventh of 34 bu. is 4 bu., with a remainder of 6 bu., equal to 24 pecks; we write the 4 bu., and add the 24 pecks to the 0 pecks in the dividend.

24 pk. and 0 pk. are 24 pk.; and one seventh of 24 pk. is 3 pk., with a remainder of 3 pk., equal to 24 quarts; we write the 3 pk., and add the 24 quarts to the 7 quarts in the dividend.

Explain the operation.

24 qt. and 7 qt. are 31 qt.; and one seventh of 31 qt. is 4 qt. with a remainder of 3 qt., equal to 6 pints; we write the 4 qt., and add the 6 pints to the 1 pint of the dividend.

6 pt. and 1 pt. are 7 pt., and one seventh of 7 pt. is 1 pt., which we write.

RULE. *Beginning at the left, divide the number of each denomination of the dividend in its order; write the quotient, and reduce the remainder, if any, to the next lower denomination, adding the same to that denomination in the dividend, before dividing it.*

PROOF. The same as in division of simple numbers.

Examples.

(2.)

	hhd.	gal.	qt.	pt.
11)	39	6	2	1
	3	34	3	$1\frac{8}{11}$

(3.)

$$\begin{array}{rll} & \text{m.} & \text{fur.} \\ 31\,) & 286 & 6\,(\,9 \\ & 279 & \\ \hline & 7 & \\ & 8 & \\ \hline & 56 & \\ & 6 & \\ \hline 31\,) & 62 & (\,2 \text{ fur.} \\ & 62 & \\ \hline \end{array}$$

Ans. 9 m. 2 fur.

When the divisor is large, it is often convenient, as in example 3, to write out the operation, after the manner of long division.

4. Divide 6 cwt. 2 qr. 4 lb. 15 oz. 15 dr. by 6.
5. Divide 131 A. 140 P. by 20. *Ans.* 6 A. 95 P.
6. Divide 131 lb. 2 oz. 15 pwt. 20 gr. by 50.
7. Divide 1 w. 3 d. 0 hr. 54 m. by 18. *Ans.* 13 h. 23 m.
8. Divide 1343 C 1 c. ft. 4 cu. ft. by 12.
9. Divide 100 y. 20 d. 13 h. 25 m. 10 sec. by 11.
Ans. 9 y. 35 d. 1 h. 13 m. $11\frac{9}{11}$ sec.

Repeat the Rule. What is the Proof? When the divisor is large, how *is it often convenient to proceed?*

258. When both dividend and divisor are denominate numbers,

Reduce them to equivalent numbers of the same denomination, and proceed as in division of simple numbers.

10. How many times 5 pwt. 9 gr. in 9 lb. 9 oz. 3 pwt. 12 gr.? *Ans.* 436.

11. How many times 17 m. 5 fur. 27 rd. in 513 m. 4 fur. 23 rd.? *Ans.* 29.

APPLICATIONS.

1. If 169 gal. 3 qt. 1 pt. be contained in equal quantities in 9 casks, how much is there in each cask? *Ans.* 18 gal. 3 qt. 1 pt.

2. If 5 hogsheads of sugar weigh 3 T. 2 cwt. 1 qr. 15 lb., what is the average weight of each?

3. If 2 dozen spoons weigh 7 lb. 6 oz. 13 pwt., what is the weight of each spoon? *Ans.* 3 oz. 15 pwt. 13 gr.

4. When a ship passes over 99° 22′ 30″ in 30 days, what is the average progress per day? *Ans.* 3° 18′ 45″.

5. If it take 4 bu. 3 pk. of wheat to make a barrel of flour, how many barrels may be made from 456 bushels? *Ans.* 96.

6. When a railroad train moves 148 m. 4 fur. in 8 hours, what is the rate per hour? *Ans.* 18 m. 4 fur. 20 rd.

7. If a man travel 12 m. 3 fur. 19 rd. per hour, in what time can he travel 174 m. 26 rd.? *Ans.* 14 hours.

8. When the weight of 27 loads of hay is 30 T. 8 cwt. 2 qr. 23 lb., what is the average weight of a load?
Ans. 1 T. 2 cwt. 2 qr. $4\frac{8}{9}$ lb.

LONGITUDE AND TIME.

259. The earth, by turning upon its axis once in 24 hours, causes $\frac{1}{24}$ of 360°, or 15°, of longitude to pass under the sun

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When both the dividend and divisor are denominate, how do you proceed

in 1 hour, and $\frac{1}{60}$ of 15°, or 15′, to pass under it in 1 minute of time, and $\frac{1}{60}$ of 15′, or 15″, to pass under it in 1 second of time. Hence, the following

Table.

15°	of longitude	equal	a difference of	1 hour in time.
15′	"	"	"	1 minute in time.
15″	"	"	"	1 second in time.

Whence, is derived the

RULE. *Divide the difference of longitude of two places by 15, and the quotient marked hours, minutes, and seconds, instead of degrees, minutes, and seconds, will express their difference of time.*

Multiply the difference of time of two places by 15, and the product marked degrees, minutes, and seconds, instead of hours, minutes, and seconds, will express their difference of longitude.

Since the earth turns from west to east, time is *later* to places *east*, and *earlier* to places *west* of any given meridian.

Examples.

1. When it is 9 o'clock, A. M., at Washington, 77° 2′ 48″ west of Greenwich, what time is it at the latter place?

Ans. 2 h. 8 m. 11$\frac{1}{5}$ sec. P. M.

2. If the time at St. Louis, longitude 90° 15′ 10″ west, is 40 m. 2$\frac{4}{5}$ sec. earlier than that of a place east, what is the longitude of the latter place? *Ans.* 80° 14′ 34″ west.

3. When it was 10 o'clock, P. M., Dec. 31, 1865, in San Francisco, longitude 48° 26′ 45″ west of New York, what time was it in New York?

Ans. 1 h. 13 m. 47 sec. A. M., Jan. 1, 1866.

4. A captain, at sea, at noon, observed that his chronometer, set to the time of Greenwich, longitude 0, pointed to 2 h. 45 m. 30 sec.; in what longitude was he? *Ans.* 41° 22′ 30″ west.

What does the earth, by turning upon its axis, cause? Repeat the Table. *The Rule. At what places is time earlier with reference to any given meridian?*

PRACTICE.

260. An **Aliquot Part** of a number is an exact divisor (Art. 97) of it. Thus,

2, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5 are aliquot parts of 10.

261. **Practice** is an abridged method of computation, by means of aliquot parts.

Table of Aliquot Parts.

Of a \$.		Of a Ton.		Of a Cwt.		Of an Acre.		Of a Month.	
Cts.	\$	cwt.	ton.	lb.	cwt.	rd.	A.	d.	m.
50	$=\frac{1}{2}$	10	$=\frac{1}{2}$	50	$=\frac{1}{2}$	80	$=\frac{1}{2}$	15	$=\frac{1}{2}$
$33\frac{1}{3}$	$=\frac{1}{3}$	5	$=\frac{1}{4}$	25	$=\frac{1}{4}$	40	$=\frac{1}{4}$	10	$=\frac{1}{3}$
25	$=\frac{1}{4}$	4	$=\frac{1}{5}$	20	$=\frac{1}{5}$	32	$=\frac{1}{5}$	$7\frac{1}{2}$	$=\frac{1}{4}$
$16\frac{2}{3}$	$=\frac{1}{6}$	$2\frac{1}{2}$	$=\frac{1}{8}$	$12\frac{1}{2}$	$=\frac{1}{8}$	20	$=\frac{1}{8}$	6	$=\frac{1}{5}$
$12\frac{1}{2}$	$=\frac{1}{8}$	2	$=\frac{1}{10}$	10	$=\frac{1}{10}$	16	$=\frac{1}{10}$	5	$=\frac{1}{6}$
10	$=\frac{1}{10}$	1	$=\frac{1}{20}$	5	$=\frac{1}{20}$	8	$=\frac{1}{20}$	3	$=\frac{1}{10}$

Exercises.

1. Let it be required to find the cost of 3632 bushels of wheat, at $\$1.87\frac{1}{2}$ a bushel.

OPERATION.

3632 bu.	at	\$1	per bu.	will cost		\$3632
"	"	.50	"	"	$\frac{1}{2}$ of \$3632 =	1816
"	"	.25	"	"	$\frac{1}{2}$ of \$1816 =	908
"	"	$.12\frac{1}{2}$	"	"	$\frac{1}{2}$ of \$908 =	454
"	"	$\$1.87\frac{1}{2}$	"	"		\$6810, *Ans.*

2. Required the cost of 3 T. 5 cwt. 2 qr. of hay, at \$24 a ton.

OPERATION.

Since 1 T.	cost		\$24.00
3 T.	will cost	\$24 × 3 =	\$72.00
5 cwt.	"	$\frac{1}{4}$ of \$24 =	6.00
2 qr.	"	$\frac{1}{10}$ of \$6 =	.60
3 T. 5 cwt. 2 qr.	"		\$78.60, *Ans.*

What is an Aliquot Part of a number? What is Practice?

3. What is the amount of a salary for 1 y. 7 mo. $22\frac{1}{2}$ d., at $1080 per year?

OPERATION.

For 1 y.	it is			$1080
" 6 mo.	"	$\frac{1}{2}$ of	$1080 =	540
" 1 mo.	"	$\frac{1}{6}$ of	$540 =	90
" 15 d.	"	$\frac{1}{2}$ of	$90 =	45
" $7\frac{1}{2}$ d.	"	$\frac{1}{2}$ of	$45 =	22.50
" 1 y. 7 mo. $22\frac{1}{2}$ d.	"			$1777.50

4. What is the value of 60 A. 120 P., at $80 per acre? *Ans.* $4860.

5. What is the cost of 13 gal. 3 qt. 1 pt. of molasses, at $.60 per gallon?

6. Required the cost of grading 10 m. 6 fur. 20 rd. of railroad, at $6490 per mile? *Ans.* 70173.12\frac{1}{2}$.

7. Required the cost of 2117 yards of cloth, at $37\frac{1}{2}$ cents per yard? *Ans.* 793.87\frac{1}{2}$.

8. What cost 120 yards of broadcloth, at 3.66\frac{2}{3}$ per yard?

9. How much must be paid for 24 bu. 2 pk. 4 qt. of corn, at $.88 per bushel? *Ans.* $21.67.

10. If a man can walk 18 m. 5 fur. 16 rd. in one day, how far can he walk in 10 d. 8 h.? *Ans.* 192 m. 7 fur. 32 rd.

11. At $240 a year, what is the rent of a house for 9 mo. 25 d.?

12. What is the cost of excavating 108 cu. yd. 18 cu. ft. of earth, at 42 cents per cu. yd.? *Ans.* $45.64.

REVIEW QUESTIONS. What is Reduction of Denominate Numbers? (244) What is the Rule for reducing from a higher to a lower denomination? (245) For reducing from a lower to a higher denomination? (247) For reducing a fraction of a given denomination to integers of lower denominations? (248) For reducing integers of lower denominations to a fraction of a higher denomination? (249)

REVIEW EXERCISES.

1. What cost 1 T. 5 cwt. 56 lb. of sugar, at 11 cents a pound? *Ans.* $281.16.

2. At 11 cents a pound, how much sugar can be bought for $281.16?

3. A farmer, having 640 acres in his farm, values one half of it at $20 per acre, and the other half at 20 cents a square rod; how much is the one half worth more than the other? *Ans.* $3840.

4. A druggist retailed $\frac{5}{8}$ of a pound Troy of a drug, at 2 cents a grain; how much did it amount to?

5. How much money can be made on 500 tons of coal bought at $5 per ton of 2240 lb., and sold at the same number of dollars per ton of 2000 lb.?

SOLUTION. 2240 *lb.* — 2000 *lb.* = 240 *lb.; if* 240 *lb. can be made on* 1 *ton*, 500 *times* 240 *lb., or* 60 *tons, can be made on* 500 *tons; and* 60 *tons, at* $5 *per ton, sell for* $300, *the money that can be made. Therefore, etc.*

6. How much can be made by buying 10 lb. of rhubarb, at $6.50 per pound avoirdupois, and selling it at the same price per lb. Troy? *Ans.* $13.99 +.

7. A laborer being 8 h. 4 m. doing a job, another man agreed to do a like piece of work in $\frac{5}{6}$ of that time; how long was that? *Ans.* 6 h. 43 m. 20 sec.

8. When $1.80 is paid for $\frac{1}{25}$ of 2 tons of hay, how much is it per hundred weight? *Ans.* 1.12\frac{1}{2}$.

9. At $1.50 per yard, in length, what will be the cost of a carpet, $\frac{5}{8}$ of a yard wide, that will cover the floor of a room 29.5 feet long by 11.25 feet wide? $88.50.

10. What must be the length of a rectangular lot, that measures 72 feet on the front, to contain 9432 square feet? *Ans.* 131 feet.

REVIEW QUESTIONS. What is the Rule for Addition of Compound Denominate Numbers? (251) Subtraction? (253) Multiplication? (256) Division? (257)

SOLUTION. *Since the 9432 must be the product of the number denoting the front and another denoting the length, the latter must be* 9432 ÷ 72, *or* 131 *feet.*

11. How wide must be a rectangular lot, which is 42.4 rods long, to contain exactly an acre?

12. How much will a man's wages amount to, at $25 per month, from April 16th to the last day of the next March?

Ans. $287.50.

13. How much money may be made on 2 bushels of chestnuts, bought at $4.80 a bushel, and retailed at 15 cents a quart, liquid measure? *Ans.* $1.56 +.

14. When tea is 75 cents a pound, how much is it per kilogram?

SOLUTION. *Since* 1 *kilogram is equal to* 2.2046 *lb., when tea is* 75 *cents a pound, it is* 2.2046 *times* 75 *cents, or* $1.65 +, *per kilogram. Therefore, etc.*

15. When a field produces 40 hectoliters per hectare, how many bushels is that per acre? *Ans.* 45.9 + bushels.

16. What is the cost of a stone wall 132 feet long, 4 feet high, and 1.5 feet thick, at $2.25 per perch of masonry?

Ans. $72.

17. A wagon is 8 feet long and 3.75 feet wide; how high must wood be piled upon it to make a cord?

SOLUTION. *Since* 128, *the number of cubic feet in a cord, must be the product of the numbers denoting the length and width, and the required hight, the latter must be* 128 ÷ (8 × 3.75), *or* 4.26 +, *feet.*

18. If a pile of wood is 4 feet wide and 6 feet high, how long is it, if it contains 4 C. 6 c. ft.?

19. What time is it in 30° west longitude, when it is 1 o'clock, A. M., July 4th, in 7° 30′ east longitude?

Ans. 10½ o'clock, P. M., July 3d.

REVIEW QUESTIONS. What is the Rule for finding the difference of time corresponding to the difference of longitude? (259) The difference of longitude corresponding to the difference of time? (259)

PERCENTAGE.

262. Per Cent., from *per*, by, or on, and *centum*, hundred, means *by* or *on a hundred.* Thus,

2 per cent. of a quantity, means 2 on a hundred, or 2 *hundredths* of the quantity.

263. Percentage is the process of computing in hundredths; and

The PERCENTAGE of a number is so many hundredths of it as is indicated by the per cent.

The BASE of percentage is the number upon which it is computed.

The RATE per cent. is the number of hundredths denoted by the per cent.

264. Since rate per cent. is a number of hundredths, it may be expressed in the form of a *decimal* or of a *common fraction.* Thus,

1 per cent.	is written	.01,	or $\frac{1}{100}$.
5 per cent.	"	.05,	" $\frac{5}{100}$.
10 per cent.	"	.10,	" $\frac{10}{100}$.
25 per cent.	"	.25,	" $\frac{25}{100}$.
100 per cent.	"	1.00,	" $\frac{100}{100}$.
125 per cent.	"	1.25,	" $\frac{125}{100}$.

$\frac{1}{2}$ per cent. is $.00\frac{1}{2}$, or .005; $\frac{1}{4}$ per cent. is $.00\frac{1}{4}$, or .0025; $\frac{1}{8}$ per cent. is $.00\frac{1}{8}$, or .00125, etc.

The *sign* % is used, in business, to stand for the word per cent.

Exercises.

Express decimally,

1. 8 %.	*Ans.* .08.	5. $\frac{1}{2}$ %.	9. 45 %.
2. 15 %.	*Ans.* .15.	6. $\frac{1}{4}$ %.	10. 90 %.
3. $12\frac{1}{2}$ %.	*Ans.* $.12\frac{1}{2}$.	7. $\frac{3}{10}$ %.	11. 150 %.
4. $1\frac{5}{8}$ %.	*Ans.* $.01\frac{5}{8}$.	8. $7\frac{3}{10}$ %.	12. 275 %.

What does Per Cent. mean? What is Percentage? The Percentage of a number or quantity? The Base? How may per cent. be expressed? *What sign for per cent. is used?*

Express as common fractions in their smallest terms,

13. $12\frac{1}{2}$ %. SOLUTION. $12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{25}{200} = \frac{1}{8}$, *Ans.*

14. 15 %. *Ans.* $\frac{3}{20}$.	16. $37\frac{1}{2}$ %.	18. $7\frac{3}{10}$ %.
15. 110 %. *Ans.* $1\frac{1}{10}$.	17. $16\frac{2}{3}$ %.	19. $62\frac{1}{2}$ %.

Express as a per cent.,

20. $\frac{5}{8}$. SOLUTION. $\frac{5}{8} = 5.00 \div 8 = .62\frac{1}{2} = 62\frac{1}{2}$ %, *Ans.*

21. $1\frac{1}{4}$. *Ans.* 125 %.	23. $\frac{1}{6}$.	25. $\frac{4}{5}$.
22. $\frac{73}{1000}$. *Ans.* $7\frac{3}{10}$ %.	24. $3\frac{1}{4}$.	26. $5\frac{2}{10}$.

CASE I.

265. To find any per cent. of a number.

1. What is 5 per cent. of 365 bushels?

OPERATION.

$$\begin{array}{r} 365 \\ .05 \\ \hline \end{array}$$

Since 5 % is .05, 5 % of 365 bu. is .05 of 365 bu., or 18.25 bu. Therefore, etc.

Ans. 18.25 bu.

RULE. *Multiply the given number by the rate per cent., expressed decimally.*

Examples.

What is

2. 5 % of 563 yards? *Ans.* 28.15 yd.	6. 13 % of \$150.50?
3. $9\frac{1}{2}$ % of 600 men? *Ans.* 57 men.	7. 19 % of \$1000.?
4. 15 % of 120 sheep? *Ans.* 18 sheep.	8. $\frac{4}{5}$ % of \$1367.50?
5. 35 % of \$34.60? *Ans.* \$12.11.	9. 103 % of 760 pounds?

10. Find 4 per cent. of 3 pk. 2 qt.

SOLUTION. 3 pk. 2 qt. = 26 qt.; 26 qt. × .04 = 1.04 qt., *Ans.*

11. Find 45 % of 12 cwt. 2 qr. 20 lb.

Ans. 5 cwt. 2 qr. $21\frac{1}{2}$ lb.

266. When the number of the per cent. is an aliquot part of 100, to find the percentage, we may

Take the same part of the base that the rate per cent. is of 100.

Explain the operation. What is the Rule? When the number per cent. *is an aliquot part of 100,* how may the percentage be found?

12. What is $33\frac{1}{3}$ % of 2880 hogsheads?

SOLUTION. $33\frac{1}{3}$ % of 2880 hhd. $= \frac{33\frac{1}{3}}{100}$, or $\frac{1}{3}$, of $2880 = 960$ hhd., *Ans.*

13. What is 25 % of $968.56? *Ans.* $242.14.

14. What is 50 % of 4 h. 25 m. 12 sec.?

Ans. 2 h. 12 m. 36 sec,

APPLICATIONS.

1. A farmer raised 3674 bushels of grain, and has sold 65 %, of it; how much has he left?

SOLUTION. *If he has sold* 65 % *of it, he has left the whole, or* 100 %, *less* 65 %, *which is* 35 %; *and* 35 % *of* 3674 *bushels is* $3674 \times .35 = 1285.9$ *bushels, Ans.*

2. If cloth will shrink in sponging $5\frac{1}{2}$ per cent. of the length, what will be the shrinkage of a piece of cloth containing 43 yards before sponging?

3. If a man's salary is $2250 a year, and his expenses $87\frac{1}{2}$ per cent. of that sum, how much can he save yearly?

Ans. $281.25.

4. A speculator bought 3160 barrels of produce, but on examining them, found $15\frac{1}{2}$ % of the whole worthless; of the remainder he reserved for his own use 5 %, and sold what was left at $3 per barrel; how much did he get for what he sold?

Ans. $7610.07.

CASE II.

267. To find what per cent. one number is of another.

1. What per cent. is 18.25 of 365?

OPERATION.

$\frac{18.25}{365} = .05$, or 5 %, *Ans.*

Since 18.25 is $\frac{18.25}{365} = .05$ of 365, and $.05 = 5$ %, 18.25 is 5 % of 365.

RULE. *Divide the number denoting the percentage by that denoting the base, and extend the division to hundredths.*

Explain the operation. Repeat the Rule.

Examples.

What per cent. is

2. 15 lb. of 50 lb.? *Ans.* 30 %.
3. 57 men of 600 men?
4. \$7.80 of \$1040? *Ans.* $\frac{3}{4}$ %.
5. \$782.80 of \$760?
6. 90.848 of 801.6? *Ans.* $11\frac{1}{3}$ %.
7. \$28.47 of \$657?
8. $199\frac{1}{2}$ rd. of 2850 rd.? *Ans.* 7 %.
9. \$235.50 of \$1500? *Ans.* $15\frac{7}{10}$ %.

10. Find what per cent. 1.04 qt. is of 3 pk. 2 qt.

SOLUTION. 3 pk. 2 qt. = 26 qt.; $1.04 \div 26 = .04 = 4$ %. *Ans.*

11. Find what per cent. 5 cwt. 2 qr. $21\frac{1}{2}$ lb. is of 12 cwt. 2 qr. 20 lb.

12. Find what per cent. $\frac{1}{8}$ is of .625? *Ans.* 20 %.

APPLICATIONS.

1. On a plantation containing 4000 trees, 20 trees died; what per cent. is that of the whole? *Ans.* $\frac{1}{2}$ %.

2. If 43 yards of cloth shrink in length, by sponging, 2.365 yards, what is the shrinkage per cent.?

3. A farmer had last year 320 sheep, and this year 344; what is the increase per cent.?

SOLUTION. *If he had last year* 320 *sheep, and this year* 344, *the increase is* 24 *sheep; and* 24 *is* $\frac{24}{320} = .07\frac{1}{2} = 7\frac{1}{2}$ % *of what he had last year.*

4. The population of a certain town was once 5600, but is now 4802; what per cent. is the decrease? *Ans.* $14\frac{1}{4}$ %.

5. Mr. Wilson has paid \$110 of a demand for \$235; what per cent. remains unpaid? *Ans.* $53\frac{9}{47}$ %.

6. A farmer gave his son Thomas 100 acres of land, and his son Arthur the same quantity; but Thomas decreased his 50 %, and Arthur increased his 50 %; what per cent. then was Thomas' land of that of Arthur? *Ans.* $33\frac{1}{3}$ %.

REVIEW QUESTIONS. What is meant by per cent.? (262) What is Percentage? (263) Percentage of a number? (263) The Base of percentage? (263) The Rate per cent.? (263)

CASE III.

268. To find a number when any per cent. of it is given.

1. $450 is 15 per cent. of what number of dollars?

OPERATION.

$$\frac{\overset{30}{\cancel{\$450}} \times 100}{\cancel{15}} = \$3000 \ \textit{Ans.}$$

or,

$$\$450 \div .15 = \$3000, \ \textit{Ans.}$$

Since 15 % of some number is $450, 1 % of it is $\frac{1}{15}$ of $450, or $30, and 100 %, or the number itself, is 100 times $30, or $3000.

Or, since 15 %, or .15, of the number is $450, that number must be $\frac{1}{.15}$ of $450 = $450 ÷ .15, which is $3000.

RULE. *Divide the given percentage by the rate per cent. expressed decimally.*

Examples.

2. Of what number is 96 bushels, 6 %? *Ans.* 1600 bushels.
3. Of what number is 57 men, $9\frac{1}{2}$ %?
4. $375 is 30 % of what number? *Ans.* $1250.
5. 1.04 qt. is 4 % of what number? *Ans.* 3 pk. 2 qt.
6. $235.50 is $15\frac{7}{10}$ % of what number?
7. $7.80 is $\frac{3}{4}$ % of what number? *Ans.* $1040.
8. 7 lb. 11.2 oz. is 5 % of what number?

Ans. 1 cwt. 2 qr. 4 lb.

269. When the number of the per cent. is an aliquot part of 100, to find the base, we may

Consider the percentage the same fraction of the base that the rate per cent. is of 100.

9. 960 hogsheads is $33\frac{1}{3}$ per cent. of what number?

SOLUTION. If $33\frac{1}{3}$ %, or $\frac{1}{3}$, of some number is 960 hogsheads, $\frac{3}{3}$, or that number, must equal 3 times 960, or 2880, hogsheads.

10. $242.14 is 25 % of what number?
11. 2 h. 12 m. 36 sec. is 50 % of what number?

Ans. 4 h. 25 m. 12 sec.

Explain the operation. What is the Rule? How may the base be found when the rate per cent. is an aliquot part of 100?

APPLICATIONS.

1. From a cask of molasses there leaked out 10.08 gallons, which was 16 per cent. of what it contained; how much did it contain at first? *Ans.* 63 gallons.

2. The tax on a house is \$8, which is $\frac{4}{10}$ % of its valuation; what is its valuation? *Ans.* \$2000.

3. If a man, by saving $37\frac{1}{2}$ % of his income, lays up \$281.25 a year, what is his income?

4. If 17 bu. 2 pk. is $7\frac{1}{2}$ % of a farmer's crop of grain, how much is the whole of it? *Ans.* 233 bu. $1\frac{1}{3}$ pk.

5. In a battle an army had 75 men killed, 93 wounded, and 112 taken prisoners, and the entire loss was $17\frac{1}{2}$ %; what was the number at first? *Ans.* 1600 men.

6. A man took from a bank \$393, which was $13\frac{1}{10}$ % of what he had deposited; how much then remained? *Ans.* \$2607.

CASE IV.

270. To find a number, which is a given per cent. more or less than a given number.

1. What number, increased by 10 per cent. of itself, is equal to 385?

OPERATION.

$$\frac{385 \times \overset{10}{\cancel{100}}}{\underset{11}{\cancel{110}}} = 350, \textit{ Ans.}$$

Or,

$$385 \div 1.10 = 350, \textit{ Ans.}$$

A number increased by 10 % of itself is equal to 110 % of itself. If 110 % of a number is equal to 385, 1 % of it is equal to $\frac{1}{110}$ of 385, or 3.50, and 100 %, or the number, is equal to 100 times 3.50, or 350.

Or, since 110 %, or 1.10, of some number is equal to 385, that number must be equal to $\frac{1}{1.10}$ of 385 = 385 ÷ 1.10, which is 350.

RULE. *Divide the given number, according as it may be more or less than the required number, by* 1, *increased or diminished by the rate per cent. expressed decimally.*

Explain the operation. Repeat the Rule.

Examples.

2. 4550 is 25 % more than what number? *Ans.* 3640.

3. 7402.5 is $17\frac{1}{2}$ % more than what number?

4. \$1047.80 is $\frac{3}{4}$ % more than what number? *Ans.* \$1040.

5. What number, diminished by 14 % of itself, is equal to 129?

SOLUTION. 100 % — 14 % = 86 %; $\frac{1}{.86}$ of 129 = 129 ÷ .86 = 150. *Ans.*

6. 534.85 yards are 5 % less than what number? *Ans.* 563 yards.

7. 1738 is $13\frac{1}{10}$ % less than what number? *Ans.* 2000.

8. 543 men are $9\frac{1}{2}$ % less than what number?

9. What fraction, diminished by 10 % of itself is equal to $\frac{18}{25}$? *Ans.* $\frac{4}{5}$.

271. It is sometimes most convenient to use the given rate per cent. expressed as a common fraction.

10. What number increased by 25 % of itself is equal to 750?

SOLUTION. $25\% = \frac{25}{100} = \frac{1}{4}$; $\frac{4}{4}$, or the number, $+ \frac{1}{4} = \frac{5}{4}$; if $\frac{5}{4} = 750$, $\frac{1}{4} = \frac{1}{5}$ of 750, or 150, and $\frac{4}{4} = 150 \times 4 = 600$, *Ans.*

11. \$620 are $33\frac{1}{3}$ % less than what number?

12. $\frac{3}{4}$ is $16\frac{2}{3}$ % more than what number? *Ans.* $\frac{9}{14}$.

APPLICATIONS.

1. A candidate for re-election received 3640 votes, which was 12 % more than he received at his first election; how many had he at the first? *Ans.* 3250 votes.

2. A regiment, which lost in a severe engagement $31\frac{1}{4}$ per cent of its men, had 440 men left; how many had it at first? *Ans.* 640 men.

REVIEW QUESTIONS. How may rate per cent. be expressed? (264) *What is used, in business, to stand for the word per cent.?* (264)

3. A man spent $66\frac{2}{3}$ % of his income, and saved \$533.33$\frac{1}{3}$; what was his income?

SOLUTION. *If he spent* $66\frac{2}{3}$ % *of his income, he saved the whole or* 100 % — $66\frac{2}{3}$ %, *which is* $33\frac{1}{3}$ %, *or* $\frac{1}{3}$; *if* \$533.33$\frac{1}{3}$ *is* $\frac{1}{3}$, $\frac{3}{3}$, *or his whole income, must be* \$533.33$\frac{1}{3}$ × 3 = \$1600, *Ans.*

4. The population of a certain village is 4059, which is $12\frac{1}{2}$ % more than it was five years ago; what was it then?

Ans. 3608.

5. A gentleman completed a journey by steamer in 26 d. 10.4 h. which was 30 % longer than would have been required to have performed it by railroad; how long would it have required by railroad? *Ans.* 20 d. 8 h.

COMMISSION AND BROKERAGE.

272. A Commission Merchant, or **Factor,** is a person who buys or sells goods for others, for a percentage.

273. A Broker is a person who buys or sells stocks, bills of credit, etc., for a fee, or a percentage.

274. Commission is the percentage paid a commission merchant, or factor, for the transaction of business.

275. Brokerage is the percentage paid a broker for the transaction of business.

The BASE is the amount of property sold or purchased, of money invested or collected, etc.

276. To find the commission or brokerage.

1. An agent sold property to the amount of \$5650; what is his commission, at 3 per cent.?

OPERATION.

\$5650 × .03 = \$169.50, *Ans.*

Since the commission is 3 %, or .03, on \$5650, it must be 3 hundredths of \$5650, or \$5650 × .03, which is \$169.50. Therefore, etc.

RULE. *Multiply the given amount by the rate per cent. expressed decimally.*

What is a Commission Merchant, or Factor? A Broker? What is Commission? *Brokerage?* The Base? Explain the operation. The Rule.

Examples.

2. An agent collects debts to the amount of \$965; what is his commission at 2 %? *Ans.* \$19.30.

3. What is the brokerage for purchasing \$10000 of gold, at $\frac{1}{4}$ %?

4. A commission merchant disposes of 718 barrels of flour at \$7.13 per barrel; what is his commission at $4\frac{1}{4}$ per cent? *Ans.* \$217.57+.

5. If a broker charges $\frac{1}{2}$ % for exchanging \$5670 of currency, how much is the brokerage? *Ans.* \$28.35.

6. If my agent charged 5 % for collecting a debt of \$564.40, how much will he have to pay over to me, after deducting his fee?

SOLUTION. \$564.40 × .05 = \$28.22; \$564.40 — \$28.22 = \$536.18, *Ans.*

7. What is the fee of a tax collector for collecting \$19500, at $1\frac{1}{2}$ %, and how much will he have to pay over? *Ans.* Fee, \$292.50; pay over, \$19207.50.

277. When the given amount includes the commission or brokerage.

The given amount divided by \$1, increased by the commission or brokerage on \$1, will denote the sum to be invested.

The sum to be invested subtracted from the given amount, will give the commission or brokerage.

8. A broker receives \$6045, with instructions to deduct his brokerage, $\frac{3}{4}$ %, and invest the balance in bank stock; what sum had he to invest, and what was his brokerage?

OPERATION.

\$6045 ÷ \$1.00$\frac{3}{4}$ = \$6000, to be invested.
\$6045 — \$6000 = \$45, brokerage.

Since the brokerage is $\frac{3}{4}$ % of the sum to be invested, the broker receives \$1.00$\frac{3}{4}$ for each dollar to be invested, and will have for investing as many dollars as \$1.00$\frac{3}{4}$ is contained times in \$6045, or \$6000; and the *difference* between the sum he receives and the sum to be invested, or *\$45, must be his* brokerage.

9. An agent receives $3838.80 to lay out in purchase of goods, deducting his commission at 5 %; what value of goods shall the owner receive? *Ans.* $3656.

10. An agent receives $581.85 to lay out in coal, deducting his commission at $2\frac{1}{2}$ %; what value of coal can he purchase?

11. I remit a commission merchant in Chicago $2050, for the purchase of flour; the flour costs $10 a barrel, and his commission is $2\frac{1}{2}$ %; after deducting his commission, how many barrels will the money enable him to send me?

Ans. 200 barrels.

12. A broker has $11000 to invest, after deducting his fees at the rate of $\frac{7}{8}$ %; what sum is there to be invested?

13. Sent an agent in New Orleans $64890 to purchase a quantity of cotton; if he deduct his commission at 3 %, and with the balance purchases 700 bales, how much is the cost per bale? *Ans.* $90.

INSURANCE.

278. Insurance is an obligation by which one party is bound to indemnify another for a loss which he may sustain.

PROPERTY INSURANCE has reference to houses, mills, ships, goods, etc.

HEALTH and LIFE INSURANCE have reference to risks on health and life.

The party taking the risk is the *Insurer* or *Underwriter*, and the party protected the *Insured.*

279. The **Policy** is the written contract made by the parties.

280. The **Base** is the amount insured; and

281. The **Premium** is the percentage charged for the insurance.

What is Insurance? Property insurance? Health or Life insurance? The Policy? The Base? The Premium?

282. To find the premium of insurance.

1. What is the premium on a policy of \$3840, at $4\frac{1}{2}$ per cent.?

OPERATION.

$\$3840 \times .04\frac{1}{2} = \$172.80$, *Ans.*

Since the premium is $4\frac{1}{2}$ %, or $.04\frac{1}{2}$, on \$3840, it must be $.04\frac{1}{2}$ of \$3840, or \$172.80.

RULE. *Multiply the amount insured by the rate per cent. of premium expressed decimally.*

Examples.

2. What is the premium for insurance of \$3560 on a dwelling house, at 2 %? *Ans.* \$71.20.

3. What is the cost of insuring a cargo of corn, worth \$5000, at 3 %, and \$1 for the policy?

4. If a person 36 years old takes out a life policy for \$5000, payable at death, or when he is 45 years old, how much will he have paid, if he shall live to the latter age, the annual premium being \$541.30? *Ans.* \$4871.70.

5. What is the cost of insuring \$7500 on a store and goods, at $2\frac{1}{2}$ %?

6. A ship and cargo are insured for \$98000, at $3\frac{1}{4}$ %; if they should be destroyed by fire or storm, what would be the actual loss of the insurers? *Ans.* \$94815.

PROFIT AND LOSS.

283. Profit and Loss are commercial terms denoting gain and loss in business transactions.

284. The **Base** is the cost price, or the quantity on the which the gain or loss accrues.

The *Examples* are like those of the four cases in Percentage, and may be solved by the application of rules already given, or by analysis.

Explain the operation. Repeat the Rule. What do the terms Profit and Loss denote? What is the Base? How may the examples be solved?

CASE I.

285. To find any given per cent. of profit or loss on a quantity.

1. If a house be bought for \$8500, and sold at a gain of 12 per cent., what was the gain?

OPERATION.

$\$8500 \times .12 = \$1020$, *Ans.*

Since the house was bought for \$8500, and sold at a gain of 12 %, the gain was 12 %, or .12, of \$8500, or \$1020. That is,

The profit or loss is equal to the base multiplied by the rate per cent., expressed decimally.

2. Mr. Edson bought flour on speculation for \$2340, but he was obliged to sell it at 15 % below cost; how much was his loss? *Ans.* \$351.

3. Sold goods for which were paid \$8500, at a gain of $21\frac{1}{2}$ per cent.; how much was the profit?

4. A man sold a farm, for which he gave \$5000, at 9 per cent. below cost; how much was his loss? *Ans.* \$450.

5. If cloth cost \$4 a yard, at what price must it be sold to gain 25 %?

SOLUTION. *25 %, or $\frac{1}{4}$, of \$4, is \$1; \$4 + \$1 = \$5 per yard, Ans.*

6. If sugar cost 12 cents a pound, at what price must it be marked to make $33\frac{1}{3}$ % profit? *Ans.* 16 cents per pound.

7. To make a profit of $12\frac{1}{2}$ % per pound, at what selling price must tea, that cost 80 cents per pound, be marked?

8. A trader having some shop-worn articles that cost him \$130, has marked them down 10 % below cost; what is his reduced price? *Ans.* \$117.

9. A merchant bought 60 casks of molasses, each containing 63 gallons, for \$1512; at what price per gallon must it be sold for him to gain 15 per cent.? *Ans.* 46 cents per gallon.

Explain the operation. To what is the profit or loss equal?

CASE II.

286. To find the rate per cent. of profit or loss.

1. If a house be bought for \$8500, and sold for \$1020 more than the cost, what was the gain per cent.?

OPERATION.

$1020 \div 8500 = .12$, or 12 %, *Ans.*

Since the gain is \$1020 on \$8500, it is $\frac{1020}{8500} = .12$, and since $.12 = 12\,\%$, the gain was 12 %. That is,

The rate per cent. of profit or loss is equal to the profit or loss divided by the base, with the division extended to hundredths.

2. From a cask containing 84 gallons, 14 have leaked out; what per cent. is the loss? *Ans.* $16\frac{2}{3}$ %.

3. If a merchant sell cloth, which cost him \$4.50, at a profit of \$.90, what is the gain per cent.?

4. When cloth, which cost \$4.50 per yard, is sold at \$6, what is the gain per cent.? *Ans.* $33\frac{1}{3}$ %.

5. Bought a horse for \$250, paid for keeping him \$10, and sold him for \$234; what was the loss per cent.? *Ans.* 10 %.

6. A merchant bought 50 barrels of flour for \$10.20 a barrel, and was obliged to lose \$85.00 in selling it; what per cent. was his loss? *Ans.* $16\frac{2}{3}$ %.

7. If I sell $\frac{1}{2}$ of a lot of land at $\frac{4}{5}$ of its cost, what is the profit per cent.? *Ans.* 60 %.

8. If I sell $\frac{3}{4}$ of a cord of wood at the cost of a cord, what is the gain per cent.? *Ans.* $33\frac{1}{3}$ %.

9. Sold a house for \$5000, and gained \$500; what would have been the loss per cent. if I had sold it at \$4000? *Ans.* $11\frac{1}{9}$ %.

CASE III.

287. To find the base of profit or loss.

1. What is the cost of a house, if in selling it at a profit of \$1020, the gain is at the rate of 12 per cent.?

Explain the operation. To what is the rate per cent. of profit or loss *equal? Explain the* operation.

OPERATION.

$$\frac{\$1020 \times 100}{12} = \$8500, \textit{Ans.}$$

Or,

$$\$1020 \div .12 = \$8500, \textit{Ans.}$$

Since 12 % of the cost is \$1020, 1 % of it is $\frac{1}{12}$ of \$1020, or \$85, and 100 %, or the whole cost, is 100 times \$85, or \$8500.

Or, since 12 %, or .12, of the cost is \$1020, the cost must be $\frac{1}{.12}$ of \$1020, or \$1020 ÷ .12 = \$8500. That is,

The base is equal to the profit or loss divided by its rate per cent., expressed decimally.

2. If by selling cloth at \$.90 per yard above cost, the gain is 20 %, what was the cost? *Ans.* \$4.50 per yard.

3. A house was sold at a loss of \$15, which was $1\frac{1}{4}$ % of the cost; what was the cost? *Ans.* \$1200.

4. Sold coal at 125 % of its cost and thereby gained \$1.75 per ton; what was the cost per ton?

SOLUTION. 125 % — 100 % = 25 %; \$1.75 ÷ .25 = \$7 per ton, *Ans.*

5. Sold cloth at 95 % of its cost, and thereby lost \$.30 on a yard; what was the cost per yard? *Ans.* \$6.

6. Sold coal at \$8.75 per ton, and thereby gained 25 %; what was the cost per ton?

SOLUTION. Since \$8.75 is 25 % more than the cost, it is equal to 125 % of the cost. If 125 %, or 1.25, of the cost is equal to \$8.75, the cost must be equal to $\frac{1}{1.25}$ of \$8.75, or \$8.75 ÷ 1.25 = \$7. That is,

The cost is equal to the selling price divided by 1 *increased by the rate per cent. of profit, or by* 1 *diminished by the rate per cent. of loss, expressed decimally.*

7. Sold a horse for \$204, and lost 15 %; what did the horse cost me? *Ans.* \$240.

8. By selling molasses at \$.55 a gallon, I suffer a loss of $8\frac{1}{3}$ %; what was the cost price?

9. By selling wheat at \$1.70 a bushel, I gain 18 %; what was the cost price? *Ans.* \$1.44+.

10. If 700 barrels of flour are sold for \$4550, and thereby at a gain of 4 %, what was the cost per barrel? *Ans.* \$6.25.

To what is the base equal? Explain the solution of example 6. To what is the cost equal?

REVIEW EXERCISES.

1. If wood, which should be cut 4 feet in length, fall short 3 inches, what per cent. should the price be reduced to correspond? *Ans.* $6\frac{1}{4}$ %.

2. A merchant paid $25 commission on sales amounting to $1250; at what rate per cent. was the commission?

3. If I pay a premium of $550 for insurance at 5 %, what is the amount for which the property is insured?

4. Sold corn at $1.00 a bushel, and gained 25 %; what per cent. should I have gained, had it been sold at $.88 a bushel?

SOLUTION. *Since* $1.00 *is* 25 % *more than the cost, it is equal to* 125 % *of the cost. If* 125 %, *or* 1.25, *of the cost is equal to* $1.00, *the cost must be equal to* $\frac{1}{1.25}$ *of* $1.00, *or* $.80.

Since the cost is $.80, *if it should be sold for* $.88, *the gain would be* $.08 *on* $.80, *and if the gain should be* $.08 *on* $.80, *it would be* $\frac{.08}{.80}$ = .10, *or* 10 %. *Therefore, etc.*

5. Sold tea at $.88 a pound, and made 10 %; what per cent. should I have made if I had sold it at $1.00 a pound?

6. Sold a horse, at a loss of 12 %, for $132; what per cent. should I have gained, if it had been sold at $159? *Ans.* 6 %.

7. Goods marked at 30 per cent. above cost, having depreciated in value, were sold at 28 % off the price as marked; what per cent. did I gain or lose?

SOLUTION. *Since* 30 % *above cost is* 130 % *of the cost,* 28 % *off the marked price must be* 28 % *of* 130 % *of the cost, or* $36\frac{2}{5}$ % *of the cost.* 130 % — $36\frac{2}{5}$ % = $93\frac{3}{5}$ %, *and if sold at* $93\frac{3}{5}$ % *of the cost, they must have been sold at* 100 %, *or cost, less* $93\frac{3}{5}$ %, *or at a loss of* $6\frac{2}{5}$ %. *Therefore, etc.*

8. How much is made or lost by selling goods marked 25 % above cost, at 20 % off? *Ans.* Nothing.

9. Sold 2 farms for $6000 each; on the one I made 25 %, and on the other I lost 25 %; what did I gain or lose on the whole? *Ans.* Lost $800.

REVIEW QUESTIONS. What is Commission? (274) Brokerage? (275) Insurance? (278) Profit and Loss? (283) What is the Base of commission or brokerage? (275) Of Insurance? (280) Of Profit and Loss? (284)

INTEREST.

288. Interest is percentage allowed for the use of money, or for value received.

The PRINCIPAL is the sum on interest.

The RATE of interest is the rate per cent. for one year, or any given time.

The AMOUNT is the sum of the principal and interest.

289. Legal Interest is that reckoned by the rate per cent. established by law.

USURY is illegal interest.

The rate per cent. of legal interest varies in different states and countries.

In most of the United States, Canadas, and Nova Scotia, the legal rate, especially where no other is mentioned, is 6 per cent. In some of the States it is 7 per cent., and in others, any rate which may be agreed upon between the parties.

The rate for one year, and at 6 per cent., is to be understood in this book when no other is named.

SIMPLE INTEREST.

290. Simple Interest is that which is reckoned on the given principal alone for the given time.

291. Since the interest of $1, at 6 per cent. for 1 year, or 12 months, is 6 cents, for 1 month it will be $\frac{1}{12}$ of 6 cents, which is 5 mills, and for 2 months, 2 times 5 mills, or 1 cent; also,

Since the interest for 1 month, or 30 days, is 5 mills, the interest for 6 days, or $\frac{1}{5}$ of 30 days, will be 1 mill, and for 2 days, 3 days, etc., as many sixths of a mill as there are days. Hence, the

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What is Interest? The Principal? The Rate? The Amount? Legal Interest? Usury? What is the legal rate in most of the United States? In some of the others? What rate is understood in this book when no other is named? What is Simple Interest?

Table.

Interest of $1, at 6 per cent.,

For	12 months,	or	1 year,		is	$0.06.
"	2 months,	"	$\frac{1}{6}$ year,		"	0.01.
"	1 month,	"	$\frac{1}{12}$ year,		"	0.005.
"	6 days,	"	$\frac{1}{5}$ month,		"	0.001.
"	1 day,	"	$\frac{1}{30}$ month,		"	$0.000\frac{1}{6}$.

292. Since the interest of $1, at 6 per cent. for 2 months is 1 cent, or $\frac{1}{100}$ of the principal, for 200 months, or 16 years and 8 months, it must be 100 cents, or equal to the principal; and as this holds true of any sum, at that rate, we have the

Table.

Interest for	200 mo.,	or	16 y. 8 mo.,	equal	the whole	Principal.
"	100 mo.,	"	8 y. 4 mo.,	"	$\frac{1}{2}$	"
"	50 mo.,	"	4 y. 2 mo.,	"	$\frac{1}{4}$	"
"	25 mo.,	"	2 y. 1 mo.,	"	$\frac{1}{8}$	"
"	20 mo.,	"	1 y. 8 mo.,	"	$\frac{1}{10}$	"
"	10 mo.,	"	$\frac{5}{6}$ y.,	"	$\frac{1}{2}$ of $\frac{1}{10}$	"
"	$6\frac{2}{3}$ mo.,	"	6 mo. 20 d.,	"	$\frac{1}{3}$ of $\frac{1}{10}$	"
"	5 mo.,	"	$\frac{5}{12}$ y.,	"	$\frac{1}{4}$ of $\frac{1}{10}$	"
"	2 mo.,	"	60 d.,	"	$\frac{1}{100}$	"
"	1 mo.,	"	30 d.,	"	$\frac{1}{2}$ of $\frac{1}{100}$	"
"	$\frac{1}{2}$ mo.,	"	15 d.,	"	$\frac{1}{4}$ of $\frac{1}{100}$	"
"	$\frac{2}{5}$ mo.,	"	12 d.,	"	$\frac{1}{5}$ of $\frac{1}{100}$	"
"	$\frac{1}{3}$ mo.,	"	10 d.,	"	$\frac{1}{6}$ of $\frac{1}{100}$	"
"	$\frac{1}{5}$ mo.,	"	6 d.,	"	$\frac{1}{1000}$	"
"	$\frac{1}{10}$ mo.,	"	3 d.,	"	$\frac{1}{2}$ of $\frac{1}{1000}$	"
"	$\frac{1}{15}$ mo.,	"	2 d.,	"	$\frac{1}{3}$ of $\frac{1}{1000}$	"
"	$\frac{1}{30}$ mo.,	"	1 d.,	"	$\frac{1}{6}$ of $\frac{1}{1000}$	"

293. In computing interest, it is customary, among business men, to regard a month as a twelfth part of a year, and any number of days less than a month, as a part of 30 days.

Repeat the Table. In what time is the interest, at 6 per cent., equal to the whole principal?

In dealing with the United States government, however, each day's interest is $\frac{1}{365}$ of the year's interest.

CASE I.

294. To find the interest, the principal, rate per cent., and time, being given.

1. Let it be required to find the interest of \$640 for 3 years 7 months and 3 days, at 7 per cent.

FIRST OPERATION.

Principal,	\$640
Rate,	.07
Int. for 1 y. =	\$44.80
	$3.59\frac{1}{6}$
	40320
	22400
	13440
	746+
Int. for $3.59\frac{1}{6}$ y. =	\$160.9066+

Since the rate is 7 %, or .07, the interest of \$640 for 1 year is .07 of \$640, or \$44.80.

If the interest for 1 year is \$44.80, for 3 years 7 months and 3 days, or $3.59\frac{1}{6}$ years, it must be $3.59\frac{1}{6}$ times \$44.80, or \$160.9066 +.

SECOND OPERATION.

Principal,		\$640
Rate,		.07
Int. for 1 yr.	=	\$44.80
		3
Int. for 3 y.	=	\$134.40
" 6 mo.	=	22.40
" 1 mo.	=	3.733+
" 3 d.	=	.373+
Int. for 3 y. 7 mo. 3 d.	=	\$160.906+

Since the interest for 1 year is 7 %, or .07 of \$640, or \$44.80, for 3 years it is 3 times \$44.80, or \$134.40.

7 months is equal to 6 months and 1 month. If the interest for 1 year is \$44.80, it must be for 6 months, or *one half* of a year, one half of \$44.80, or \$22.40; for 1 month, or *one sixth* of six months, one sixth of \$22.40, or \$3.733+; and for 3 days, or *one tenth* of a month, one tenth of \$3.733+, or \$.373+.

How is it customary to regard each month and each day? In dealing with the United States government, what is each day's interest? Explain the first operation. The second.

If the interest for 3 years is $134.40, for 6 months $22.40, for 1 month $3.733+, and for 3 days $.373+, it must be for 3 years, 7 months, 3 days, the sum of those interests, or $160.906+.

GENERAL RULE. *Multiply the principal by the rate, expressed decimally, and the product by the number expressing the time in years.* Or,

Find the interest for one year by multiplying the principal by the rate, expressed decimally, and then, for the given time, by aliquot parts.

To find the *amount*, add the principal and interest together.

Examples.

Find the interest of

2. $960.50 for 2 years, at 8 %. *Ans.* $153.68.
3. $150.40 for 4 years, at 5 %. *Ans.* $30.08.
4. $1700 for 5 years, at 6 %.
5. $8000 for 3 years, at $7\frac{3}{10}$ %. *Ans.* $1752.
6. $9080 for 2 years 6 months, at $3\frac{1}{2}$ %. *Ans.* $794.50.
7. $71.20 for 1 year 8 months, at $4\frac{1}{4}$ %. *Ans.* $5.043+.
8. $30.16 for 1 year 10 months, at 7 %.
9. $56.78 for 3 years 11 months, at 10 %. *Ans.* $22.23.
10. $300 for 2 years 7 months 15 days, at 6 %. *Ans.* $47.25.
11. $444 for 6 years 5 months 7 days, at $5\frac{1}{2}$ %. *Ans.* $157.16.
12. $19000 for 2 years 2 months 2 days, at 9 %.
13. $2000 for 5 years 4 months 10 days, at $7\frac{3}{10}$ %. *Ans.* $782.72+.

14. Find the amount of $575 for 2 years 6 months 15 days, at 6 %. *Ans.* $662.687+.

15. Find the amount of $1234.56 for 8 years 9 months 10 days, at 7 %. *Ans.* $1993.128+.

295. It has been shown (Art. 292), that interest upon any sum at 6 per cent., in 200 months, is equal to the principal; in 2 months, to 1 *hundredth* of the principal; in 6 days, to 1 *thousandth* of the principal, etc. Hence, to find interest at 6 per cent., we may

---

Repeat the Rule. What has been shown with regard to the relation of interest at 6 per cent. to 200 months? To 2 months? To 6 days?

Take such aliquot parts of the principal as the number expressing the time is of 200 *months.* Or,

Multiply the principal by the number expressing one half of the entire months of the time as hundredths, and one sixth of the days as thousandths.

16. What is the interest of \$2440 for 3 years, 11 months, 21 days, at 6 %?

FIRST OPERATION.

Principal,			\$2440.
Int. for 2 y. 1 mo.	$= \frac{1}{8}$ of Principal	=	\$305
" 1 y. 8 mo.	$= \frac{1}{10}$ "	=	244
" 2 mo.	$= \frac{1}{100}$ "	=	24.40
" 15 d.	$= \frac{1}{4}$ of $\frac{1}{100}$ "	=	6.10
" 6 d.	$= \frac{1}{1000}$ "	=	2.44
Int. for 3 y. 11 mo. 21 d.		=	\$581.94

SECOND OPERATION.

Principal,	\$2440
	$.238\frac{1}{2}$
	19520
	7320
	4880
	1220
Int. for 3 y. 11 mo. 21 d.,	\$581.94

The second operation may be analyzed thus:—

Since the interest of any sum at 6 %, for 2 months, is equal to .01 of the principal, for 3 years and 11 months, or 47 months, it must be as many times .01 of the principal as 2 months are contained times in 47 months, which are $23\frac{1}{2}$, and $23\frac{1}{2}$ times .01 are .235.

Since the interest of any sum at 6 % for 6 days is equal to .001 of the principal, for 21 days it must be as many times .001 as 6 days are contained times in 21 days, which are $3\frac{1}{2}$ times, and $3\frac{1}{2}$ times .001 are $.003\frac{1}{2}$.

How may we find interest at 6 per cent.? Explain the second operation.

Since the interest for 3 years and 11 months is .235 of the principal, and for 21 days, $.003\frac{1}{2}$, it must be for 3 years 11 months 21 days, $.235 + .003\frac{1}{2}$, or $.238\frac{1}{2}$, of the principal, and $.238\frac{1}{2}$ of $2440 is $581.94.

Find the interest of

17. $64.24 for 5 months and 6 days, at 6 %. *Ans.* $1.67.
18. $19.60 for 2 years and 11 months, at 6 %.
19. $75 for 1 year 4 months 20 days, at 6 %. *Ans.* $6.25.
20. $1000 for 2 years 4 months 9 days, at 6 %. *Ans.* $141.50.
21. $2000 for 8 months, at 6 %. *Ans.* $80.
22. $600.80 for 15 months, at 6 %. *Ans.* $45.06.
23. $36400 for 42 days, at 6 %.

SOLUTION. $36400 × .007 = $254.80, *Ans.*

24. $1200 for 25 days, at 6 %. *Ans.* $5.
25. $3540 for 57 days, at 6 %. *Ans.* $33.63.

296. When the time is short, we may, as a convenient method of finding interest at 6 per cent.,

Take one hundredth of the principal, by removing the decimal point two places toward the left, for the interest for two months, or sixty days; or one thousandth of the principal by removing the decimal point three places toward the left, for the interest for six days; and then find the interest for the given time by aliquot parts.

26. Required the interest of $4800 for 93 days, at 6 %.

FIRST OPERATION.

Principal,		$4800
Int. for 60 d.	$= \frac{1}{100}$ of the principal $=$	48.00
" 30 d.	$= \frac{1}{2}$ of $\frac{1}{100}$ " $=$	24.00
" 3 d.	$= \frac{1}{20}$ of $\frac{1}{100}$ " $=$	2.40
Int. for 93 d.	$=$	$74.40

What is a convenient method of finding interest at 6 per cent. when the time is short?

SECOND OPERATION.

$$
\begin{array}{lr}
\text{Principal,} & \$4800 \\
\text{Int. for 6 d.} = \frac{1}{1000} \text{ of the principal} = & 4.80 \\
\qquad 93 \text{ d.} \div 6 \text{ d.} \qquad\qquad = & 15\frac{1}{2} \\
 & 2400 \\
 & 480 \\
 & 240 \\
\text{Int. for 93 d.} = \text{int. for 6 d.} \times 15\frac{1}{2} = & \$74.40
\end{array}
$$

In the second operation, since the interest for 6 days is .001 of the principal, or \$4.80, for 93 days it must be as many times \$4.80 as 6 days are contained times in 93 days, which are $15\frac{1}{2}$; and $15\frac{1}{2}$ times \$4.80 are \$74.40.

Required the interest of

27. \$140 for 123 days, at 6 %. *Ans.* \$2.87.
28. \$44.80 for 4 months and 9 days, at 6 %. *Ans.* \$.963.
29. \$3000 for 63 days, at 6 %.
30. \$1120.60 for 6 months and 20 days, at 6 %. *Ans.* \$37.35.
31. \$8000 for 15 days, at 6 %. *Ans.* \$20.
32. \$1880.85 for 1 month and 3 days, at 6 %. *Ans.* \$10.34+.

297. For any other rate than 6 per cent., we may, when more convenient than to apply the rule (Art. 294),

Find the interest at 6 per cent., and then increase or diminish this interest by such a part of itself, as will give the interest at the given rate.

It will be observed, that interest at 3 % is $\frac{1}{2}$ of the interest at 6 %, at 5 % is equal to the interest at 6 % — $\frac{1}{6}$ of itself, at 7 % is equal to the interest at 6 % + $\frac{1}{6}$ of itself, etc.

What is the interest of

33. \$1385.50 for 23 days, at 7 % ? *Ans.* \$6.196+.
34. \$3600 for 66 days, at 7 % ? *Ans.* \$46.20.
35. \$1600 for 21 days, at 5 % ? *Ans.* \$4.66.
36. \$15600 for 13 days, at 5 % ?

Explain the second operation. How may we find the interest for any other rate than at 6 per cent.?

37. $21.40, for 11 months, at 7 %? *Ans.* $1.37.

38. $3.70 for 14 months, at 5 %? *Ans.* $.216.

39. $300 for 1 year and 6 months, at 4 %? *Ans.* $18.

40. $750.40 for 2 years and 3 months, at 9 %? *Ans.* $151.956.

41. $344.45 for 2 years, 2 months, 3 days, at 7 %? *Ans.* $52.44+.

42. $68.75 for 1 year, 4 months, 10 days, at 7 %?

43. $3976.18 for 2 years, 4 months, 8 days, at 8 %?

What is the amount of

44. $80 for 1 year, 5 months, 12 days, at 6 %? *Ans.* $86.96.

45. $241.20 for 6 months and 20 days, at 7 %? *Ans.* $250.58.

46. $500 from May 16th to November 3d following, at 10 %? *Ans.* $523.33+.

Here, by Art. 255, we find, by taking the number of *entire* calendar months from the first date, and the *exact* number of days left, that the time is 5 months and 18 days.

47. $345.94 from December 8th to March 4th following, allowing the intervening February to have 28 days, at 7 %? *Ans.* $351.59.

48. $800 from June 20, 1866, to July 8, 1867, at 5 %? *Ans.* $842.

49. $1000 from August 31, 1865, to September 15, 1868, at 6 %? *Ans.* $1182.50.

298. When it is required to compute interest very exactly, count the actual number of days in each calendar month included in the time, and make each day's interest $\frac{1}{365}$ of a year's interest. That is,

Multiply the interest of the principal for one year at the given rate by the number of days in the time, and divide by 365.

How is the time found in example 46? When it is required to compute interest very exactly, how must the time be counted? What must each day's interest be made? How is the interest found?

50. What is the interest on a government bond of $500 for 100 days, at 6 %?

Solution. The interest of $500 for 1 year at 6 % is $30. If the interest for 1 year is $30, for 100 days, or $\frac{100}{365}$ of 1 year, it must be $\frac{100}{365}$ of $30, or $\frac{\$30 \times 100}{365} = \$8.22$.

51. What is the interest on a note of $1000 from June 15 to October 31 following, at $7\frac{3}{10}$ %? *Ans.* $27.60.

52. What is the interest of $6400 from February 24, 1864, to January 30, 1865, at 5 %? *Ans.* $298.95.

CASE II.

299. To find the rate, the principal, interest, and time, being given.

1. At what rate per cent. must $960.50 be on interest to gain $153.68 in 2 years?

OPERATION.

Int. of $960.50 at 1 % = $19.21
$153.68 ÷ $19.21 = 8.

Since the interest of $960.50 for 2 years, at 1 %, is $19.21, it must require as many times 1 % to gain $153.68 as $19.21 is contained times in $153.68, which is 8. Therefore, etc.

Rule. *Divide the given interest by the interest of the principal for the given time at* 1 *per cent.*

Examples.

At what rate of interest will

2. $75 gain $6.25, in 1 year, 4 months, 20 days? *Ans.* 6 %.
3. $3000 gain $525, in 2 years and 4 months?
4. $3600 gain $46.20, in 66 days? *Ans.* 7 %.
5. $150 gain $30, in 4 years? *Ans.* 5 %.
6. $444 gain $156.695, in 6 years, 5 months? *Ans.* $5\frac{1}{2}$ %.

CASE III.

300. To find the time, the principal, interest, and rate, being given.

1. In what time will the interest of $960.50 be $153.68, at 8 per cent.?

Explain the operation. Repeat the Rule.

OPERATION.

Int. of \$960.50 for 1 year = \$76.84.
\$153.68 ÷ \$76.84 = 2.

Since the interest of \$960.50 for 1 year, at 8 per cent., is \$76.84, it must require as many years to gain \$153.68, as \$76.84 is contained times in \$153.68, which is 2. Therefore, etc.

RULE. *Divide the given interest by the interest of the principal for* 1 *year, at the given rate.*

Examples.

In what time will the interest of

2. \$3000 be \$525, at 7 %? *Ans.* 2 y. 6 mo.
3. \$700 be \$63, at 6 %?
4. \$4080 be \$668.10, at 5 %? *Ans.* 3 y. 3 mo. 9 d.
5. \$444 be \$157.16, at $5\frac{1}{2}$ %? *Ans.* 6 y. 5 mo. 7 d.
6. \$225 be \$77.40, at 6 %? *Ans.* 5 y. 8 mo. 24 d.

CASE IV.

301. To find the principal, the interest, time, and rate, being given.

1. What principal will gain \$153.68, in 2 years, at 8 per cent.?

OPERATION.

Int. of \$1 for 2 years = \$.16.
\$153.68 ÷ \$.16 = 960.50.

Since the interest of \$1 for 2 years, at 8 %, is \$.16, it must require as many dollars of principal to gain \$153.68 as \$.16 is contained times in \$153.68, or 960.50. Therefore, etc.

RULE. *Divide the given interest by the interest of* \$1 *for the given time, at the given rate.*

Examples.

What principal will gain

2. \$63, in 1 year and 6 months, at 6 %? *Ans.* \$700.

Explain the operation. Repeat the Rule. Explain the operation of Case IV. Repeat the Rule.

3. $1752, in 3 years, at $7\frac{3}{10}$ % ? *Ans.* $8000.

4. $581.94, in 3 years, 11 months, 21 days, at 6 % ?

5. $9.38, in 6 months and 20 days, at 7 % ? *Ans.* $241.20.

6. $151.875, in 2 years and 3 months, at 9 % ? *Ans.* $750.

APPLICATIONS.

1. If a man gives his note July 15, 1866, for $400, on interest, what sum will pay it January 21, 1867? *Ans.* $412.40.

2. If a capitalist has $20000 loaned, one half at 6 %, and the other at 7 %, what interest will it gain in 2 years, 2 months, 12 days? *Ans.* $2860.

3. How much must be paid for the use of $1200, for 2 months and 3 days, at 1 % a month?

SOLUTION. *If the interest at* 1 % *for* 1 *month is* .01 *of the principal, for* 2 *months and* 3 *days, or* $2\frac{1}{10}$ *months, it will be* $2\frac{1}{10}$ *times* .01, *or* $.02\frac{1}{10}$ *of the principal, and* $.02\frac{1}{10}$ *of* $1200 *is* $25.20.

4. If you lend $250 at 2 % a month, and are paid at the end of 15 days, what amount must you receive? *Ans.* $252.50.

5. Borrowed $200 the 20th July, at $1\frac{1}{2}$ % a month; how much was the interest on the 4th of the following October? *Ans.* $7.50.

6. When $4.268 is paid for the use of $194 for 4 months and 12 days, what is the rate of interest? *Ans.* 6 %.

7. Lent $114 at 7 % interest; on its return it had gained $13.30; how long had it been on interest? *Ans.* 1 y. 8 m.

8. In what time will any sum double itself by simple interest, at 6 %?

SOLUTION. *Since the interest in one year is* 6 % *of the principal, it must require, for a sum at interest to double, or for the interest to equal* 100 % *of the principal, as many years as* 6 % *is contained times in* 100 %, *or* $16\frac{2}{3}$ *years, which is* 16 *years and* 8 *months. Therefore, etc.*

9. In what time will any sum double itself by simple interest, at 7 %? At $7\frac{3}{10}$ %?

Give the solution of example 3. How many per cent. must the interest be to equal the principal? Give the solution to example 8.

10. If I am paid $37.26 as the interest due on money lent for 2 years and 17 days, at 7 %, what was the sum lent?

Ans. $260.00.

11. What would be the difference between the interest of $10000 from July 1st to January 1st following, computed first by months, and then by days, counting 365 days to the year?

Ans. $2.465+ more by the latter method.

PRESENT WORTH.

302. The **Present Worth** of a sum of money, payable at a future time without interest, is such a sum as, being placed at interest, at the given rate, will amount to the debt when it becomes due.

The DISCOUNT is the interest on the present worth, deducted or abated from the *apparent* value of the debt, for present payment. It is the difference between the *real* and *apparent* value of the debt, and, for distinction, is called ***true discount.***

303. To find the present worth of any sum.

1. Find the present worth of $480, due in 4 years, without interest, money being worth 5 %.

OPERATION.

Amount of $1 for 4 y. = $1.20
$480 ÷ $1.20 = 400

Since $1, at 5 % interest, in 4 years, amounts to $1.20, it will require as many dollars to amount to $480, at the same rate, for the same time, as $1.20 is contained times in $480, or 400.

RULE. *Divide the given sum by the amount of* $1 *for the given time, at the given rate.*

To find the discount, subtract the present worth from the given sum.

Examples.

Find the present worth of

2. $250, due in 6 months, at 6 %. *Ans.* $242.71+.
3. $900, due in 72 days, at 7 %. *Ans.* $887.57+.

What is Present Worth? Discount? Explain the operation. Repeat the Rule. How do you find the discount?

4. $650, due in 1 year and 4 months, at 8 %.

5. $347.25, due in 2 years, 7 months, 15 days, at 6 %.
Ans. $300.

What is the discount on

6. $672, due 2 years hence, at 6 % ? *Ans.* $72.

7. $350.75, due in 93 days, at 6 % ? *Ans.* $5.36+.

8. $750, due in 2 years, 3 months, 20 days, at 7 % ?
Ans. $104.23+.

304. Since the *present worth* corresponds to the *principal*, the *debt* to the *amount*, and the *discount* to the *interest* of the principal for the time and at the rate given, the rule also applies, when *the time, rate, and amount are given, to find the principal.*

What principal will amount to

9. $1114.18, in 2 years, at 8 % ? *Ans.* $960.50.

10. $3641.20, in 66 days, at 7 % ? *Ans.* $3595.04+.

11. $145.67, in 123 days, at 6 % ?

12. $4748.10, in 3 years, 3 months, 9 days, at 5 % ?
Ans. $4080.

APPLICATIONS.

1. What is the present value of a note for $385, payable in 9 months without interest, money being worth 6 % ?
Ans. $368.42.

2. What is the difference between the interest and discount of $1050, due 10 months hence, at 6 % ? *Ans.* $2.50.

3. Bought goods for $1831.53 cash, and at once sold them for $1986.48, on credit of 6 months; how much did I make, money being worth 5 % ? *Ans.* $106.49.

4. A man was offered a horse for $225, cash in hand, or for $230, payable in 9 months; if he accepts the latter, when money is worth 7 %, how much will he gain by the choice?
Ans. $6.48.

To what does the present worth correspond? The debt? The discount? To what does the rule apply?

BANK DISCOUNT.

305. A **Bank** is a joint stock company, or an incorporation, for the purpose of receiving deposits, loaning money, or issuing notes or bills for circulation.

306. A **Promissory Note** is a written promise to pay absolutely a certain sum of money, for value received.

The FACE of a note is the sum made payable.

DAYS OF GRACE are the three days usually allowed for the payment of a note, after the expiration of the time named on its face.

A note is *nominally* due at the expiration of the time named in it, and is *legally* due at *maturity*, or at the expiration of the days of grace.

When the last day of grace occurs on Sunday, or a holiday, the note is payable the day before.

The time when a note is nominally and when legally due may be indicated by writing the number of the days with a line between them. Thus, May $^{16}|_{19}$.

A note is *discounted* when bought for less than its face.

The *time to run*, or *term of discount*, of a note, is the time from the day of discounting to the maturity.

307. **Bank Discount** is an allowance to a bank for payment of money on a note before it is due.

It is the interest on the face of the note for the term of discount.

The PROCEEDS, or AVAILS, of a note is the sum paid for it, or the face of the note less the discount.

CASE I.

308. To find the bank discount or proceeds of a note.

1. What is the bank discount and proceeds of a note for $500, for 90 days, at 6 %?

What is a Bank? A Promissory Note? The Face of a note? Days of Grace? When is a note legally due? What is Bank Discount? The Proceeds? The time to run?

OPERATION.

Interest of \$500 for 60 d. = \$5.00
" " " 30 d. = 2.50
" " " 3 d. = .25

Interest of \$500 for 93 d. = \$7.75, discount.
\$500 — \$7.75 = \$492.25, proceeds.

RULE. *Find the interest on the face of the note for the term of discount, at the given rate, and it will give the discount.*

The discount subtracted from the face of the note will give the proceeds.

Notes discounted usually draw no interest till maturity. If, however, a note is *at interest*, the face of the note is the *amount* at maturity.

The difference between bank and true discount is equal to the interest on the true discount.

Examples.

2. A sixty days' note for \$600 was dated and discounted on the same day, at 6 %; required the discount and proceeds.
Ans. Discount \$6.30; proceeds \$593.70.

3. A note for \$250, in 4 months, was dated and discounted Dec. 31, at 1 % a month; required when due, and the amount of proceeds. *Ans.* Due April 30 | May 3; proceeds \$239.75.

Find when due, time to run, discount, and proceeds, of the following notes: —

4. \$1650$\frac{40}{100}$. New York, July 5, 1866.

Four months after date, for value received, I promise to pay Horatio Sheridan, or order, one thousand six hundred fifty $\frac{40}{100}$ dollars at the Manhattan National Bank.

[Stamp]. Charles N. Thayer.

Discounted Sept. 5, at 7 %.

Ans. Due Nov. 5 | 8; to run 2 mo. 3 d.; discount \$20.217; proceeds \$1630.183.

Explain the operation. Repeat the Rule. How does bank discount compare with true discount? To what is their difference equal?

5. $5000. St. Louis, June 10, 1866.

Ninety days after date, I promise to pay at the order of S. Clark & Co. five thousand dollars, value received.

[Stamp.] William Kaspar.

Discounted July 13, at 6 %.

Ans. Due Sept. $^8|_{11}$; to run 60 d.; discount $50; proceeds $4950.

CASE II.

309. To find the face of a note, its proceeds being given.

1. What must be the face of a note at 90 days, which, when discounted at 6 %, will give $492.25?

OPERATION.

Proceeds of $1 for 93 d. = $.9845

$492.25 ÷ $.9845 = 500

Since $1 discounted for the given time and rate gives $.9845 proceeds, there must be required to give $492.25 proceeds, as many dollars as $.9845 is contained times in $492.25, or 500.

RULE. *Divide the proceeds of the note by the proceeds of $1, for the given time and rate.*

Examples.

2. Find the face of a four months' note which, when discounted at 1 % a month, yields $239.75. *Ans.* $250.

3. The proceeds of a sixty days' note discounted at 6 %, are $593.70; required the face of the note.

4. I wish to obtain $3755 from a bank; what must be the face of the note, payable in 90 days, at 7 %? *Ans.* $3824.15+.

5. Find the face of a two months' note which, when discounted at 2 % a month, yields $576.

6. A merchant owing $994.50, gave a 30 days' note, which was discounted at 6 %; required the face of the note to pay the exact debt. *Ans.* $1000.

Explain the operation. Repeat the Rule.

ANNUAL INTEREST.

310. Annual Interest is simple interest on the principal and on each year's interest of the principal due and unpaid, when the note is written "with interest annually."

This interest is sanctioned by the courts of some States, in the nature of damages for the detention and use of interest after it is due.

311. To compute annual interest.

1. What is the interest due on a note of $800, interest payable yearly, on which no payments have been made, at the end of 3 years and 9 months?

OPERATION.

Int. of $800 for 3 y. 9 mo.	= $180.00
Int. of $800 for 1 y. = $48; and	
Int. of $48 for 2y. 9 mo. + 1y. 9mo. + 9mo., or for 5y. 3mo. =	15.12
	Annual interest, $195.12

Here, the interest of the principal for the time is $180, and for each year is $48. The first year's interest, after becoming due, remains unpaid 2 y. 9 mo., the second year's, 1 y. 9 mo., and the third year's 9 mo. The interest of $48 for 2 y. 9 mo. + 1 y. 9 mo. + 9 mo., is equal to the interest of $48 for 5 y. 3 mo., or $15.12. Hence, the annual interest is $180 + $15.12, or $195.12.

RULE. *Find the interest of the principal for the whole time; find the interest of one year's interest for the sum of the times each year's interest remains unpaid, and the sum of these interests will be the annual interest.*

Examples.

2. William Norton has L. Dixon's note, dated June 1, 1864, for $500, with interest to be paid annually, at 6 %; what was due June 1, 1867? *Ans.* $595.40.

3. What is the interest due on a note of $200, interest pay-

What is Annual Interest? Explain the operation. Repeat the Rule.

able annually, on which no payments have been made, at the end of 2 years, 6 months, 3 days? *Ans.* $31.55.

4. What was due July 1, 1867, on a note dated May 1, 1863, for $780, with interest payable annually, at 6 %?

5. What was due January 1, 1868, on a note dated July 1, 1865, for $1000, with interest payable annually, at 7 %? *Ans.* $1184.80.

PARTIAL PAYMENTS.

312. **Partial Payments** are payments in part of a note, bond, or like obligation.

The payments, being receipted for by writing upon the back of the obligation, are called INDORSEMENTS.

313. When settlements are made at the end of short periods, or within a year, and on notes for any time, in States where there are no prescribed methods of reckoning interest in case of partial payments, it is customary to compute by the

MERCANTILE RULE.

Find the amount of the principal from the time it began to draw interest, and the amount of each indorsement from the time it was made until settlement.

Subtract the sum of the amounts of the payments from the amount of the principal, and the remainder will be the balance due.

In *mercantile accounts*, the rule may be applied to each specified period for which they are allowed to run, so that a new principal may be formed at the end of every three, six, twelve months, etc., according to the custom of merchants in balancing their ledgers. This method has, even in application to notes, been sanctioned by the courts of some of the States.

What are Partial Payments? What are called Indorsements? By what Rule is it customary to compute when settlements are made within a year? Repeat the Mercantile Rule.

Examples.

(1.) $1728. Chicago, January 1, 1863.

For value received, I promise to pay Raymond, Morris, and Co., or order, on demand, one thousand seven hundred and twenty-eight dollars, with interest.

[Stamp.] Rufus Ogden.

INDORSEMENTS. March 1, 1863, $300; May 16, 1863, $150; September 1, 1863, $270; December 11, 1863, $135.

What is due Dec. 16, 1863?

OPERATION.

Principal,		$1728.00
Interest for 11 months and 15 days,		99.36
Amount,		1827.36
First payment,	$300.00	
Interest for 9 months and 15 days,	14.25	
Second payment,	150.00	
Interest for 7 months,	5.25	
Third payment,	270.00	
Interest for 3 months and 15 days,	4.72	
Fourth payment,	135.00	
Interest for 5 days,	.11	
		879.33
Balance due December 16, 1863,		$948.03

(2.) $700. Montville, February 4, 1864.

For value received, we jointly and severally promise to pay Joseph Perry or order, on demand, seven hundred dollars, with interest.

Jackson Fields.

[Stamp.] Edwin Reed.

REVIEW QUESTIONS. What is Interest? (288) The Principal? (288) The Rate of interest? (288) The Amount? (288) Legal Interest? (289) Usury? (289)

INDORSEMENTS. December 18, 1864, $164; June 24, 1865, $200; September 11, 1865, $120; July 5, 1866, $60.

What was due on this note November 28, 1866?

Ans. $227.87+.

(3.) $500. Milwaukee, January 1, 1866.

For value received, three months after date, I promise to pay to the order of Andrew Benson, five hundred dollars.

[Stamp.] James Lockwood.

INDORSEMENT. January 1, 1867, $200.

What was due April 1, 1867, at 7 % interest?

Ans. $331.50.

314. The United States Courts have adopted the following, called the

UNITED STATES RULE:

Find the amount of the principal to the time of the first payment; if the payment equals or exceeds the interest, subtract it from the amount, and regard the remainder as a new principal.

If any payment be less than the interest due, find the amount of the same principal up to the time when the sum of the payments shall first equal or exceed the interest due, and subtract the sum of the payments from the amount; the remainder regard as a new principal, with which proceed as before.

Examples.

(1.) $1000. Washington, January 1, 1866.

For value received, I promise to pay Albert White, or order, one thousand dollars, on demand, with interest.

[Stamp.] George W. Reves.

INDORSEMENTS. April 1, 1866, $24; August 1, 1866, $4; December 1, 1866, $6; February 1, 1867, $60; July 1, 1867, $40.

What will be due June 1, 1870? *Ans.* $1121.90.

Repeat the United States Rule.

OPERATION.

Principal,		$1000.00
Interest to April 1, 1866, 3 mo.,		15.00
Amount,		$1015.00
First payment, April 1, 1866,		24.00
Remainder for a new principal,		$991.00
Interest to Feb. 1, 1867, 10 mo.,		49.55
Amount,		$1040.55
Second pay't, Aug. 1, 1866, less than int. due,	$4.00	
Third " Dec. 1, 1866, " "	6.00	
Fourth " Feb. 1, 1867, exceeds "	60.00	
		70.00
Remainder for new principal,		$970.55
Interest to July 1, 1867, 5 mo.,		24.26+
Amount,		$994.81+
Fifth payment, July 1, 1867,		40.00
Remainder for a new principal,		$954.81+
Interest to June 1, 1870, 2 y. 11 mo.,		167.09+
Amount due June 1, 1870,		$1121.90+

(2.) 625\frac{50}{100}$. Boston, October 1, 1864.

For value received, we promise to pay Madison Wells, or order, on demand, six hundred twenty-five $\frac{50}{100}$ dollars, with interest.

[Stamp.] Bancroft, Stetson, & Co.

INDORSEMENTS. January 1, 1865, $200; Nov. 1, 1865, $20; January 1, 1866, $300.

How much was there due May 1, 1866? *Ans.* $143.79+.

REVIEW QUESTIONS. What is Simple Interest? (290) In computing interest, how is it customary to regard months and days? (293) What is each day's interest in dealing with the United States Government? (293)

(3.) $2400. New York, May 16, 1864.

For value received, I promise to pay J. L. Weston and Company, or order, on demand, twenty-four hundred dollars, with interest after three months.

J. M. Meigs.

[Stamp.]

INDORSEMENTS. August 16, 1865, $400; May 31, 1866, $67.89.

How much was due November 30, 1866, interest at 7 %?

Ans. $2295.71.

(4.) $5660. New Orleans, May 1, 1863.

For value received, I promise to pay to the order of Louis De Bois five thousand six hundred and sixty dollars, on demand, with interest. John Vincent.

[Stamp.]

INDORSEMENTS. June 16, 1864, $578⅛; Jan. 31, 1865, $160; June 16, 1866, $420.

How much was due February 16, 1867, interest at 5 %?

Ans. $5538.71.

315. The preceding rule modified, so as not to allow of computing interest between payments for any period less than a year, is the

CONNECTICUT RULE.

When at least a year's interest has accrued at the time of a payment, and also in case of the last payment, proceed according to the United States Rule.

When less than a year's interest has accrued at the time of any payment, except the last, take the difference between the amount of the principal for a whole year, and the amount of the payment for the remainder of the year after it was made, for a new principal.

When the interest which has accrued at the time of a payment exceeds the payment, find the interest only upon the principal.

REVIEW QUESTIONS. What is the General Rule for computing simple interest? (294) The Rule for interest at 6 per cent.? (295)

(1.) $1000. New Haven, July 1, 1864.

For value received, I promise to pay to the order of H. B. Bacon, one thousand dollars, on demand, with interest.

[Stamp.] Richard Russell.

INDORSEMENTS. January 1, 1865, $100; September 1, 1866, $223.99; December 25, 1866, $12.

How much was due January 1, 1867? *Ans.* $804.

316. The General Assembly of Vermont, in 1866, by law, established the following as the

VERMONT RULE.

On all notes, bills, or other similar obligations, whether made payable on demand or at a specified time, WITH INTEREST, *where payments are made, such payments shall be applied: first, to liquidate the interest that has accrued at the time of such payments; and, secondly, to the extinguishment of the principal.*

On all notes, bills, or other similar obligations, whether made payable on demand or at a specified time, WITH INTEREST ANNUALLY, *the annual interests that remain unpaid shall be subject to simple interest, from the time they become due to the time of final settlement; but if in any year reckoning from the time such annual interest began to accrue, payments have been made, the amount of such payments at the end of such year with interest thereon from the time of payment shall be applied: first, to liquidate the simple interest that has accrued from the unpaid annual interests; secondly, to liquidate the annual interests that have become due; and thirdly, to the extinguishment of the principal.*

Examples.

(1.) $5000. Montpelier, Vt., Dec. 1, 1867.

For value received, we promise to pay to the order of James Mason, five thousand dollars, on demand, with interest.

[Stamp.] Richardson, Bentley & Co.

INDORSEMENTS. June 1, 1869, $400; December 1, 1869, $2200.

What was due June 1, 1870? *Ans.* $3090

(2.) $1000. Pomfret, Vt., Oct. 1, 1862.

For value received, I promise to pay Andrew Baldwin, or order, one thousand dollars, three years from date, with interest annually. Charles Dayton.

INDORSEMENTS. April 1, 1864, $50; June 1, 1865, $400; August 1, 1865, $200.

What was due at maturity? *Ans.* $526.43.

COMPOUND INTEREST.

317. Compound Interest is interest upon principal and interest, the two being combined at regular intervals of time and converted into a new principal.

The interest may be made a part of the principal, or compounded, annually, semi-annually, quarterly, etc., according to agreement.

318. To find the compound interest of any sum.

1. What is the compound interest of $600 for 2 years and 6 months, at 6 %?

OPERATION.

Principal,	$600.00
Interest for 1st year,	36.00
Amount, or 2d principal,	$636.00
Interest for 2d year,	38.16
Amount, or 3d principal,	$674.16
Interest for 6 months,	20.2248
Amount for 2 y. and 6 mo.,	$694.3848
Given principal,	600
Compound interest for 2 y. and 6 mo.,	$94.3848

What is Compound Interest? How often may the interest be compounded? Explain the operation.

The interest for the first year is \$36, and the amount \$636, which is made a second principal.

The interest for the second year is \$38.16, and the amount, \$674.16, which is made a third principal.

The interest for 6 months, the time at the end of the entire years, is \$20.2248, and the amount \$694.3848.

Subtracting the given principal from the last amount, the compound interest is \$94.3848.

RULE. *Find the amount of the principal for the first interval, and make it the principal of the second interval; then, the amount of the second principal for the second interval, and make it the principal of the third, and so on for all the entire intervals.*

If there be a part of the time more than the entire intervals, find the amount for it.

The last amount will be the amount at compound interest, and it, less the given principal, will be the compound interest.

Examples.

2. What is the amount of \$100, at 6 % compound interest, for 3 years? *Ans.* \$119.10+.

3. What is the compound interest of \$600.50, at 5 %, for 2 years?

4. What is the compound interest of \$300, at 7 %, for 3 years, 4 months, and 15 days? *Ans.* \$77.15+.

5. What is the amount of \$860, at 4 % half yearly compound interest, for 3 years? *Ans.* \$1088.17+.

6. What is the amount of \$500, at 5 % compound interest, for 4 years, 2 months, and 15 days? *Ans.* \$614.08+.

319. The process of computing compound interest may be abridged by means of the following

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Repeat the Rule. How may the process of computing compound interest be abridged?

Table,

SHOWING THE AMOUNT OF $1 FROM 1 TO 20 YEARS, AT 2½, 3, 5, 6, AND 7 PER CENT., COMPOUND INTEREST.

Years.	2½ per cent.	3 per cent.	5 per cent.	6 per cent.	7 per cent.	Years.
1	1.025000	1.030000	1.050000	1.060000	1.070000	1
2	1.050625	1.060900	1.102500	1.123600	1.144900	2
3	1.076890	1.092727	1.157625	1.191016	1.225043	3
4	1.103812	1.125508	1.215506	1.262476	1.310796	4
5	1.131408	1.159274	1.276281	1.338225	1.402552	5
6	1.159693	1.194052	1.340095	1.418519	1.500730	6
7	1.188685	1.229873	1.407100	1.503630	1.605781	7
8	1.218402	1.266770	1.477455	1.593848	1.718186	8
9	1.248862	1.304773	1.551328	1.689478	1.838459	9
10	1.280084	1.343916	1.628894	1.790847	1.967151	10
11	1.312086	1.384233	1.710339	1.898298	2.104852	11
12	1.344888	1.425760	1.795856	2.012196	2.252191	12
13	1.378511	1.468533	1.885649	2.132928	2.409845	13
14	1.412973	1.512589	1.979931	2.260903	2.578534	14
15	1.448298	1.557967	2.078928	2.396558	2.750032	15
16	1.484505	1.604706	2.182874	2.540351	2.952164	16
17	1.521618	1.652847	2.292018	2.692772	3.158815	17
18	1.559658	1.702433	2.406619	2.854339	3.379932	18
19	1.598650	1.753506	2.526950	3.025599	3.616527	19
20	1.638616	1.806111	2.653297	3.207135	3.869685	20

7. What is the compound interest of $400, at 6 %, for 20 years and 6 months?

SOLUTION. Amount of $1 for 20 years = $3.207135; interest of $1 for 6 months = $.03; $3.207135 × .03 = $.09621405 = interest of amount for 6 months; $3.207135 — $1 = $2.207135 = compound interest of $1 for 20 years; $2.207135 + $.096214 = $2.303349 = compound interest of $1 for 20 years and 6 months; and $2.303349 × 400 = $921.3396, or $921.33+ = compound interest of $400 for 20 years and 6 months.

8. What is the amount of $100, at a semi-annual compound interest of 2½ %, for 10 years? *Ans.* $163.86+.

9. What is the amount of \$50, at 7 % compound interest, for 30 years? *Ans.* \$380.61+.

Here, find the amount for 20 years, and then the amount of that sum for 10 years, by aid of the table.

REVIEW EXERCISES.

1. If on settlement with a merchant I give my note for \$5400 payable in 6 months, with 6 % interest, how much must be paid when the note becomes due? *Ans.* \$5562.

2. A certain sum lent at 6 % produced \$250, between July 5, 1865, and December 6, 1866; what was the sum? *Ans.* \$2935.42+.

3. My money at interest doubles itself, I find, in just $14\frac{2}{7}$ years; what is the rate per cent.? *Ans.* 7 %.

4. On the 15th of July, 1866, I paid \$65, the interest due on a note of \$250, at 6 %; from what date did the interest commence? *Ans.* March 15, 1862.

5. How much more is the bank than the true discount on \$800, for 3 years, 4 months, and 18 days? *Ans.* \$27.80+.

6. Having a gold watch to sell, one man offers \$220 payable in two years, and another offers me \$200 cash in hand; which is the better offer, and how much? *Ans.* \$200 cash in hand, by \$3.58+.

7. I have received a note dated April 10, 1866, for \$500, payable six months after date; required when it becomes due, the time to run if discounted Aug. 11, and the proceeds at 6 %.

Ans. Due Oct. $^{10}|_{13}$; time to run, 63 days; proceeds, \$494 75.

8. How much more is the compound than the annual interest of \$1300, at 6 %, for 4 years? *Ans.* \$1.13+

Review Questions. What is the Present Worth of any sum? (302) The Discount? (302) The Rule for finding the present worth? (303) What is a Promissory Note? (306) The Face of a Note? (306) Days of Grace? (306) Bank Discount? (307) Rule for finding bank discount? (308)

RATIO AND PROPORTION.

320. Ratio is the measure of the relation of two like quantities.

It is determined by dividing the first quantity by the second. Thus,

The ratio of 6 to 3 is 2, or of $8 to $2 is 4.

321. The **Terms** of a ratio are the two quantities compared.

The ANTECEDENT is the first term of the ratio.

The CONSEQUENT is the second term of the ratio.

322. The relation of antecedent to consequent may be indicated by writing : , or the sign of division, between two numbers. Thus,

$6 : 3$, or $6 \div 3$, indicate the ratio of 6 to 3.

The sign : is an abbreviated form of $\div$, and has a like meaning.

Some few American authors determine ratio by dividing the consequent by the antecedent, after the *old* method of La Croix, which has become quite obsolete in the country where it originated.

323. A **Simple Ratio** is a single ratio of two terms. Thus,

$8 : 2$ expresses a simple ratio.

324. A **Compound Ratio** is the product of two or more simple ratios. Thus,

$(6 : 5) \times (2 : 3)$, or $\frac{6}{5} \times \frac{2}{3}$, expresses a compound ratio.

325. From the definition of ratio, follow the

PRINCIPLES.

1. *Ratio can only exist between quantities of the same name and kind.*

2. *The ratio is equal to the quotient of the antecedent divided by the consequent.*

What is Ratio? Terms of a ratio? The Antecedent? The Consequent? How may the relation of antecedent to consequent be indicated? What is a Simple Ratio? A Compound Ratio? Give the first Principle. The second.

3. *The antecedent is equal to the product of the consequent and ratio.*

4. *The consequent is equal to the quotient of the antecedent divided by the ratio.*

Also, since ratio may be expressed by a fraction:

5. *The ratio is not changed, if both the antecedent and consequent are multiplied or divided by the same number.*

Exercises.

Write the ratio of

1. 3 to 5. *Ans.* 3 : 5.	4. 3×2 to 4×3. *Ans.* $(3 \times 2) : (4 \times 3)$.
2. 8 to 7. *Ans.* 8 : 7.	5. $\frac{1}{2}$ to $\frac{1}{4}$. *Ans.* $\frac{1}{2} : \frac{1}{4}$.
3. 6 to 4.	6. 2 to $\frac{1}{2}$.

What is the ratio of

7. 12 to 6? *Ans.* 2.	10. $\frac{9}{12}$ to $\frac{1}{4}$? *Ans.* 3.
8. 2 to 3? *Ans.* $\frac{2}{3}$.	11. $\frac{5}{7}$ to $\frac{1}{10}$? *Ans.* $7\frac{1}{7}$.
9. \$16 to \$4?	12. 3 yd. to $\frac{1}{4}$ yd.?

13. Reduce the ratio 6 : 30 to its smallest terms. *Ans.* $\frac{1}{5}$.

14. Reduce to a simple ratio $8 \times 3 : 6 \times 2$. *Ans.* 2.

15. Reduce to a simple ratio $\frac{4}{9} \times \frac{3}{4} : \frac{5}{6} \times \frac{12}{7}$. *Ans.* $\frac{7}{30}$.

16. Find the ratio of 6 h. 20 m. to 2 h. *Ans.* $3\frac{1}{6}$.

17. If the antecedent is 15.6 and the ratio 6, what is the consequent? *Ans.* 2.6.

18. If the consequent is $\frac{16}{24}$ and the ratio $\frac{1}{5}$, what is the antecedent? *Ans.* $\frac{2}{15}$.

PROPORTION.

326. A **Proportion** is an equality of ratios. Thus,

$$8 : 2 = 16 : 4 \text{ is a proportion.}$$

The equality is generally indicated by writing :: between the ratios. Thus,

$$8 : 2 :: 16 : 4 \text{ indicates a proportion,}$$

Give the third Principle. The fourth. The fifth. What is Proportion? How is the equality generally indicated?

and may be read, the ratio of 8 : 2 is equal to the ratio of 16 to 4, or 8 is to 2 as 16 is to 4.

327. The **Terms** of a proportion are those of its ratios.

The EXTREMES are the first and fourth terms.

The MEANS are the second and third terms.

A PROPORTIONAL is any one of the terms.

A MEAN PROPORTIONAL is a term repeated between the other two. Thus,

In 12 : 6 :: 6 : 3, 6 is a mean proportional.

PRINCIPLES.

328. 1. *In every proportion the product of the means is equal to the product of the extremes.*

For, in the proportion 6 : 3 :: 4 : 2, since the ratios are equal (Art. 326), we have $\frac{6}{3} = \frac{4}{2}$. Now, these equal fractions reduced to equivalent fractions having a common denominator, give $\frac{6 \times 2}{3 \times 2} = \frac{4 \times 3}{2 \times 3}$, and by dropping the common denominator, $6 \times 2 = 4 \times 3$. Hence,

2. *Either extreme is equal to the product of the means divided by the other extreme.*

3. *Either mean is equal to the product of the extremes divided by the other mean.*

4. *The fourth term is equal to the quotient of the third term divided by the ratio of the first to the second.*

Exercises.

Find the missing term in

1. 27 : 3 :: 54 : ( ). *Ans.* 6.
2. 12 yd. : 4 yd. :: $9 : ( ). *Ans.* $3.
3. 20 rd. : 25 rd. :: ( ) : $10. *Ans.* $8.
4. $5\frac{1}{2}$: ( ) :: 16 : 32.
5. $\frac{5}{16}$: $\frac{1}{4}$ :: 15 : ( ). *Ans.* 12.
6. ( ) : $\frac{2}{7}$ :: $\frac{7}{8}$: $\frac{3}{4}$. *Ans.* $\frac{1}{3}$.
7. $1.50 : $7.50 :: ( ) : 3 bu. *Ans.* $\frac{3}{5}$ bu.
8. 2 gal. 2 qt. : ( ) :: $4 : $80.

Here, in example 8, the 2 gal. 2 qt. must be reduced to an equivalent single denomination before proceeding to find the missing term.

What are the terms of a proportion? The Extremes? The Means? A Proportional? A Mean Proportional? The Principles?

SIMPLE PROPORTION.

329. In **Simple Proportion** the terms of two equal simple ratios are compared.

It applies to the solution of questions, in which three given quantities are so related that a fourth may be determined from them, by equality of ratios.

Of the three given quantities, two of them must be of the same name, and constitute a ratio, and the third must be of like name with the required quantity, so as to constitute with it a second ratio.

330. To solve questions by simple proportion.

1. If 8 yards of cloth cost $66, what will 32 yards cost?

OPERATION.

yd. yd. \$ \$

$8 : 32 :: 66 :$ cost of 32 yd.

$$\frac{\$66 \times \overset{4}{\cancel{32}}}{\cancel{8}} = \$264 = \text{cost of } 32\,\text{yd.}$$

It is evident that 8 yards bear the same relation to 32 yards, that the cost of 8 yards bears to the cost of 32 yards; hence, the proportion 8 yd. : 32 yd. : : $66 : cost of 32 yd.

Since the product of the means divided by either extreme must give the other extreme (Art. 328), the required extreme is equal to ($66 $\times$ 32) $\div$ 8, or $264.

Or, if 8 yards of cloth cost $66, 32 yards, which are 4 times 8 yards, must cost 4 times $66, or $264.

RULE. *Arrange the given terms so that, from the nature of the question, the ratios shall be equal.*

Find, then, the required term by dividing the product of the second and third terms by the first; or, by dividing the third term by the ratio of the first term to the second.

All questions in proportion may also be solved by *Analysis*. It is recommended to the learner to solve the examples that follow by both methods.

What are compared in Simple Proportion? To what questions does Simple Proportion apply? Explain the operation. Repeat the Rule.

Examples.

2. If 12 bushels of wheat cost $16, what will 30 bushels cost? *Ans.* $40.

3. If the rent of a farm of 183 acres is $273, what will be the rent of a farm of 61 acres? *Ans.* $91.

4. If 98 bushels of potatoes cost $56, how many bushels can be had for $16? *Ans.* 28 bushels.

5. When 12 yards of cloth can be bought for $16, how many can be bought for $72? *Ans.* 54 yards.

6. A person completed a journey of 40 miles in five hours; how far, at the same rate, can he travel in 45 hours? *Ans.* 360 miles.

7. If 5 yards of cloth cost $$6\frac{3}{8}$, what will $12\frac{2}{5}$ yards cost? *Ans.* $15.81.

8. If 385 kilos of sugar cost $63, how many may be bought for $18?

9. If I can complete a piece of work in 32 days, by working 8 hours a day, in what time can I do the same by working only 6 hours a day? *Ans.* $42\frac{2}{3}$ days.

10. If I borrowed of a friend $300 for 8 months, for how long a time should I lend him $200 in return? *Ans.* 12 months.

11. If 100 workmen can do a piece of work in 12 days, how many can do the same in 8 days?

12. How many yards of flannel $\frac{3}{4}$ of a yard wide are required to line a cloak which has in it 12 yards of cloth $\frac{1}{2}$ of a yard wide? *Ans.* 8 yards.

13. If I give $2 for .4 of a cord of wood, how much must I give for $\frac{1}{4}$ of a cord? *Ans.* $1.25.

14. If $\frac{3}{16}$ of a ship cost $9750, what will $\frac{21}{26}$ of it cost? *Ans.* $42000.

REVIEW QUESTIONS. What is Ratio? (320) Terms of a ratio? (321) A Simple Ratio? (323) A Compound Ratio? (324) Principles of ratio? *(325)*

15. Two numbers are to each other as 15 to 34, and the smaller is 75; what is the greater? *Ans.* 170.

16. Two numbers are to each other as 3 to 2, and the greater is 210; what is the smaller?

17. If 3 cords 5 cord feet of wood will purchase 1 T. 5 cwt. 3 qr. of hay, what quantity of wood will be required to purchase 1 ton of hay? *Ans.* 2 C. 6+ c. ft.

18. If a railway train moves 150 miles in $5\frac{1}{2}$ hours, in what time will it move 225 miles? *Ans.* 8 h. 15 m.

19. If 8 boarders consume a certain quantity of provisions in 10 days, how many days would it have lasted if 2 more had been in the company? *Ans.* 8 days.

20. A besieged fortress has provisions for 3 weeks, at the rate of 14 ounces a day for each man; at what rate per day must the provisions be distributed, so that the place may hold out 5 weeks? *Ans.* $8\frac{2}{5}$ oz.

21. If the fore wheel of a coach, which is 7 feet 6 inches in circumference, turns round 70400 times in going a hundred miles, how often will the hind wheel, which is 9 feet 2 inches, turn round in going the same distance? *Ans.* 57600 times.

22. If 125 bushels of wheat grow on 4 acres 84 square rods, how much land will be required to produce 650 bushels?

Ans. 23 A. 84.8 P.

23. If a stick 7 feet high cast a shadow 5 feet in length, what is the hight of a spire that casts a shadow 129 feet in length? *Ans.* 180 ft. $7\frac{1}{5}$ in.

24. A besieged town, containing 22400 inhabitants, has provisions to last 3 weeks; how many must be sent away that they may be able to hold out 7 weeks? *Ans.* 12800.

331. When it is required to divide a quantity into parts which are proportional to given numbers, we may.

Add together the proportional numbers; then, the sum of

REVIEW QUESTIONS. What is Proportion? (326) Terms of a Proportion? (327) *Principles?* (328)

these numbers will be to any one of them, as the quantity to be divided is to the part corresponding to that number.

25. A gentleman bequeathed \$18000 to his three sons, A, B, and C, in the proportion of 3, 4, and 5; how much was the part of each?

OPERATION.

$3+4+5=12$. Then,
12 : 3 :: \$18000 : \$4500, A's part.
12 : 4 :: \$18000 : \$6000, B's part.
12 : 5 :: \$18000 : \$7500, C's part.

Since the given sum is to be bequeathed to the three sons, in the proportion of 3, 4, and 5, if we divide \$18000 into $3+4+5$, or 12, equal parts, A will have 3, B 4, and C 5, of these parts. Hence, A's part is $\frac{3}{12}$ of \$18000, or \$4500, B's $\frac{4}{12}$ of \$18000, or \$6000, and C's $\frac{5}{12}$ of \$18000, or \$7500.

26. Two men own a field of 640 acres in common, and their respective shares of it are in the ratio of 7 to 9; if divided, how many acres would belong to each?

27. Divide 4720 into 3 parts which shall be in the proportion of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{5}$. *Ans.* 1200, 1600, and 1920.

28. In a certain town of 4500 inhabitants, the youth are to the adults in the ratio of 13 to 12; required the number of each. *Ans.* Youth, 2340; adults, 2160.

COMPOUND PROPORTION.

332. Compound Proportion is an equality between a compound and a simple ratio. Thus,

$$\begin{matrix} 3 : 4 \\ 5 : 8 \end{matrix} :: 45 : 96 \text{ is a compound proportion.}$$

It applies to the solution of questions which would require several simple proportions.

333. To solve questions by compound proportion.

1. If 4 men can earn \$96 in 8 days, how much can 10 men earn in 6 days?

How may we proceed when it is required to divide a quantity into parts which are proportional to given numbers? Explain the operation. What is Compound Proportion? To what does it apply?

OPERATION.

$$\begin{matrix} 4 : 10 \\ 8 : 6 \end{matrix} :: \$96 : \text{amount required; or}$$

$$4 \times 8 : 10 \times 6 :: \$96 : \text{amount required.}$$

$$\text{Then, } \frac{\overset{3}{\cancel{\$96}} \times 10 \times 6}{\cancel{4} \times \cancel{8}} = \$180 = \text{amount required.}$$

If 4 men can earn \$96 in 8 days, 1 man can earn in the same time $\frac{1}{4}$ of \$96, or \$24, and 10 men can earn 10 times \$24, or \$240.

If 10 men can earn \$240 in 8 days, in 1 day they can earn $\frac{1}{8}$ of \$240 or \$30, and in 6 days 6 times \$30, \$180.

Or, if 4 men can earn \$96 in 8 days, 10 men in the same time will earn $\frac{10}{4}$ of \$96, and in 6 days, $\frac{6}{8}$ of $\frac{10}{4}$ of \$96, or \$180.

RULE. *Write the given number of the same kind as the answer for the third term; and arrange the remaining numbers, each pair of the same kind, as in simple proportion.*

Find the required term by dividing the product of the third and second terms by the product of the first terms.

It is recommended to solve the questions by both compound proportion and analysis.

Examples.

2. If a pasture of 16 acres will feed 6 horses for 4 months, how many acres will feed 12 horses for 9 months? *Ans.* 72 acres.

3. If 36 yards of cloth, 7 quarters wide, be worth \$98, what is the value of 120 yards of equal quality, but only 5 quarters wide?

4. If 6 men can mow 16 acres in 4 days, how long will it take 10 men to mow 40 acres, at the same rate? *Ans.* 6 days.

5. If a man travel 90 miles in 3 days, by walking 8 hours a day, in what time will he travel 540 miles, by walking 6 hours a day? *Ans.* 24 days.

6. If a family of 9 spend \$600 in 8 months, how much will serve a family of 24 for 16 months? *Ans.* \$3200.

Explain the operation. Repeat the Rule.

7. If 12 horses in 5 days plow 11 acres, how many horses would plow 33 acres in 18 days? *Ans.* 10 horses.

8. If \$2000 will support a garrison of 150 men 3 months, how long will \$6000 support 4 times as many men?

9. The interest on \$200 for 4 months being \$4, what will be the interest on \$590 for 1 year and 3 months? *Ans.* \$44.25.

10. When 12 men can put $\frac{2}{5}$ of a cargo on board of a ship in $\frac{3}{4}$ of a day, how long would it require 5 men to load 3 such vessels? *Ans.* $13\frac{1}{2}$ days.

11. If \$750 will defray the expenses of 12 men for 14 weeks 2 days, how much will 5 men require for 22 weeks 6 days? *Ans.* \$500.

12. If 18 men can dig a trench 30 yards long in 24 days, by working 8 hours a day, how many will dig a trench 60 yards long in 64 days, working 6 hours a day? *Ans.* 18 men.

13. If 29 men in 5 days of 12 hours each can reap 32 acres, how many acres can 20 men reap in $8\frac{19}{52}$ days of 13 hours each? *Ans.* 40 acres.

PARTNERSHIP.

334. Partnership is the association of two or more persons in business, under a certain name, with an agreement to share the profits and losses.

The PARTNERS are the individuals associated.

The COMPANY, or FIRM, is the business association.

The CAPITAL, or STOCK, is the money, or property used in business.

A DIVIDEND is the profits to be divided.

The profits and losses of a firm are usually distributed among the partners in proportion to their respective shares in the business.

REVIEW QUESTIONS. A Simple Proportion? (329) The Rule? (330) What is Partnership? The Partners? The Company or Firm? The Capital or Stock? A Dividend?

CASE I.

335. When the capital of each partner is employed for equal time.

1. A, B, and C enter into partnership; A furnishes $5000, B $2500, and C $7500; they gain $600; what is each partner's share?

OPERATION.

A's	stock =	$5000	$= \frac{5000}{15000}$	$= \frac{1}{3}$	of entire stock;
B's	" =	2500	$= \frac{2500}{15000}$	$= \frac{1}{6}$	" "
C's	" =	7500	$= \frac{7500}{15000}$	$= \frac{1}{2}$	" "

Entire stock = $15000

Hence, A's share $= \frac{1}{3}$ of $600 = $200
B's " $= \frac{1}{6}$ of $600 = 100
C's " $= \frac{1}{2}$ of $600 = 300

Entire gain = $600

Since A's stock is equal to $\frac{1}{3}$, B's to $\frac{1}{6}$, and C's to $\frac{1}{2}$, of the entire stock, A must have $\frac{1}{3}$, B $\frac{1}{6}$, and C $\frac{1}{2}$, of the gain; hence, A's share is equal to $\frac{1}{3}$ of the $600, or $200, B's to $\frac{1}{6}$ of the $600, or $100, and C's to $\frac{1}{2}$ of the $600, or $300.

RULE. *Apportion to each partner such a part of the gain or loss as his stock is of the entire stock.*

Examples.

2. A, B, and C form a joint capital for conducting a business, of which A contributes $1500, B $1950, and C $2100. At the end of a year the profits are $1665; what share should each receive? *Ans.* A $450, B $585, and C $630.

3. A, B, and C traded together. A put into the business $240, B $360, and C $120. They gained $350; what was each partner's share of the gain?

4. A, B, and C had 108 tons of freight on board a ship, of which A had 48 tons, B 36, and C 24; but in a storm 45 tons were washed away; what was each man's share of the loss?

What is the Rule?

5. A and B, with equal stocks, clear in trade $2000; A is to have 3 parts of the profits, and B 2 parts, because A managed the business; what is each one's share of the profits, and how much did A receive for his services?

Ans. A's share $1200; B's share $800; and A for services $400.

336. The rule applies to distributing the assets of bankrupts, and other like apportionments.

6. A bankrupt, whose property is only worth $6000, owes A $8000 and B $12000; what should each of these creditors receive? *Ans.* A $2400, and B $3600.

7. Three persons rent a pasture for the summer; the first puts in 21 horses, the second 17, and the third 47. The rent is $307; what part of it must each pay?

Ans. The first $75.84+, the second $61.40, and the third $169.75+.

8. A gentleman left by will to his wife $5000, to his elder son $3000, and to his younger son $2500. But it was found that the property, after paying debts, was only $7475; how should this be apportioned under the will?

Ans. To the wife $3559.52+; to the elder son $2135.71+; and to the younger son $1779.76.

CASE II.

337. When the capital of the partners is employed for unequal times.

1. A and B trade in company; A puts in $500 for 8 months, and B $600 for 10 months. They gain $240; what is each partner's share of it?

OPERATION.

A's	$500	for	8 mo. =	$4000	for	1 mo.;
B's	$600	"	10 mo. =	6000	"	1 mo.

Entire stock the same as $10000 for 1 mo.

How does the Rule apply?

$$\text{Hence, A's share} = \tfrac{4000}{10000} = \tfrac{2}{5} \text{ of } \$240 = \$96$$
$$\text{B's share} = \tfrac{6000}{10000} = \tfrac{3}{5} \text{ of } \$240 = \$144$$
$$\text{Entire gain} = \$240$$

$500 for 8 months is the same as $4000 for 1 month, and $600 for 10 months is the same as $6000 for 1 month; hence the entire stock is the same as $4000 + $6000, or $10000, for 1 month.

If $10000 gain in a certain time $240, $4000, or $\frac{2}{5}$ of that sum, must gain $\frac{2}{5}$ of $240, or $96, and $6000, or $\frac{3}{5}$ of the sum, $\frac{3}{5}$ of $240, or $144.

RULE. *Multiply each partner's stock by the time it was invested, and apportion the gain or loss in proportion to the products.*

Examples.

2. A, B, and C commenced trade together the first of June, on $6000 put in by A; the first of August B put in $9000, and the first of September C put in $12000. At the end of the year their gains amounted to $4500; what was each partner's share? *Ans.* A's $1400, B's $1500, and C's $1600.

3. A, B, and C enter into partnership; A put in $500 for 18 months; B $380 for 13 months; C $270 for 9 months. They lost $818.50; what was each man's share?
Ans. A's $450, B's $247, and C's $121.50.

4. Jones and Smith rent a pasture for $275; Jones puts in 80 sheep and Smith 100, but at the end of 6 months they each dispose of half their stock, and allow Hall to put in 50 sheep; what should each pay toward the rent at the end of the year?
Ans. Jones $103.12½, Smith $128.90⅝, and Hall $42.96⅞.

5. A and B entered into partnership for 1 year. A at first put in $500, and at the end of 5 months he put in $150 more; B at first put in $600, and at the end of 9 months took out $200. Their year's profits were $682.50; what was each man's share? *Ans.* A's $352.50, and B's $330.

Explain the operation. Repeat the Rule.

6. A builds a mill at a cost of $35000; 2 months after its completion B buys stock in it of A to the amount of $11000; and in 3 months more C purchases also of A $4000 worth of stock. They run the mill for 7 months, and gain during that time $9700; what portion of this belongs to each?

Ans. A's share $7205.71+, B's share $2177.55+, and C's share $316.73+.

7. S, T, and Y entered into partnership. S kept his stock in 1 year; T put in $\frac{1}{8}$ as much as S, and for 10 months; Y put in $\frac{3}{4}$ as much as S, and for 4 months. They gained $3400; what was each one's share of the profit?

Ans. S's share $2400, T's share $400, and Y's share $600.

EQUATION OF PAYMENTS.

338. Equation of Payments is the process of finding the average or equitable time for paying several sums due at different times.

339. The **Equated Time** is the date at which the items due at different times may be justly paid together.

340. The **Average Term of Credit** is the time that must elapse before the equated time.

CASE I.

341. To find the equated time when the terms of credit begin at the same date.

1. I owe, July 1, to John Wentworth, $600, of which $200 is due in 2 months, $300 due in 4 months, and $100 in 8 months; required the equated time of paying the several items at once.

REVIEW QUESTIONS. What is a Compound Proportion? (332) The Rule? (333) Partnership? (334) The Rule when the capital of each partner is employed equal times? (335) When for unequal times? (337) — What is Equation of Payments? The Equated Time? The Average Term of Credit?

OPERATION.

$$\begin{array}{rcl} 2 \text{ mo.} \times 200 &=& 400 \text{ mo.} \\ 4 \text{ “} \times 300 &=& 1200 \text{ “} \\ 8 \text{ “} \times \underline{100} &=& \underline{800} \text{ “} \\ 600 &)& \underline{2400} \text{ mo.} \\ && 4 \text{ mo.} \end{array}$$

July 1 + 4 mo. = November 1, *Ans.*

A credit on \$200 for 2 mo. is equal to a credit on \$1 for 200 times 2 mo., or 400 mo.; a credit on \$300 for 4 mo., to a credit on \$1 for 300 times 4 mo., or 1200 mo.; and a credit on \$100 for 8 mo., to a credit on \$1 for 100 times 8 mo., or 800 mo.

Hence, the entire credit is equal to a credit on \$1 for 2400 mo.; and a credit on \$1 for 2400 mo. is equal to a credit on \$600 for $\frac{1}{600}$ of 2400 mo., or 4 mo.; hence, 4 mo. from July 1, or November 1, is the equated time

RULE. *Multiply each term of credit by the number denoting its debt, and divide the sum of the products by the number denoting the sum of the debts; the quotient will be the average term of credit.*

The average term of credit, added to the date of the debts, will give the equated time.

When any of the items have cents, if 50 or more, reckon them as *one* dollar, but if less than 50 cents, neglect them. Also, when any result has a fraction of a day, if it is $\frac{1}{2}$ or more, reckon it *one* day, otherwise neglect it.

Examples.

2. Required the average credit for the payment of \$500 payable in 2 months, \$1000 in 5 months, and 1500 in 8 months. *Ans.* 6 months.

3. I owe \$1600 payable now, and \$800 in 90 days; what is the average term of credit? *Ans.* 30 days.

4. Required the equated time from March 1st, at which to pay \$200, of which \$40 is due in 3 months, \$60 in 5 months, and the remainder in 10 months. *Ans.* October 4th.

5. May 16, 1866, Albert Day owes \$199.50 payable in 30 days, \$150.15 in 60 days, and \$300 in 90 days; what is the equated time? *Ans.* July 20, 1866.

Explain the operation. Repeat the Rule. How do you proceed when any of the items have cents? When any result has a fraction of a day?

CASE II.

342. When the terms of credit begin at different dates.

1. New Haven, January 1, 1867.

Alexander English

1866. Bought of James Miles & Co.

Oct. 7,	Merchandise,	on 90 days,	$1000.
Nov. 15,	"	net cash,	800.
Dec. 20,	"	on 60 days,	600.

What is the equated time of the payment of this bill?

OPERATION.

$$
\begin{array}{llrcrl}
\text{Due Nov. 15,} & 0 \text{ days} \times & 800 & = & 0 & \text{days.} \\
\text{" Jan. 5,} & 51 \text{ "} \times & 1000 & = & 51000 & \text{"} \\
\text{" Feb. 18,} & 95 \text{ "} \times & \underline{600} & & \underline{57000} & \text{"} \\
 & & 2400 &) & \underline{108000} & \text{days.} \\
 & & & & 45 & \text{days.}
\end{array}
$$

Nov. 15, 1866 + 45 days = Dec. 30, 1866, *Ans.*

Reckoned, for convenience, from the earliest date that any one of the debts is due, the credits of the $800, $1000, and $600, are, respectively, 0, 51, and 95 days, from Nov 15.

The average term of these credits, by Case I., is 45 days; hence, 45 days from Nov. 15, 1866, or Dec. 30, 1866, is the equated time.

RULE. *Select the earliest date at which any one of the debts became due, and therefrom reckon the terms of credit.*

Multiply the terms of credit of each item by the number denoting its item, and divide the sum of the products by the number denoting the sum of the items; the quotient will be the average term of credit.

The average term of credit, added to the selected date, will give the equated time.

Examples.

2. Purchased the following bills of goods: July 1, a bill of $200, on 2 months; July 20th, a bill of $600, on 60 days; Aug. 1, a bill of $1000, on 30 days. What is the equated time of payment? *Ans.* September 6th.

Explain the operation. Repeat the Rule.

3. I owe Oliver Bates as follows: April 1, for cash, $1400; May 1, for merchandise $500; June 1, for flour, $1100. What is the average date of the items? *Ans.* April 28th.

4. R. Hicks & Co. have sold a merchant the following bills: Jan. 1, merchandise, $735; Jan. 21, corn, on 30 days, $649.50; Feb. 1, lumber, $100; March 12, merchandise, on 30 days, $200. If they should receive in settlement for the whole, a note, from what date ought it to draw interest?

Ans. February 3d.

343. When the items have the same term of credit, we may *First find their average date, and then add the common term of credit, for the equated time.*

5. Purchased goods, on 4 months, as follows:—April 1, a bill of $1450; May 7, a bill of $1250; June 5th, a bill of $850. Required the equated time of payment.

Ans. August 29th.

6. Sold the following bills of goods, on 6 months: Jan. 15, a bill of $3750; Feb. 10, a bill of $3000; March 6, a bill of $2400; June 8, a bill of $2250. At what time should a note be made payable, that will settle for the whole?

Ans. Sept. 2d.

AVERAGING OF ACCOUNTS.

344. The **Balance of an account** is the difference between its debtor and creditor sides.

Accounts are subject to *interest* after the expiration of the term of credit.

345. The **Averaging of an Account** is the process of finding the equated time of paying the balance, or the date at which the balance of the account becomes due, or subject to interest.

How may we proceed when the items have a common term of credit? What is the Balance of an Account? When are accounts subject to interest? What is the Averaging of an Account?

346. To find the equated time of the balance of an account.

1. When does the balance of the following account become due?

Dr. Franklin Fuller. Cr.

1866.			1866.		
March 10,	To Mdse. on 4 mo.,	$200.00	April 11,	By Cash,	$60.00
" 28,	" " " 6 "	160.00	July 10,	" "	140.00
April 1,	" " net,	140.00	Aug. 29,	" "	100.00

OPERATION.

Due April 1,	$0 \times 140 =$		Due April 11,	$10 \times 60 =$	600
" July 10,	$100 \times 200 =$	20000	" July 10,	$100 \times 140 =$	14000
" Sept. 28,	$180 \times 160 =$	28800	" Aug. 29,	$150 \times 100 =$	15000
	500	48800		300	29600
	300	29600	$19200 \div 200 = 96$ da.		
Balances,	200	19200	April 1 + 96 days = July 6, *Ans.*		

We select, for convenience, April 1, the earliest date at which any of the items of account is due, as the point of reckoning, and find the aggregate of the terms of credit of the debit items, with reference to the selected date, to be equal to the credit of $1 for 48800 days; and the aggregate of the terms of credit of the credit items to be equal to the credit of $1 for 29600 days.

Striking the balance, it appears that at the selected date, $200 subject to a credit equal to the credit of $1 for 19200 days, is against Franklin Fuller. But the credit of $1 for 19200 days is equal to that of $200, for $\frac{1}{200}$ of $19200 = 96$ days; hence, the $200 is not due in equity till 96 days *after* April 1, or till July 6.

If, however, the balance of items and of terms of credit had been on different sides of the account, the balance of items would have been due *before*, instead of after, the selected date.

RULE. *Select the earliest date at which any of the items of account become due, and therefrom reckon the terms of credit.*

Multiply each term of credit by the number denoting the corresponding item, and divide the balance of the sums of the products by the balance of the sums of the items of the account.

The quotient will be the time, which must be ADDED *to the selected date, when the two balances are on the same side of account, but* SUBTRACTED *from the selected date, when the balances are on different sides, to obtain the equated time of* THE ACCOUNT.

Examples.

2. What is the balance of the following account, and, allowing each item to be on 30 days, when does it become due?

Dr. Robert Smith & Co. Cr.

1866.				1866.			
June 30,	To Merchandise,	$550	00	July 1,	By Merchandise,	$400	00
July 15,	" "	850	00	July 5,	" "	300	00

Ans. Balance $700; due Aug. 15.

3. Required the date, at which a note, given for the balance of the following account, should begin to draw interest.

Dr. Strickland & Hooper. Cr.

1866.				1866.			
Nov. 3,	To Merchandise,	$500	00	Nov. 13,	By Cash,	$700	00
Dec. 23,	" "	600	00				

Ans. Dec. 31, 1866.

4. Required the face of a note which must be given for the balance of the following account, and the date at which it should begin to draw interest.

Dr. J. F. Gould. Cr.

1866.				1866.			
May 16,	To Mdse. on 60 d.,	$300	00	May 20,	By Mdse. on 30 d.,	$200	00
June 3,	" " " 60 "	49	60	July 19,	" " " 60 "	200	00
July 1,	" " " 30 "	150	00				

Ans. Face of note $99.60; on interest from June 2.

What is the Rule?

SETTLEMENT OF ACCOUNTS

347. Merchandise Balance is the balance of the items without interest.

348. Interest Balance is the balance of the interest of the items of the two sides of an account.

349. Cash or Net Balance is the balance when the merchandise and interest balances have been added to the proper sides of the account.

350. The **Settlement** of an account is ascertaining the balance at any specified time.

351. Since the merchandise balance is understood to be subject to interest from the date of its being due (Art. 344), it follows, that

The cash or net balance, at any date subsequent to the equated time, may be found by ADDING *to the merchandise balance its interest up to date; or at any date previous to the equated time, by* SUBTRACTING *from the merchandise balance its interest for the time intervening.*

INTEREST METHOD.

352. The cash balance of an account drawing interest may be determined by means of the interest on the items of the two sides.

1. Let it be required, on January 1, 1867, to find the cash balance of the following account; and, also, the equated time, allowing each item to draw interest from date, at 6 per cent.

Dr. P. T. Montgomery & Co. Cr.

1866.				1866.			
Nov. 2,	To Merchandise,	$600	00	Nov. 17,	By Cash,	$800	00
Dec. 2,	" "	700	00	Dec. 2,	" "	200	00
				1867.			
" 17,	" "	1000	00	Jan. 1,	" "	100	00

What is Merchandise Balance? Interest Balance? Cash or Net Balance? The Settlement of an Account? How may the cash or net balance *be found?*

OPERATION.

Int. on $600 for 60 da.	= $6.00	Int. on $800 for 45 da.	= $6.00
" 700 " 30 "	= 3.50	" 200 " 30 "	= 1.00
" 1000 " 15 "	= 2.50	" 100 " 0 "	= 0.00
$2300	$12.00	$1100	$7.00
1100	7.00		
Mdse bal. $1200	Int. bal. $5.00		

Cash bal. = $1200 + $5 = $1205, *Ans.*

Computing the interest on each of these items, from the time of becoming due to the given date, January 1, we find the balance of the interest to be $5, and of the merchandise $1200, and both on the debit side; hence, the cash balance, January 1, 1867, is $1200 + $5, or $1205.

RULE. *Compute the interest of each item for the time intervening between its being due and the time of settlement.*

Find the sum of the interest on each side, and also the sum of the items; and if the interest balance and the merchandise balance fall on the same side of the account, the cash balance is their SUM; *but if on different sides, it is their* DIFFERENCE.

When an item is not due till after settlement, its interest must be subtracted from the interest of its side, or added to that on the other side.

Examples.

2. E. Holmes owes C. Simmons $600 due in 60 days from September 1, and $200 due in 30 days from December 2; and Simmons owes Holmes $700 due in 30 days from August 3; how much balance ought Holmes in equity to pay, should the account be closed on January 1, the rate of interest being 7 %? *Ans.* $90.77.

3. I owe James Conant the following bills: May 16, for merchandise, on 60 days, $300; June 3, for flour, on 60 days, $50; July 1, for labor, on 30 days, $150; and he owes me a bill dated May 30, on 30 days, of $300, and another dated July 19, on 60 days, of $200; required the cash balance I owe him on September 1, the rate of interest being 6 %. *Ans.* $.78.

Explain the operation. Repeat the Rule.

TAXES.

353. A **Tax** is a sum of money assessed upon a person, or upon property, for government or public use.

A POLL TAX is a sum assessed upon each male citizen, of a certain age, without regard to property.

354. Real Estate is immovable property, such as lands, houses, etc.

355. Personal Property is movable property, such as money, stocks, cattle, etc.

356. Assessors are officers appointed to take an inventory of taxable property, and a list of taxable polls, and to apportion to each person the tax to be raised.

357. To assess, or apportion, a town, or other tax.

1. The town and county tax of Middlebury, for 1866, was \$12200. The real estate of the town is valued at \$1040000, and the personal property at \$560000. There are 500 polls, each taxed \$2.00. How many mills is the tax on a dollar; and what is A's tax, who has \$5000 of real estate and \$1500 of personal property, and pays 1 poll tax?

OPERATION.

\$2 × 500 = \$1000, amount of poll taxes.
\$12200 — \$1000 = \$11200, amount of property tax.
\$1040000 + \$560000 = \$1600000, amt. of taxable property.
\$11200 ÷ \$1600000 = 7 mills, tax on \$1 of property.
\$5000 + \$1500 = \$6500, A's taxable property.
\$6500 × .007 = \$45.50, A's property tax.
\$45.50 + \$2.00 = \$47.50, A's entire tax. Hence,

The tax on one poll, multiplied by the number of polls, will give the amount of poll taxes.

What is a Tax? A Poll Tax? Real Estate? Personal Property? Assessors?

The amount of poll taxes subtracted from the whole amount of tax to be apportioned, will give the amount of property tax.

The amount of property tax divided by the whole taxable property, will denote the tax on one dollar of property.

Each individual's taxable property multiplied by the number denoting the tax on one dollar, with his poll tax added, if any, will give his tax.

Having determined the number of mills the tax on property is on \$1, assessors usually facilitate computation, by finding the tax on \$2, \$3, etc., and arranging the same for use in a table. Suppose the tax on \$1 is 16 mills, then we could have the following

Table,

Showing Taxes at the rate of 16 mills on \$1.

\$1	pays	\$.016	\$11	pays	\$.176	\$25	pays	\$.40
2	"	.032	12	"	.192	35	"	.56
3	"	.048	13	"	.208	45	"	.72
4	"	.064	14	"	.224	55	"	.88
5	"	.080	15	"	.240	65	"	1.04
6	"	.096	16	"	.256	75	"	1.20
7	"	.112	17	"	.272	85	"	1.36
8	"	.128	18	"	.288	95	"	1.52
9	"	.144	19	"	.304	100	"	1.60
10	"	.160	20	"	.320	1000	"	16.00

2. Find by the Table A's tax, his valuation being \$1950, and he paying for 2 polls, at \$1.50 each.

OPERATION.

\$1900	pays	\$30.40
50	"	.80
\$1950	"	\$31.20
2 poll taxes,		3.00
A's tax,		\$34.20

The tax on \$19 is \$.304, and removing the decimal point two places to the right, we have \$30.40, the tax on \$1900.

The tax on \$5 is \$.08, and removing the decimal point one place to the right, we have \$.80, the tax on \$50.

The tax on 2 polls, at \$1.50 each, is \$3.00. Adding the results, we have A's tax on \$1900 + \$50, or \$1950, and for 2 polls, \$34.20.

How do we find the amount of poll taxes? The tax on one dollar? Each individual's whole tax? Explain the operation.

DUTIES.

358. Duties are taxes or imposts levied by government upon commodities.

359. Excise Duties are imposts upon articles produced, and used or consumed in the country, and, also, on licenses.

A SPECIAL TAX is a duty, which is payable without regard to the amount of an individual's income.

STAMPS are marks to be attached to articles, as evidence that the duty upon them is paid.

INTERNAL REVENUE is the income of the government derived from excise duties, special taxes, stamps, etc.

CUSTOMS.

360. Customs are duties upon imports, and upon the tonnage of vessels.

361. An **Invoice** is a bill of merchandise, from the seller to the importer, giving the price and quantity of the goods.

362. An **Ad Valorem Duty** is a certain per cent. upon the value of an article estimated from the invoice.

363. A **Specific Duty** is a certain sum upon an article, without regard to its value.

364. Tare is an allowance made for the weight of the cask, box, bag, etc., containing the goods.

When the tare cannot be satisfactorily determined from the invoice, it may be estimated by actual weighing, or, on some articles, by a schedule furnished by the treasury department.

LEAKAGE is an allowance for waste of liquors in casks, and BREAKAGE is an allowance on liquors in bottles.

GROSS WEIGHT is the weight before any allowances are made, and NET WEIGHT is the weight of the goods alone.

What are Duties? Excise Duties? Stamps? Internal Revenue? Customs? An Invoice? An Ad Valorem Duty? A Specific Duty? *Tare? Leakage?* Gross Weight?

Exercises.

1. Holderman & Co. have imported 5000 weight of raisins in quarter boxes; what is the duty, allowing for tare 29 %, and the rate of duty to be 5 cents a pound?

SOLUTION. $5000 \times .29 = 1450$; $5000 - 1450 = 3550$; $3550 \times .05 = \$177.50$ *duty, Ans.*

2. What is the amount of ad valorem duty at 30 % on goods invoiced at $5600?

3. I have imported 200 bags of Rio coffee, at 25 kilograms each; if the tare is 2 %, and the duty 5 cents a pound, how much will be the duty? *Ans.* $540.13.

4. A merchant has imported 6000 pounds of cassia in mats; allowing 9 % tare, and the rate of duty to be 20 cents a pound, how much duty will he be required to pay? *Ans.* $1092.

5. What is the duty, at 20 % ad valorem, upon 5 tons (of 2240 pounds) of steel invoiced at 22 cents a pound?

STOCKS.

365. A **Corporation** is an incorporated body, or company, authorized by law to act and to be considered as an individual.

366. A **Share** is one of the equal parts into which the property of a corporation is divided.

367. **Bonds** are obligations securing the payment of a certain sum of money at a specified time.

The principal bonds of the United States are, —

6's of '81, payable in 1881, bearing interest at 6 %, in gold.

5's of '81, payable in 1881, bearing interest at 5 %, in gold.

5-20's, redeemable after 5 years, payable after 20 years, bearing interest at 6 %, in gold.

What is a Corporation? A Share? Bonds?

10-40's, redeemable after 10 years, payable after 20 years, bearing interest at 5 %, in gold.

4½'s of 1886, payable after 1886, bearing interest at 4½ %, in gold.

4's of 1901, payable after 1901, bearing interest at 4 %, in gold.

368. Stocks is a general term applicable to the bonds of governments, and the bonds and shares of incorporated companies.

Stocks are *at par*, or at their face value, when quoted at 100; *above par* when quoted at more than 100; and *below par* when quoted at less than 100. Thus,

A stock quoted at 96, is at 96 % of its face value.

A COUPON is an interest certificate attached to a bond.

BROKERAGE for buying or selling stock is reckoned on the par value of the stock, and usually at the rate of ⅛ to ¼ %.

Exercises.

1. How much must be paid for $5000, U. S. 10-40's, at 110, and brokerage at ¼ %? *Ans.* $5512.50.

2. What amount of U. S. 10-40's, at 110 and brokerage, can be bought for $5512.50?

3. How many $100 U. S. 4 per cent. bonds, at 91, can be bought with $5460? *Ans.* 60.

4. How much is the yearly income, in currency, from $20000 in U. S. 5's, when gold is at 105¾? *Ans.* 1057.50.

5. What rate of interest will an investment in a 6 % manufacturing stock, at 80, yield? *Ans.* 7½ %.

6. At what rate must a 10 % bank stock be bought, to pay 8 % on the investment? *Ans.* 125.

7. At what rate must a 7 % stock be bought, to pay 6 % on the investment?

8. What sum must be invested in an 8 % railroad stock, at 112, to yield a semi-annual income of $600? *Ans.* $16800.

What are Stocks? When are Stocks at par? When below par? When *above* par? What is a coupon? On what is the brokerage of stocks *reckoned?*

EXCHANGE.

369. **Exchange** is the method of remitting money from one place to another, by drafts, or written orders.

370. **A Bill of Exchange, or Draft,** is a written order directing one person to pay money to another, or to his order.

The DRAWER is the person who signs the bill, or draft; the DRAWEE is the person directed to pay the same; the PAYEE is the person to whom the money is directed to be paid; the INDORSER is the person who transfers his right to a bill, or draft, by indorsing it; and the HOLDER is the person who has legal possession of it.

371. The **Indorsement** of a bill, or draft, is done by the payee writing his name upon its back.

372. The **Acceptance** of a bill, or draft, is done by the drawee writing his name after the word "Accepted" across its face.

The drawer, acceptor, and each indorser of a bill, or draft, are liable for its payment. If the drawee declines to pay or accept a bill, or an acceptor to make payment, it is customary for the holder to employ a Notary Public to give a notice called a *Protest*, to the drawer and each of the indorsers, so as to hold them legally for the payment.

Three days of grace are usually allowed on bills and drafts, after the time specified has expired.

373. The **Par of Exchange** is the comparative value of the money of two places. Hence,

Exchange is said to be at *par*, when a bill sells for its face; at a *premium*, or *above par*, when for more than its face; and at a *discount*, or *below par*, when for less.

374. The **Rate of Exchange** is a rate per cent. of the face of a bill.

375. The **Course of Exchange** is the par of exchange as affected by the rate of exchange.

What is Exchange? A Bill of Exchange? How is the Indorsement of a bill done? The Acceptance? Who are liable for the payment of a bill or draft? How many days of grace are allowed? The Par of Exchange? The Rate of Exchange? The Course of Exchange?

DOMESTIC OR INLAND EXCHANGE.

376. Domestic or Inland Exchange is between drawer and drawee in the same country.

CASE I.

377. To find the cost of an inland bill or draft.

1. What is the cost of the following draft, at $1\frac{1}{2}$% discount?

$1000. *New York, August 1, 1866.*

At sight, pay to Thomas R. Sherwood, or order, One Thousand Dollars, and place to the account of

Julian McCulloch.

To Albert Prince,
St. Louis.

OPERATION.

$1 × .985 = $.985, cost of $1.
$.985 × 1000 = $985, " draft.

At $1\frac{1}{2}$% discount, the cost of $1 of exchange will be $.985, and of $1000, 1000 times $.985, or $985.

2. What must be paid in Cleveland for a draft of $2000 on Philadelphia, at 30 days, when exchange is at 2 % premium?

OPERATION.

$1 × 1.02 = $1.02, cost of $1 at sight.
$1.02 × 2000 = $2040, " $2000 "
$2000 × .0055 = $11, bank dis. for 33 da.
$2029, cost of draft.

At 2% premium, the cost of $1 of exchange at sight will be $1.02.

If the cost of $1 at sight is $1.02, the cost of $2000 will be 2000 times $1.02, or $2040; and, since the bank discount on the face of the draft, for the given rate and time is $11, the cost of the draft must be $2040 — $11, or $2029.

RULE. *If the bill is at sight, multiply the cost of $1 of exchange, by the number denoting the face of the bill.*

What is Domestic or Inland Exchange? Repeat the Rule.

If the bill is payable after sight, reckon the bank discount of the face of the draft for the time and three days' grace, and deduct it from the product.

Examples.

3. What must be paid in Detroit, where the rate of interest is 7 %, for a draft of \$500, at 60 days, on Nashville, at ½ % discount? *Ans.* \$491.37½.

4. What must be paid at Mobile for a draft of \$1940, on Providence, exchange being at 1¼ % premium? *Ans.* \$1964.25.

5. What is the cost of a draft of \$920, for 90 days, at 8 %, exchange being at a discount of ¼ % ?

6. A merchant in Portland wishes to discharge a debt of \$3000 in New Orleans; required the cost of a draft for that sum, for 2 months, at 6 %, exchange being at 1 % premium. *Ans.* \$2998.50.

CASE II.

378. To find the face of an inland bill or draft.

1. What is the face of a draft on Cincinnati for 60 days, at 1 % premium, which can be bought for \$1000?

OPERATION.

$\$1 + \$.01 = \$1.01,$ cost of \$1 at sight.
$\$1 \times .0105 = \underline{\quad .0105,}$ bank dis. for 63 da.
$\$.9995,$ cost of \$1 at 60 da.

At 1% premium, \$1 of exchange for 60 days can be bought for \$.9995. If \$1 of exchange can be bought for \$.9995, as many dollars of exchange can be bought for \$1000, as \$.9995 are contained times in \$1000, or \$1000.50.

RULE. *Divide the given sum by the cost of \$1 of exchange.*

Examples.

2. Required the face of a draft that may be purchased for \$6075, exchange being at 1¼ % premium. *Ans.* \$6000.

Explain the operation. Repeat the Rule of Case II.

3. I wish to remit to a merchant in Charleston, $19490; what is the face of a draft for that amount for 30 days, at 2 % discount? *Ans.* $20000.

FOREIGN EXCHANGE.

379. Foreign Exchange is between drawer and drawee in different countries.

Foreign bills are usually drawn in *sets*, and to prevent loss or delay each bill of the set is remitted in a different manner, and when one of the set has been paid, the others become worthless.

380. The **Intrinsic Par** of **Exchange** between countries is the mint value of their coins.

381. The LEGAL VALUE of some foreign coins, as fixed by present laws of the United States, is shown in the following

Table.

Places.	Denominations of Money.	Value.
Great Britain,	1 pound (£) = 20 shillings, or 10 florins = $	4.84
France,	1 franc = 100 centimes =	.186
Belgium,	1 franc = 100 centimes =	.186
Turin, Genoa,	1 lira = 100 centesimi =	.186
Amsterdam, Hague,	1 florin or guilder = 100 cents =	.40
Russia,	1 silver ruble = 100 copecks =	.75
Havana,	1 dollar = 8 real plate = 20 real vellon =	1.00
Spain,	1 real plate = 2 real vellon =	.10
Constantinople,	1 piaster = 100 aspers =	.05
Portugal,	1 millrea = 1000 reas,	1.12
Canton,	1 tael = 10 mace = 100 candarins = 1000 cash =	1.48

382. British money is usually expressed in pounds, shillings, pence, and farthings; but, by means of the new coin, the *florin*, the *decimal* system of money-reckoning has been commenced, and is made use of by the Bank of England, and by other large business establishments.

What is Foreign Exchange? The Intrinsic Par of Exchange? The Legal Value? What is the value of a pound? A franc? A lira of Turin? A florin of Amsterdam? A silver ruble? A dollar of Havana? A real plate of Spain? A piaster of Constantinople? A millrea? A tael of Canton? How is British money usually expressed?

Of British or English currency, 4 farthings (far.) make 1 penny (d.), 12 pence 1 shilling (s.), and 20 shillings 1 pound; also, 2 shillings 1 florin (fl.), 10 florins 1 pound, or sovereign.

A shilling may be written 5 tenths of a florin, or as 5 hundredths of a pound, and 6 pence as 25 hundredths of a florin, or 25 thousandths of a pound. Thus,

3 £. 7 fl. 1 s. 6 d. may be expressed £3.775

383. Exchange in this country on England is reckoned at a certain per cent. on the former value of a £ sterling, which was at the rate of £9 = \$40, or £1 = \$4.44$\frac{4}{9}$, instead of the present value of \$4.84. Hence,

The present *commercial par* of exchange is at about 9 per cent. above the old, or nominal par. Thus,

The nominal par of £1.	= \$4.444+
Add 9 per cent.,	= .399+
The nominal par at 9 per cent. premium	= \$4.843+, or \$4.84

The intrinsic value of a *new* English sovereign of recent coinage is \$4.86, so that the intrinsic par of exchange on England is about 9$\frac{3}{8}$ per cent. premium on the old or nominal par.

384. To compute foreign exchange.

1. Required the cost of the following bill, at 9$\frac{1}{2}$ per cent. premium:

£3000. Boston, May, 16, 1866.

At sight of this first of exchange (second and third unpaid), pay to the order of Frank Fuller, in London, three thousand pounds sterling, value received, and charge the same to the account of

Holden, Wells & Co.

To Baring & Bro.,
London.

How is exchange in this country on England usually reckoned? What is the *present commercial par*? Intrinsic value?

OPERATION.

$$£9 = \$40 \times 1.095,$$

$$£1 = \frac{\$40 \times 1.095}{9} = \$4.86\tfrac{2}{3}.$$

$$£3000 = \$4.86\tfrac{2}{3} \times 3000 = \$14600.$$

At $9\frac{1}{2}$% premium, the cost of £9 will be \$40 $\times$ 1.095, and the cost of £1 will be $\frac{\$40 \times 1.095}{9}$, or \$4.86$\frac{2}{3}$.

If £1 costs \$4.86$\frac{2}{3}$, £3000 will cost \$4.86$\frac{2}{3}$ $\times$ 3000, or \$14600.

2. If J. P. Clay, of Baltimore, should purchase a bill of £2200, at 8 % premium, what would it cost in United States money? *Ans.* \$10560.

3. How much must be paid in New York city for a bill on Liverpool, for 1173 £ 5 s., exchange being at $8\frac{1}{2}$ % premium? *Ans.* \$5657.67.

4. What will be the cost, in Washington, of a bill on Havre for 2570 francs, exchange being at 5.14 francs to a dollar?

SOLUTION. $2570 \div 5.14 = \$500$, *Ans.*

5. What must be paid for a bill on Amsterdam for 2626 florins, at 1 % premium? *Ans.* \$1060.90

6. If I pay \$14600 for a bill on London, when exchange is at $9\frac{1}{2}$ % premium, what is the face of the bill in English money?

OPERATION.

$$£9 = \$40 \times 1.095$$

$$£1 = \frac{\$40}{9} \times 1.095 = \$4.86\tfrac{2}{3}.$$

$$\$14600 \div \$4.86\tfrac{2}{3} = 3000$$

At $9\frac{1}{2}$ % premium, £1 can be remitted for \$4.86$\frac{2}{3}$.

If £1 can be remitted for \$4.86$\frac{2}{3}$, as many pounds can be remitted for \$14600, as \$4.86$\frac{2}{3}$ is contained times in \$14600, or 3000.

7. At 5.14 francs to a dollar, how many francs in Paris will be required to remit \$500?

8. When exchange is at $8\frac{1}{2}$ % premium, what is the face, in English money, of a bill on London, which can be bought for \$5657.67? *Ans.* 1173 £ 5 s.

REVIEW QUESTIONS. What is Exchange? (369) A Bill of Exchange? (370) Par of Exchange? (373) Course of Exchange? (375)

REVIEW EXERCISES.

1. What is the ratio of 6 weeks to three days? *Ans.* 14.

2. Reduce the ratio 8 : 72 to its smallest terms. *Ans.* $\frac{1}{9}$.

3. If the antecedent of the ratio 2.11 is 31.65, what is the consequent? *Ans.* 15.

4. If the consequent of the ratio 2.11 is 15, what is the antecedent?

5. If the product of the means of a proportion is $2\frac{2}{3}$, and one of the extremes is $\frac{3}{5}$, what is the other? *Ans.* $4\frac{4}{9}$.

6. Which is the greater ratio, 9 : 10 or 3 : 4? *Ans.* 9 : 10.

7. Required the simple ratio equivalent to $(2:3) \times (5:7) \times (1:7.)$ *Ans.* 10 : 147.

8. If a ship sails 155 miles in 12 hours, how far will it sail in 60 hours? *Ans.* 775 miles.

9. If 2 men can build 803 rods of fence in 22 days, how long will it take them to build 73 rods?

10. If a hound makes 27 leaps while a hare makes 25, and their leaps are of equal length, how many leaps must the hound make to overtake the hare, if the latter has 50 leaps the start? *Ans.* 675.

11. A, B, and C trade in company; A put in \$3000, B \$2000, and C \$1000; they gained $12\frac{1}{2}$ % on the whole capital; what was each partner's share of the gain?
Ans. A's \$375, B's \$250, and C's \$125.

12. If Hendricks, William, Arthur, and Frank should share an estate of \$54000, in the proportion of the numbers of the letters in their first names, how much will be the share of each?
Ans. Hendricks' \$18000, William's \$14000, Arthur's \$12000, Frank's \$10000.

13. If $3\frac{1}{2}$ yards of carpeting $\frac{3}{4}$ of a yard wide cost $\$7\frac{5}{8}$, what should $9\frac{3}{4}$ yards $\frac{7}{8}$ of a yard wide cost? *Ans.* \$24.78+.

REVIEW QUESTIONS. What is Equation of Payments? (338) Equated Time? (339)

14. A commenced business January 1, with a capital of \$9000; April 1, he is joined by B with a capital of \$10000; at the expiration of the year they had gained \$1320; what was each partner's share of the gain? *Ans.* A's \$720, B's \$600.

15. What is the equated time of paying \$3000, if \$500 of it is payable in 4 months, \$1000 in eight months, and the remainder in 16 months? *Ans.* 11 mo. 10 da.

16. In 1866, Samuel Ashton charged me for cash May 15, \$400, and Nov. 2, \$1000; and I charged him for cash, April 3, \$800, and August 31, \$900; allowing interest at 6 %, what will be the balance due me January 1, 1867? *Ans.* \$329.45.

17. A certain corporation has laid a tax of \$1500 upon its capital stock of \$75000; how much will A, who owns \$1200 of stock, be required to pay?

18. What is the amount of duty, at 30 % ad valorem, on an importation of woollens, invoiced 500 yards at 15 s. per yard, allowing \$4.84 as the value of a pound sterling? *Ans.* \$544.50.

19. When exchange is at 9½ % premium, what is the face of a bill on London that can be bought in Cincinnati for \$2182.94. *Ans.* 448 £ 11 s.

Exercises in Analysis.

1. If \$120 will purchase 4½ acres of land, how many acres will \$480 purchase?

SOLUTION. \$480 *are* 4 *times* \$120; *hence, if* \$120 *will purchase* 4½ *acres*, \$480 *will purchase* 4 *times* 4½ *acres, or* 18 *acres. Therefore, etc.*

2. If the income from 49 shares in a certain mill is \$735, how much will it be from 7 shares?

3. A and B trade in company. A paid in 6 times as much as B, and they gain \$1974; what was each one's share of the gain? *Ans.* B's \$282, A's \$1692.

4. A, B, and C trade in company; A put in ⅓ of the stock,

REVIEW QUESTIONS. What is the Rule for finding the equated time when the terms of credit begin at the same date? (341) When the terms of *credit begin at* different dates? (342)

B $\frac{1}{2}$ of it, and C the rest; on dividing, C's share of the gain was \$495; what was the gain of each of the other partners?

SOLUTION. *If A put in $\frac{1}{5}$ of the stock, B $\frac{1}{2}$, and C the rest, C must have put in* $1 - (\frac{1}{5} + \frac{1}{2})$, *or* $\frac{3}{10}$, *of the stock.*

If $\frac{3}{10}$ *of the stock gain* \$495, $\frac{1}{10}$ *of the stock must gain* $\frac{1}{3}$ *of* \$495, *or* \$165.

If $\frac{1}{10}$ *of the stock gain* \$165, *since A put in* $\frac{1}{5}$, *or* $\frac{2}{10}$, *of the stock, his share of the gain must be* 2 *times* \$165, *or* \$330; *and since B put in* $\frac{1}{2}$, *or* $\frac{5}{10}$, *of the stock, his share of the gain must be* 5 *times* \$165, *or* \$825.

Therefore, etc.

5. A ship worth \$60200 was entirely lost, $\frac{1}{8}$ of her belonging to A, $\frac{1}{4}$ to B, and the rest to C; what loss will each sustain, if \$35000 of her was insured?

Ans. A \$3150, B \$6300, and C \$15750.

6. A merchant owes a certain sum, $\frac{1}{4}$ of which is due in 2 months, $\frac{1}{3}$ in 4 months, $\frac{1}{6}$ in 5 months, and the balance in 6 months; what is the average term of credit?

Ans. 4 months 5 days.

7. A borrowed \$2000 for 6 months; 4 months before it was due he paid \$1000, and 2 months before it was due he paid \$600; how long after the expiration of the 6 months may the remaining \$400 remain unpaid?

SOLUTION. *A credit on* \$1000 *for* 4 *months is equivalent to a credit on* \$1 *for* 4000 *months, and a credit on* \$600 *for* 2 *months is equivalent to a credit on* \$1 *for* 1200 *months.*

Hence, the whole credit is equivalent to the credit of \$1 *for* 4000 + 1200, *or* 5200, *months; and the credit of* \$1 *for* 5200 *months is equivalent to the credit of* \$400 *for* $\frac{1}{400}$ *of* 5200, *or* 13 *months.*

Therefore, etc.

8. Sold, March 5, 1866, goods to H. Mitchell amounting to \$1600 on 6 months; April 5, he paid \$200, and on August 5, \$800. When in equity should the balance be paid?

Ans. December 5, 1866.

9. If I owe \$2500 payable in 4 months, but, to accommo-

REVIEW QUESTIONS. What is the Balance of an Account? (344) Averaging of an Account? (345)

date, pay $1500 down, how long in equity should I be permitted to keep the remainder after the expiration of the 4 months?

Ans. 6 months.

10. A hare starts 25 of its leaps in advance of a hound, and takes 4 leaps to the hound's 3; but 2 of the hound's leaps are equal to 3 of the hare's; how many leaps must the hound take to overtake the hare?

SOLUTION. *If* 2 *of the hound's leaps equal* 3 *of the hare's,* 1 *of the hound's is equal to* $1\frac{1}{2}$ *of the hare's, and if the hound takes* 3 *leaps to the hare's* 4, *he takes* 1 *leap to the hare's* $1\frac{1}{3}$.

If 1 *of the hound's leaps is equal to* $1\frac{1}{2}$ *of the hare's, and he takes* 1 *leap to the hare's* $1\frac{1}{3}$, *he gains in taking* 1 *leap,* $1\frac{1}{2} - 1\frac{1}{3}$, *or* $\frac{1}{6}$ *of a hare's leap.*

If he gains $\frac{1}{6}$ *of a hare's leap in taking* 1 *leap, he will gain* 25 *leaps of the hare, or overtake the hare, in taking as many leaps, as* $\frac{1}{6}$ *is contained times in* 25, *which are* 150.

Therefore, etc.

11. A thief having gone 51 miles, an officer set out to overtake him, and for 16 miles traveled by the thief, the officer travels 19 miles. How far will the officer have traveled before the thief is overtaken? *Ans.* 323 miles.

12. A starts from Boston toward a town 16 miles distant, walking at the rate of $2\frac{1}{4}$ miles an hour; and 2 hours after, B starts from Boston upon the same route, by coach driven at the rate of 9 miles an hour; in what time, and how far from Boston will B overtake A?

Ans. In 40 minutes, and 6 miles from Boston.

13. If 12 barrels of corn will pay for 10 cords of wood, and 48 cords of wood will pay for 8 tons of hay, how many barrels of corn will pay for 15 tons of hay?

SOLUTION. *If* 8 *tons of hay can be paid for by* 48 *cords of wood,* 15 *tons can be paid for by* $\frac{15}{8}$ *of* 48 *cords, or* 90 *cords.*

If 10 *cords of wood can be paid for by* 12 *barrels of corn,* 90 *cords*

REVIEW QUESTIONS. What is the Rule for finding the equated balance of an account? (346) What is Merchandise Balance? (347) Interest Balance? (348) Cash Balance? (349)

of wood, or 15 *tons of hay, can be paid for by* 9 *times* 12 *barrels, or* 108 *barrels.*

Therefore, etc.

14. If 10 calves are worth as much as 9 colts, and 90 colts are worth as much as 112 sheep, how many sheep are worth as much as 50 calves? *Ans.* 56.

15. If 8 men can do the work of 32 women, and 2 women can do the work of 3 boys, how many men can do the work of 24 boys? *Ans.* 4 men.

16. If the relative value of oak wood to spruce is as 2 to 1, and that of spruce to pine as 7 to 8, how many cords, composed of spruce and pine in equal parts, will equal 60 cords of oak? *Ans.* 112 cords.

17. A, B, and C together can do a piece of work in 20 days, A alone can do it in 60 days, and B alone can do it in 80 days; in what time could C working alone do it?

SOLUTION. *If A, B, and C together can do the work in* 20 *days, in* 1 *day they can do* $\frac{1}{20}$ *of it.*

If A working alone can do the work in 60 *days, in* 1 *day he can do* $\frac{1}{60}$ *of it, and if B working alone can do it in* 80 *days, in* 1 *day he can do* $\frac{1}{80}$; *hence, A and B working together can do* $\frac{1}{60}+\frac{1}{80}$, *or* $\frac{7}{240}$, *of the work in* 1 *day.*

If A and B working together can do $\frac{7}{240}$ *of the work in* 1 *day, since A, B, and C working together can do* $\frac{1}{20}$ *of it in* 1 *day, C must do the difference between* $\frac{1}{20}$ *and* $\frac{7}{240}$, *or* $\frac{5}{240}=\frac{1}{48}$ *of the work in* 1 *day.*

If C can do $\frac{1}{48}$ *of the work in* 1 *day, working alone, he can do the whole of it in as many days as* $\frac{1}{48}$ *is contained times in* 1, *or in* 48 *days.*

Therefore, etc.

18. A piece of work can be done in a day of $11\frac{1}{2}$ hours by 2 men, or 5 women, or 12 boys; in what time could it be done by 1 man, 2 women, and 3 boys, working together? *Ans.* In 10 hours.

REVIEW QUESTIONS. What are Taxes? (353) A Poll Tax? (353) How is a town tax assessed or apportioned? (357) What are Duties? (358) Internal Revenue? (361) Customs? (363) Exchange? (369) Domestic Exchange? (376) Foreign Exchange? (379)

19. A carpenter is offered \$325 to do a job of work, which he can do in $12\frac{1}{2}$ days, his journeyman in $18\frac{3}{4}$ days, and his apprentice in 25 days. If they should do it together, in what time could it be completed, and how much would each earn?

Ans. In $5\frac{10}{13}$ days; the carpenter \$150, the journeyman \$100, and the apprentice \$75.

INVOLUTION.

385. A **Power** of a number is either the number itself, or the product obtained by taking the number several times as a factor. Thus,

$2^1 = 2$, is the *first* power of 2.
$2^2 = 2 \times 2 = 4$, is the *second* power, or *square* of 2.
$2^3 = 2 \times 2 \times 2 = 8$, is the *third* power, or *cube* of 2.
$2^4 = 2 \times 2 \times 2 \times 2 = 16$, is the *fourth* power of 2,

and so on, the *exponent* (Art. 103) of the power denoting the number of times the number 2 is taken as a factor.

386. Involution is the process of raising a given number to a required power.

This may be effected, as is evident from the definition of a power, by the simple process of multiplying. That is, to raise a number to any power,

Find the product of the number taken as a factor as many times as is denoted by the exponent of the required power.

Exercises.

Raise to the powers denoted by their respective exponents:

1. 25^2.	*Ans.* 625.	5. $(\frac{3}{4})^3$.	*Ans.* $\frac{27}{64}$.
2. 22^2.	*Ans.* 484.	6. $(2\frac{2}{3})^4$.	*Ans.* $50\frac{46}{81}$.
3. 81^3.	*Ans.* 531441.	7. 1.2^3.	*Ans.* 1.728.
4. 5^5.	*Ans.* 3125.	8. $.13^3$.	*Ans.* .002197.

What is a Power? What does the exponent of the power denote? What is *Involution*? How is a number raised to any power?

9. What is the square of 78? *Ans.* 6084.
10. What is the cube of .09? *Ans.* .000729.

387. From the definition of a power may be derived the following

PRINCIPLES.

1. *The product of two or more powers of a number is that power denoted by the sum of their exponents.*

For, since $2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2) = 2^5$, $2^3 \times 2^2 = 2^{3+2} = 2^5$.

2. *Any power of a number raised to a power is that power of the number denoted by the product of the exponents.*

For, since $(2^3)^2 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^6$, $(2^3)^2 = 2^{3 \times 2} = 2^6$.

3. *Any power of a number divided by a power of the same number is that power denoted by the difference of their exponents.*

For, since $2^5 \div 2^2 = (2 \times 2 \times 2 \times 2 \times 2) \div (2 \times 2) = 2^3$, $2^5 \div 2^2 = 2^{5-2} = 2^3$.

Exercises.

1. What power of 2 is $2^5 \times 2^2$? *Ans.* 2^7.
2. What is the product of $3^3 \times 3^3$? *Ans.* 729.
3. What is the 7th power of 2?

SOLUTION. $2^7 = 2^4 \times 2^3 = 16 \times 8 = 128$, *Ans.*

4. What is the 8th power of 5? *Ans.* 390625.
5. What power of 15 is 15^2 raised to the 3d power? *Ans.* 15^6.
6. Find the 2d power of 5^3. *Ans.* 15625.
7. What power of 17 is $17^5 \div 17^4$? *Ans.* 17^1.
8. Find the quotient of $10^6 \div 10^4$. *Ans.* 100.
9. What is the 8th power of $\frac{1}{4}$? *Ans.* $\frac{1}{65536}$.
10. Required the value of $(4^3)^3$. *Ans.* 262144.
11. Required the 10th power of 3. *Ans.* 59049.

What is the first Principle? The second? The third?

EVOLUTION.

388. A **Root** of a number is one of the equal factors taken to form the number.

389. The **Second,** or **Square Root** of a number is one of its two equal factors. Thus,

The square root of $25 = 5$; since $5 \times 5 = 25$.

The **Third,** or **Cube Root** of a number is one of its three equal factors. Thus,

The cube root of $125 = 5$; since $5 \times 5 \times 5 = 125$.

390. The **Radical Sign,** $\sqrt{\ }$, or *Fractional Exponents*, are used to denote roots. Thus,

$\sqrt[2]{4}$, or $4^{\frac{1}{2}}$, denotes the *second* or *square* root of 4;

$\sqrt[3]{8}$, or $8^{\frac{1}{3}}$, denotes the *third* or *cube* root of 8;

$\sqrt[4]{16}$, or $16^{\frac{1}{4}}$, denotes the *fourth* root of 16;

and so on, the figure or figures, called the *Index*, written over the radical sign, or the denominator of the fractional exponent, denoting the degree or name of the roots.

The index is usually omitted in denoting the square root.

391. **Evolution** is the process of finding or extracting the roots of numbers. It is the reverse of Involution.

A *Perfect Power* is a number whose root can be exactly obtained; and an *Imperfect Power*, or *Surd*, a number whose root cannot be exactly obtained.

392. The **Root** corresponding to any perfect power may be found by factoring the power. Thus,

By resolving 196 into its prime factors, 2, 2, 7, and 7, we find its square root is 14, since one of its two equal factors is $7 \times 2 = 14$.

$\sqrt[3]{27000} = 30$; since the factors of 27000 are 2, 2, 3, 3, 5 and 5; and one of the three equal factors is $2 \times 3 \times 5 = 30$.

What is a Root? The Square Root of a number? The Cube Root? By what are roots denoted? The degree or name of the roots? What is Evolution? A Perfect Power? An Imperfect Power?

In finding the approximate root of an imperfect power, and, for convenience, in finding the root of large numbers, other methods are used.

SQUARE ROOT.

393. A method of extracting the square root of numbers is derived from the following

PRINCIPLES.

1. *The square of any integral number is expressed by twice as many places of figures as the number, or by twice as many less one.*

For, the first ten numbers are

1, 2, 3, 4, 5, 6, 7, 8, 9, 10,

and their squares are

1, 4, 9, 16, 25, 36, 49, 64, 81, 100;

also, the square of 99 is 9801, of 100 is 10000, of 999 is 998001, of 1000 is 1000000, and so on. Hence,

2. *If a point be placed over every second figure in any integral number, beginning with units, the groups or periods of figures thus formed will correspond respectively to the* UNITS, TENS, HUNDREDS, *etc., in its square root.*

3. *Every integral number expressed by more than two places of figures is equal to the square of the tens in its* ROOT, *plus the product of twice the tens plus the units, multiplied by the units.*

For, take any number composed of tens and units, as 36, separate it into its tens and units, square it, keeping the products distinct, and we have

$36 = 30 + 6$, and $36^2 = (30 + 6) \times (30 + 6)$

$$30 \times 30 = 30^2 \qquad\qquad = 900$$

$$\left.\begin{array}{r} 6 \times 30 \\ 30 \times 6 \end{array}\right\} = \quad (30 \times 6) \times 2 \qquad\qquad = 360$$

$$6 \times 6 = \qquad 6^2 \qquad\qquad = 36$$

$$36^2 = 30^2 + (30 \times 6) \times 2 + 6^2 = 30^2 + (60 + 6) \times 6 = 1296$$

What is the first Principle? The second? The third?

If, now, we denote the tens by t, and the units by u, we have the formula,

$$(t+u)^2 = t^2 + 2\,(t \times u) + u^2 = t^2 + (2\,t + u) \times u,$$

or that which was to be proved.

This formula is general, since all numbers expressed by more than two places of figures may be considered as composed of tens and units.

394. To extract the square root of a number.

1. Let it be required to find the square root of 1296.

OPERATION.

$$\left.\begin{array}{lr|l}
 & \dot{1}2\dot{9}6 & 30+6 \\
30^2 = & 900 & \\ \hline
30 \times 2 = 60 & 396 & \\
\quad\;\; \underline{6} & & \\
\quad 66 \times 6 = & 396 & \\ \hline
\end{array}\right\} \text{ or } \left\{\begin{array}{lr|l}
 & \dot{1}2\dot{9}6 & 36 \\
 & 9 & \\ \hline
60 & 396 & \\
\underline{6} & & \\
66 \times 6 = & 396 & \\ \hline
\end{array}\right.$$

Since the square of any integral number is expressed by twice as many places of figures as the number, or by twice as many less one, the root of 1296 must be expressed by two places of figures.

If the root is expressed by two places of figures, it must be composed of tens and units; hence, 1296 must be equal to the square of the tens in its root, plus the product of twice the tens plus the units, multiplied by the units.

Since the square of tens is always hundreds, we seek the greatest number of tens whose square is contained in the 12 hundreds, which is 3 tens.

We write the 3 as the tens' figure in the required root.

3 tens squared, or 9 hundreds, subtracted from 1296, leaves 396, which must contain the product of twice the tens plus the units, multiplied by the units.

If the 396 contained only the product of twice the tens multiplied by the units, dividing by twice the tens must give the units; and making a trial divisor of twice the tens, or 60, we find the probable units of the root to be 6.

The 6 units added to twice the 3 tens, and multiplied by 6 units, or $(60+6) \times 6$, equals 396, and completes the square. We write 6 as the true units' figure of the root.

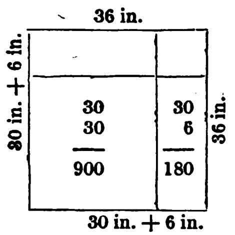
How many places of figures express the square of any integral number? *What is always the* square of tens? Explain the operation.

Therefore, the square root of 1296 is 3 tens + 6 units, or 36.

Had the root been expressed by more than two places of figures, the principle applied in the operation would have been the same, since we may have tens and units of *units*, tens and units of *tens*, tens and units of *hundreds*, etc.

The operation may be illustrated by diagrams. Thus,

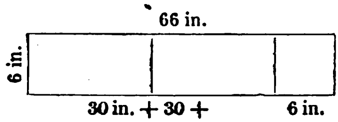
Let there be a square of 1296 square inches, whose side is required.



The greatest square of the tens in 1296 square inches is 900 square inches, representing a square whose side is 3 tens, or 30, inches.

900 square inches taken out of 1296 square inches leaves 396 square inches, which belong to two of the sides, so as to preserve the form of a square.

If 396 square inches belong to two sides, the width would be as many inches as the number denoting the length of two equal sides, or 60, is contained times in 396, which is 6. But the whole length of the additional portion is that



of twice one of the equal sides plus the side of a small square, whose side is the width of the additional portion, or 6 inches; $(60 + 6)\ 6$, or 66×6 is 396; and 396 square inches subtracted, leaves no remainder.

Therefore, 1296 is a square whose root is 36.

RULE. *Separate the given number into periods, by pointing every second figure, beginning with the units' figure, proceeding to the left if whole numbers, and to the right if decimals.*

Find the greatest number whose square is contained in the left-hand period, and write it in the root; subtract the square of this from the left-hand period, and to the remainder annex the next period for a dividend.

Take twice the root found, regarded as tens, for a trial divisor; divide the dividend by it, and write the result as a second part of the root.

Had the root been expressed by more than two figures, would the principle applied in the operation still apply? Why? Explain the operation as illustrated by diagrams. Recite the Rule.

To the trial divisor add the last part of the root; multiply the result by the last part of the root, and subtract the product from the dividend.

If there are more periods, continue the operation as before.

The trial divisor being an *incomplete* divisor, the figure of the root may sometimes be too large; in such a case, substitute for it a root 1 less.

Examples.

2. Extract the square root of 1866.24.

OPERATION.

	1866.24	43.2
	16	
80	266	
3		
83 × 3 =	249	
86.0	17.24	
2		
86.2 × .2 =	17.24	

3. Extract the square root of 10291264.

OPERATION.

	10291264	3208
	9	
60	129	
2		
62 × 2 =	124	
6400	51264	
8		
6408 × 8 =	51264	

In the operation of the 3d example, a 0 occurring in the root, we annex a cipher to the trial divisor, and to the dividend another period, and proceed as before.

Find the square root

4. Of 77841. *Ans.* 279.
5. Of 2916.
6. Of 10.4976. *Ans.* 3.24.
7. Of 11664. *Ans.* 108.
8. Of .459684. *Ans.* .678.
9. Of 31640625. *Ans.* 5625.
10. What is the square root of .0003272481? *Ans.* .01809.
11. What is the square root of .00001849? *Ans.* .0043.

395. When there is a *remainder* after using all the periods, it indicates that the given number has not an exact square root; but the approximation may be continued, by annexing periods of decimal ciphers.

If a figure of the root prove to be too large, what is to be done? How do we proceed when a 0 occurs in the root? What does a remainder, after using all the periods, indicate? How may the approximation be continued?

12. Extract the square root of 12 to three decimal places. *Ans.* 3.464+.

13. Extract the square root of 1.6 to two decimal places.

14. Extract the square root of .002 to four decimal places. *Ans.* .0447+.

15. Extract the square root of 5. *Ans.* 2.236+.

16. What is the square root of .5 to four decimal places? *Ans.* .7071+.

396. When the given number is a *common fraction*, or a *mixed number*, reduce it to its simplest form, and if the numerator and denominator are both perfect squares, extract the square root of each separately; but, if not, reduce the fraction to an equivalent decimal, and extract its root.

What is the square root

17. Of $\frac{121}{169}$? *Ans.* $\frac{11}{13}$.
18. Of $\frac{7056}{9216}$? *Ans.* $\frac{7}{8}$.
19. Of $\frac{450}{2048}$? *Ans.* $\frac{15}{32}$.
20. Of $37\frac{36}{49}$? *Ans.* $6\frac{1}{7}$.
21. Of $\frac{7}{8}$? *Ans.* .9354+.
22. Of $17\frac{23}{100}$? *Ans.* 4.1509+.

23. What is the value of $\sqrt{42025}$? *Ans.* 205.

24. What is the value of $\sqrt{\frac{478}{549}}$? *Ans.* .93309+.

APPLICATIONS.

1. A certain number of boys spent \$3.61, each spending as many cents as there were boys; what was the number of boys? *Ans.* 19.

2. A square pavement contains 20736 square stones, all of the same size; what number compose one of its sides?

3. If 3969 hills of corn, each hill an equal distance from another, are planted in a square, how many hills are there in each row? *Ans.* 63 hills.

How do we proceed when the given number is a common fraction or a mixed number?

REVIEW QUESTIONS. What is a Power? (385) Involution? (386) A Root? (388) Evolution? (391) The Square Root of a number? (389)

4. A general has an army of 141376 men; how many must he place in rank and file to form them into a square? *Ans.* 376.

5. What is the length of one of the equal sides of a square acre? *Ans.* 12.64+ rods.

6. How much will it cost to inclose 10 acres of land in the form of a square, at $.60 a rod? *Ans.* $96.

7. How much will it cost to inclose a hectare of land, in the form of a square, at $.25 a meter? *Ans.* $100.

8. A general, trying to mass his army of 15410 men into a square, found he had 34 men over; required the number in rank and file. *Ans.* 124.

CUBE ROOT.

397. A method of extracting the cube root of numbers is derived from the following

PRINCIPLES.

1. *The cube of any integral number consists of three times as many places of figures as the number, or of three times as many less one or two.*

For, the first ten numbers are

1, 2, 3, 4, 5, 6, 7, 8, 9, 10,

and their cubes are

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000;

also the cube of 99 is 970299, of 100 is 1000000, of 999 is 997002999, of 1000 is 1000000000, and so on. Hence,

2. *If a point be placed over every third figure in any integral number, beginning with units, the groups or periods of figures thus formed will correspond respectively to the* UNITS, TENS, HUNDREDS, *etc., in its cube root.*

3. *Every integral number expressed by more than three places of figures is equal to the cube of the tens in its root, plus the sum of three times the square of the tens, three times*

What is the Cube or Third Root? What is the first Principle? The second? The third?

the product of the tens and units, and the square of the units, multiplied by the units.

For, let 36 be any number composed of tens and units,

Then, $36 = 30 + 6$, and, by Art. 392, we have

$$36^2 = 30^2 + (30 \times 2) \times 6 + 6^2.$$

Multiplying this square by $36 = 30 + 6$, and keeping the products distinct, we obtain

$$36^3 = 30^3 + (30^2 \times 3) \times 6 + (30 \times 3) \times 6^2 + 6^3$$
$$= 30^3 + (30^2 \times 3 + 30 \times 3 \times 6 + 6^2) \times 6$$

And $30^3 = 27000$

$$\begin{aligned} 30^2 \times 3 &= 2700 \\ 30 \times 3 \times 6 &= 540 \\ 6^2 &= 36 \end{aligned}$$

$$(30^2 \times 3 + 30 \times 3 \times 6 + 6^2) \times 6 = 3276 \times 6 = 19656$$

$$36^3 = 30^3 + (30^2 \times 3 + 30 \times 3 \times 6 + 6^2) \times 6 = 46656$$

If, now, we denote the tens by t, and the units by u, we have the formula,

$$(t + u)^3 = t^3 + (t^2 \times 3) \times u + (t \times 3) \times u^2 + u^3$$
$$= t^3 + (t^2 \times 3 + t \times u \times 3 + u^2) \times u,$$

or that which was to be proved.

This formula is general, since all integral numbers expressed by two or more places of figures, may be considered as composed of tens and units.

398. To extract the cube root of a number.

1. Let it be required to find the cube root of 405224.

OPERATION.

$$405\dot{2}2\dot{4} \mid 74$$

$$70^3 = 343000$$

$$62224$$

$$\begin{aligned} 70^2 \times 3 &= 14700 \\ 70 \times 3 \times 4 &= 840 \\ 4^2 &= 16 \end{aligned}$$

$$(70^2 \times 3 + 70 \times 3 \times 4 + 4^2) \times 4 = 15556 \times 4 = 62224$$

Since the cube of any integral number is expressed by three times as many figures as the number, or three times as many less one or two,

What is the formula? Why is it general?

the root of 405224 must be expressed by two places of figures, and, therefore, must consist of tens and units.

Since the cube of tens is always thousands, we seek the greatest number of tens whose cube is contained in 405 thousands, which is 7 tens. We write the 7 as the tens' figure in the required root.

7 tens cubed, or 343 thousands, subtracted from 405224, leaves 62224, which must contain the product of three times the square of the tens, three times the tens by the units, and the square of the units, multiplied by the units.

If the 62224 contained only three times the square of the tens by the units, dividing by three times the square of the tens must give the units; and making a trial divisor of three times the square of the tens, or 14700, we find the probable units of the root to be 4.

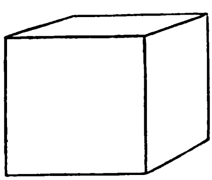
Adding to three times the square of the tens, or 14700, the product of three times the tens by the units, or 840, and the square of the units, or 16, we have the sum 15556, and this, multiplied by the units, equals 62224, which completes the cube. We write 4, as the true units' figure of the root.

Therefore, the cube root of 405224 is 74.

Had the root been expressed by more than two places of figures, the principle applied in the operation would have been the same, since we may have tens and units of *units*, tens and units of *tens*, tens and units of *hundreds*, etc.

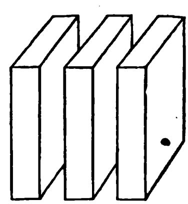
The operation may be illustrated by diagrams. Thus,

Let there be a cube of 405224 solid inches, whose edge is required.



The greatest cube of the tens in 405224 solid inches is 343000 solid inches, representing a cube whose edge is 7 tens, or 70 inches.

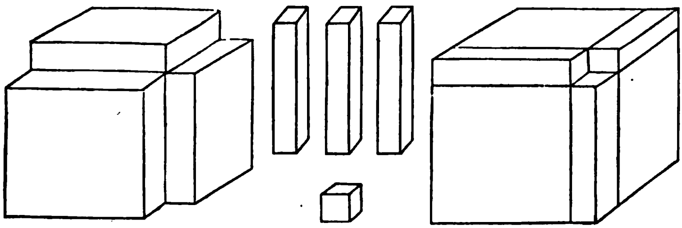
343000 solid inches taken out of 405224 solid inches, leaves 62224 solid inches, which belong to three sides of the solid, so as to preserve the form of a cube.



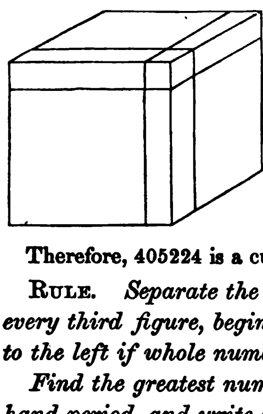
The addition upon one side of the cube must be 70 inches long by 70 inches wide, and if 1 inch thick, will be 4900 solid inches, and upon the three sides will be 3 times 4900 or 14700 solid inches.

How many places of figures express the cube of any integral number? If the root is expressed by two places of figures, of what must it be composed? What is always the cube of tens? Explain the operation. How may the operation be illustrated?

If 14700 solid inches will make 1 inch of thickness, 62224 solid inches will make as many inches of thickness, as 14700 solid inches are contained times in 62224 solid inches, or 4 inches.



But, besides the three additions to the sides, there are required, upon three edges, three other additions of 70 inches long, 4 inches wide, and if 1 inch thick, there will be for the three 840 solid inches.



To complete the cube an addition is required, upon one corner, 4 inches long, 4 inches wide, and if 1 inch thick, will be 16 solid inches.

But all the additions are 4 inches thick, and $(14700 + 840 + 16) \times 4 = 62224$ solid inches, which completes the cube.

Therefore, 405224 is a cube whose root is 74.

RULE. *Separate the given number into periods, by pointing every third figure, beginning with the units' figure, proceeding to the left if whole numbers, and to the right if decimals.*

Find the greatest number whose cube is contained in the left-hand period, and write it in the root; subtract the cube of this root from the left-hand period, and to the remainder annex the next period for a dividend.

Take three times the square of the root found, regarded as tens, for a trial divisor; divide the dividend by it, and write the result as a second part of the root.

To the trial divisor add three times the first part of the root, regarded as tens, multiplied by the last part, also the square of the last part; multiply the result by the last part, and subtract the product from the dividend.

Give the illustration. Repeat the Rule.

If there are more periods, continue the operation as before.

If the number expressed by any figure of the root should prove to be too large, a smaller number must be taken.

Examples.

2. Find the cube root of 705919.947264.

OPERATION.

	705919.947264	89.04
	512	
19200	193919	
2160		
81		
21441 $\times$ 9 =	192969	
237630000	950947264	
106800		
16		
237736816 $\times$ 4 =	950947264	

In the operation, a 0 occurring in the root, we annex two ciphers to the trial divisor, and to the dividend annex another period, and proceed as before.

Find the cube root

3. Of 54872. *Ans.* 38.
4. Of 636056.
5. Of 64964808. *Ans.* 402.
6. Of 444194.947. *Ans.* 76.3.
7. Of .000001728. *Ans.* .012.
8. Of .001906624. *Ans.* .124.
9. Of 1076890625. *Ans.* 1025.
10. Of 80.677568161. *Ans.* 4.321.

399. When the given number has not an exact cube root, the operation may be extended by annexing periods of decimal ciphers.

11. What is the cube root of 26.2 to two places of decimals? *Ans.* 2.97+.

If the number expressed by any figure of the root prove too large, what is to be done? How do we proceed when a 0 occurs in the root? How may the operation be extended when the given number has not an exact cube root?

12. What is the cube root of 2 to three places of decimals? *Ans.* 1.259+.

13. What is the cube root of 517 to three places of decimals? *Ans.* 8.025+.

400. When the given number is a *common fraction*, or a *mixed number*, reduce it to its simplest form, and, if the numerator and denominator are both perfect cubes, extract the cube root of each separately; but, if not, reduce the fraction to an equivalent decimal, and extract the root of it.

What is the cube root

14. Of $\frac{189}{875}$? *Ans.* $\frac{3}{5}$.

15. Of $\frac{4}{9}$? *Ans.* .763+.

16. Of $\frac{136}{12393}$? *Ans.* $\frac{2}{9}$.

17. Of $\frac{6}{7}$? *Ans.* .949+.

18. Of $30\frac{265}{512}$? *Ans.* $3\frac{1}{8}$.

19. Of $7\frac{3}{5}$? *Ans.* 1.966+.

20. What is the value of $\sqrt[3]{405\frac{28}{125}}$? *Ans.* $7\frac{2}{5}$.

21. What is the value of $\sqrt[3]{15.32}$? *Ans.* 2.483+.

APPLICATIONS.

1. A block of granite in the form of a cube contains 103823 cubic inches; what is the measure of one of its equal edges? *Ans.* 47 inches.

2. Required the depth of a cubic box that shall exactly hold a bushel. *Ans.* 12.9+ inches.

3. There is a range of wood $21\frac{1}{3}$ feet long, 6 feet high, and 4 feet wide; how long a cubic pile will it make? *Ans.* 8 feet.

4. Find the area of a side of a cube containing 474552 liters. *Ans.* 60.84 square meters.

5. There is a cistern of a cubical form, which contains 1331 cubic feet; what are the length, breadth, and depth of it?

6. What must be the depth of a cubical cistern that shall contain 576 gallons? *Ans.* 4.25+ feet.

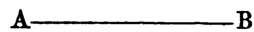
How do we proceed when the given number is a common fraction or a mixed number?

MENSURATION.

401. A **Point** is that which has only position.

402. A **Line** is that which has only length.

A STRAIGHT LINE is one that has all its parts in the same direction. Thus,



The line A B is a straight line.

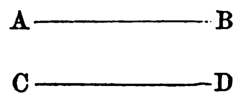
A CURVED LINE is one which continually changes its direction. Thus,

The line C D is a curved line.

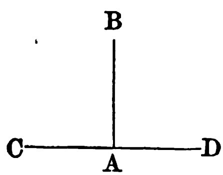
403. A **Plane** is a surface (Art. 199) in which any two points being taken, the straight line that joins them will lie wholly in the surface.

A CURVED SURFACE is one of which no part is plane.

404. **Parallel Lines** are such as, being in the same plane, have the same direction with each other. Thus,



A B and C D are parallel lines.

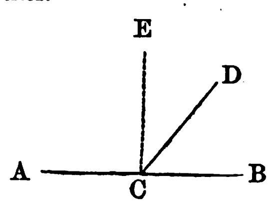


405. Two straight lines are said to be PERPENDICULAR to each other, when their meeting forms equal adjacent angles (Art. 208). Thus,

The lines A B and C D are perpendicular to each other.

406. A **Right Angle** is one formed by a straight line and a perpendicular to it. Thus,

The angle A C E is a right angle.



An **Acute Angle** is one which is less than a right angle; as the angle B C D.

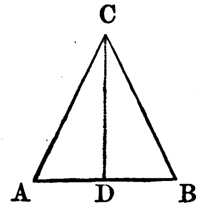
What is a Point? A Line? A Straight Line? A Curved Line? A Plane? *A Curved* Surface? Parallel Lines? A Right Angle? An Acute Angle?

An **Obtuse Angle** is one which is greater than a right angle; as the angle A C D.

The *sides* of an angle are the lines forming it, and the *vertex* of an angle the point of their meeting. Thus,

In the angle A C E, A C and C E are the sides, and C the vertex.

407. A **Plane Figure** is a plane bounded by a line or lines.

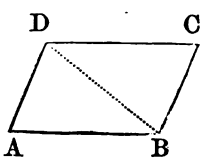


The PERIMETER of a plane figure is its boundary.

The BASE of a figure is the line upon which it is supposed to stand.

The ALTITUDE of a figure is its perpendicular hight. Thus,

In the plane figure A C B, the line A B is the base, and the line C D the altitude.



The DIAGONAL of a figure is a straight line joining any two of its angles, which are not adjacent to each other. Thus,

In the figure A B C D, the line D B is a diagonal.

The SIDES of a figure bounded by straight lines are the bounding lines. Thus,

A B, B C, C D, D A, are sides of the figure A B C D.

408. A **Polygon** is a plane figure bounded by straight lines.

A REGULAR POLYGON has equal sides and equal angles.

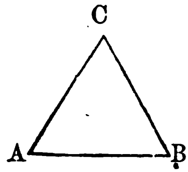
A polygon of three sides is called a *Triangle*, of four sides a *Quadrilateral*, of five sides a *Pentagon*, of six sides a *Hexagon*, of seven sides a *Heptagon*, of eight sides an *Octagon*, etc.

409. **Mensuration** treats of the measurement of lines, planes, and solids or volumes (Art. 199).

What is an Obtuse Angle? A Plane Figure? The Perimeter of a plane figure? The Base? The Altitude? The Diagonal? What is a Polygon? A *Regular Polygon*? Mensuration?

TRIANGLES.

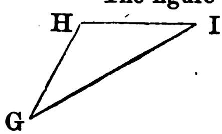
410. **A Triangle** is a polygon having three sides, and, therefore, three angles. Thus,



The figure A B C is a triangle.

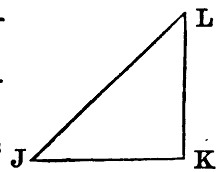
An ACUTE-ANGLED TRIANGLE has three acute angles. Thus,

The figure A B C is an acute angled triangle.



An OBTUSE-ANGLED TRIANGLE has one obtuse angle. Thus,

The figure G H I is an obtuse-angled triangle.



A RIGHT-ANGLED TRIANGLE has one right angle. Thus,

The figure J K L is a right angled triangle.

The side opposite the right angle is called the HYPOTHENUSE, and the side perpendicular to the base, the PERPENDICULAR. Thus,

In the figure J K L, the side J L is the HYPOTHENUSE, and K L the perpendicular.

411. By Geometry there may be readily demonstrated the following

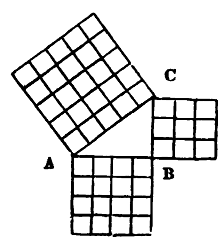
PRINCIPLES.

1. *The square of the hypothenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.* Thus,

Let h denote the hypothenuse, b the base, and p the perpendicular of a right-angled triangle, and we have the formula,

$$h^2 = b^2 + p^2,$$

which is illustrated by the diagram.



What is a Triangle? An Acute-Angled Triangle? An Obtuse-Angled *Triangle*? A Right-Angled Triangle? What are the sides called? What *is the first* Principle?

2. *The hypothenuse of a right-angled triangle is equal to the square root of the sum of the squares of the other two sides;* and

3. *Either of the two shorter sides of a right-angled triangle is equal to the square root of the difference of the squares of the hypothenuse and the other side.*

Exercises.

1. If the base of a right-angled triangle is 60 feet, and the perpendicular 45 feet, what is the hypothenuse?

OPERATION.

$60^2 + 45^2 = 3600 + 2025 = 5625$; $\sqrt{5625} = 75$, feet, *Ans.*

2. If the hypothenuse of a right-angled triangle is 75 feet, and one of the other sides 60 feet, what is the third side?

OPERATION.

$75^2 - 60^2 = 5625 - 3600 = 2025$; $\sqrt{2025} = 45$, feet, *Ans.*

3. A fort which is 15 feet high is surrounded by a moat 20 feet wide; what must be the length of a ladder that will just reach from the outer edge of the moat to the top of the fort? *Ans.* 25 feet.

4. Two men travel from the same place, one due east, and the other due north. One travels the first day 60 miles, and the other 80 miles. How far apart are they at the end of the day? *Ans.* 100 miles.

5. A line 36 meters long will exactly reach from the top of a perpendicular tower standing on the brink of a river, known to be 24 meters broad, to the opposite bank; what is the hight of the tower? *Ans.* 26.83+ meters.

6. A tree broken off 30 feet from the ground and resting on the stump, touches the ground 40 feet from the stump; what was the hight of the tree? *Ans.* 80 feet.

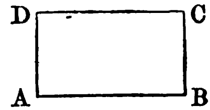
7. The rafters of a house, each 25 feet long, meet at the edge of the roof 15 feet above the attic floor; required the width of the house. *Ans.* 40 feet.

What is the second Principle? The third?

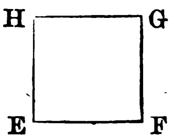
QUADRILATERALS.

412. A **Quadrilateral** is a polygon having four sides, and therefore four angles.

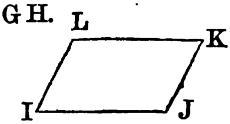
413. A **Parallelogram** is a quadrilateral having its opposite sides parallel.



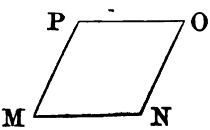
A RECTANGLE is a right-angled parallelogram; as the figure A B C D.



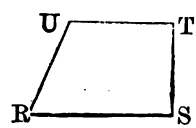
A SQUARE is a rectangle having equal sides; as the figure E F G H.



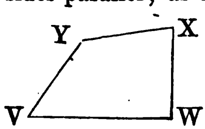
A RHOMBOID is a parallelogram having no right angles; as the figure I J K L.



A RHOMBUS is a rhomboid having equal sides; as the figure M N O P.



414. A **Trapezoid** is a quadrilateral having only two of its sides parallel; as the figure R S T U.



415. A **Trapezium** is a quadrilateral having no two of its sides parallel; as the figure V W X Y.

AREAS OF TRIANGLES AND QUADRILATERALS.

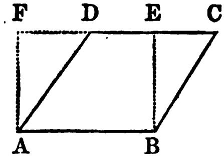
416. By Geometry, may be proved, in relation to areas, the following

PRINCIPLES.

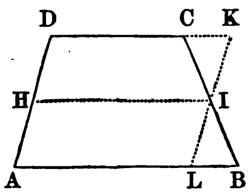
1. *The area of a* PARALLELOGRAM *is equal to the product of the base by the altitude.*

What is a Quadrilateral? A Parallelogram? A Rectangle? A Rhomboid? A Rhombus? A Trapezoid? A Trapezium? To what is the area of a parallelogram equal?

This has been shown to be the case with a rectangle (Art. 210), and that it applies equally to a rhomboid or rhombus, appears from the diagram, in which the rhomboid A B C D is equivalent to the rectangle A B E F, of the same base and altitude.



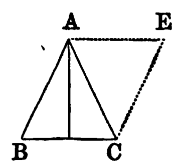
2. *The area of a* TRAPEZOID *is equal to the product of half the sum of the parallel sides by the altitude.*



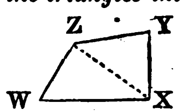
For, any trapezoid A B C D is equivalent to a parallelogram A L K D of the same altitude, and whose base A L is equal to H I, which is half of A B + C D.

3. *The area of a* TRIANGLE *is equal to the product of half the base by the altitude, or of half the altitude by the base.*

For, any triangle A B C is equivalent to one half of the parallelogram B C E A, of the same base and altitude.



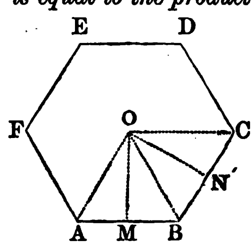
4. *The area of a* TRAPEZIUM, *or of any polygon, is equal to the sum of the areas of the triangles into which it may be resolved.*



Thus, the trapezium W X Y Z is equal to the triangle W X Z plus the triangle X Y Z, made by the diagonal X Z.

5. *The area of a* REGULAR POLYGON *is equal to the product of the perimeter by half the perpendicular drawn from the center to any one of the sides.*

For, any regular polygon, A B C D E F, may be resolved into as many equal triangles as it has sides, by drawing from the center, O, the lines O A, O B, O C, etc.



To what is the area of a trapezoid equal? Of a triangle? Of a trapezium? Of a regular polygon?

Exercises.

1. What is the area of a board 18.8 feet long and 2.7 feet wide? *Ans.* 50.76 sq. ft.

2. What is the area of a board 28 feet long and 15 inches broad? *Ans.* 35 sq. ft.

3. If the base of a gable of a house be 40 feet long and its perpendicular hight 20 feet, how many square feet of boards will be required to cover it? *Ans.* 400 sq. ft.

4. How many acres in a triangular lot, one side measuring 32 rods, and the shortest distance from this side to the opposite angle being 14 rods? *Ans.* 1 A. 64 sq. rd.

5. If the parallel sides of a lot be 75 and 33 yards, and its breadth 20 yards, what is the area in square rods? *Ans.* 35.7+ sq. rd.

6. How many hectares in a rectangular meadow 640 meters long and 240 meters wide? *Ans.* 15 hectares and 36 ares.

7. One of the diagonals of a field in the form of a trapezium is 160 rods long, and the perpendiculars from the opposite angles to that diagonal are 70 and 50 rods; what is the area? *Ans.* 60 acres.

417. When the three sides of a triangle are given, we may, to find the area,

Take half the sum of the three sides, subtract therefrom each side separately, multiply together the four results, and extract the square root of the product.

8. The sides of a triangle are 13, 84, and 85 rods, respectively; what is its area? *Ans.* 3 A. 66 sq. rd.

9. The sides of a certain field in the form of a trapezium measure 30, 35, 40, and 25 rods, respectively, and the diagonal which forms a triangle with the first two sides, 45 rods; what is the area? *Ans.* 6 A. 61.8 sq. rd.

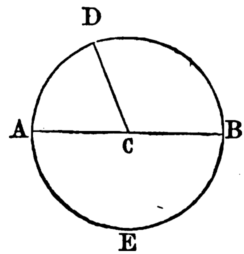
10. What is the area of a regular hexagon, whose sides

When the three sides of a triangle are given, how may the area be found?

are each 14.6 feet, and the perpendicular from the center to a side 12.64 feet? *Ans.* 553.63+ sq. ft.

CIRCLES.

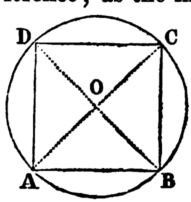
418. A **Circle** is a plane figure bounded by a curved line, all the points of which are equally distant from a point within, called the center.



The CIRCUMFERENCE is the bounding line; as the line A E B D.

The DIAMETER is any straight line drawn through the center and terminating in the circumference; as the line A B.

The RADIUS is any straight line drawn from the center to the circumference; as the lines C A, C B, or C D.



419. A *Square* is said to be *inscribed* in a circle when the vertices of its angles are in the circumference. Thus,

The square A B C D is inscribed in a circle.

420. By Geometry there may be proved the following

PRINCIPLES.

1. *The* CIRCUMFERENCE *of every circle is nearly* 3.1416 *times its diameter.* Hence,

2. *The* CIRCUMFERENCE *is equal to the product of the diameter by* 3.1416; and

3. *The* DIAMETER *is equal to the quotient of the circumference divided by* 3.1416.

What is a Circle? The Circumference? The Diameter? The Radius? How many times the diameter is the circumference? To what is the circumference equal? *The diameter?*

4. *The* AREA *is equal to the product of the circumference by one half of the radius, or by one fourth of the diameter.* Hence,

5. *The* AREA *is equal to the product of the square of the diameter by* .7854; and

6. *The* DIAMETER *is equal to the square root of the quotient of the area divided by* .7854.

7. *The* SIDE *of every square inscribed in a circle is nearly* .7071 *times the diameter, or* .2251 *times the circumference;* also,

8. *The* SIDE *of every square inscribed in a circle is equal to the square root of half the square of the diameter.*

9. *The* SIDE *of a square equal in area to a given circle is equal to the product of the diameter by* .8862.

Exercises.

1. What is the circumference of a circle whose diameter is 20 feet? *Ans.* 62.83+ feet.

2. What is the diameter of a circle whose circumference is 142 yards? *Ans.* 45.19+ yards.

3. What is the area of a circle whose diameter is 100 yards? *Ans.* 7854 sq. yd.

4. What must be the side of a square stick of timber that can be hewn from a round stick 24 inches in diameter? *Ans.* 16.97 inches.

5. A wheel is 5 feet in diameter; what is the length of its tire?

6. The area of a circle is 5 acres 146 square rods; what is the diameter? *Ans.* 34.7 rods.

7. What is the surface in ares of a circular fish-pond which is 50 meters in diameter? *Ans.* 19 ares 63.5 centares.

To what is the area equal? The side of every inscribed square? The side of a square equal in area to a given circle?

8. What is the side of a square equal in area to a circular plat 50 feet in diameter? *Ans.* 44.31 feet.

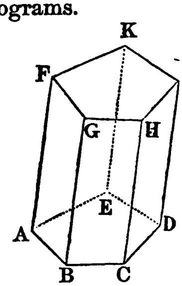
9. What must be the length of a tether fastened to a horse's neck, that it may sweep over just one acre? *Ans.* 7.136+ rods.

10. How large a square can be cut out of a circular piece of plank 300 inches in circumference?

11. How many rods in length must be a rope, such as, with one end fastened to a stake in a meadow, and the other to the nose of a cow, will allow her to graze over just 2 acres?

PRISMS AND CYLINDERS.

421. A **Prism** is a volume having two faces equal and parallel polygons, and the other faces parallelograms.

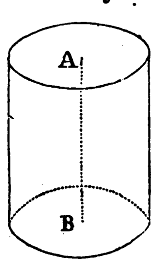


The BASES of a prism are its equal and parallel polygons.

The CONVEX SURFACE of a prism is formed of its lateral faces, or parallelograms.

The prism is *triangular, quadrangular,* etc., according as its base is a triangle, quadrilateral, etc. Thus,

The diagram represents a pentangular prism, whose bases are A B C D E, and F G H I K, and whose convex surface is formed by the faces A B G F, B C H G, etc.



422. A **Cylinder** is a round body of uniform diameter, whose bases are equal and parallel circles.

The ALTITUDE of a cylinder, or of a prism, is the straight line joining the centers of the two bases. Thus,

The diagram represents a cylinder, of which A B is the altitude.

What is a Prism? The Bases of a prism? The Convex Surface? A Cylinder? The Altitude?

423. By Geometry, there may be established the following

PRINCIPLES.

1. *The* CONVEX SURFACE *of a prism is equal to the product of the perimeter of the base by the altitude.*

2. *The* CONVEX SURFACE *of a cylinder is equal to the product of the circumference of the base by the altitude.*

3. *The* ENTIRE SURFACE *of a prism or cylinder is equal to the convex surface plus the area of the bases.*

4. *The* CONTENTS *of a prism or cylinder are equal to the product of the area of the base by the altitude.*

Exercises.

1. What is the entire surface of a square prism whose side is 4 feet wide and length 30 feet?

SOLUTION. $30 \times 4 \times 4 = 480$; $4 \times 4 \times 2 = 32$; $480 + 32 = 512$ sq. ft., *Ans.*

2. Required the convex surface of a roller 4 feet in diameter and 10 feet long? *Ans.* 125.66+ sq. ft.

3. Required the contents of a cylinder 90 centimeters in diameter and 10 meters in length. *Ans.* 6.36+ cubic meters.

4. If each side of the base of a triangular prism be 2 inches and its length 14 inches, what are its contents?

5. What are the contents of a stick of timber 22 feet 7 inches long, 1 foot 5 inches broad, and $6\frac{1}{2}$ inches thick? *Ans.* 17.329+ cu. ft.

PYRAMIDS AND CONES.

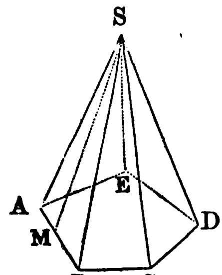
424. A **Pyramid** is a body whose base is any polygon, and whose other faces are triangles meeting at a common point.

To what is the convex surface of a prism equal? The convex surface of a cylinder? The entire surface? The contents of a prism or cylinder? What is a Pyramid?

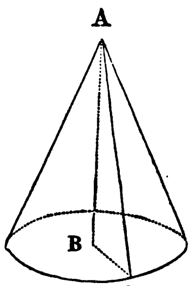
The VERTEX is the common point at which the triangular faces meet.

The CONVEX SURFACE is formed of the triangular faces.

The diagram represents a pentangular pyramid, whose vertex is S, and whose convex surface is formed by the faces A S B, B S C, C S D, etc.



425. A **Cone** is a body whose base is a circle, and whose convex surface tapers uniformly to a point at the top, or vertex.



The ALTITUDE of a pyramid or cone is a straight line drawn from the vertex perpendicular to the base. Thus,

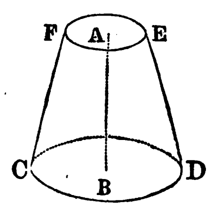
In the diagram the line A B represents the altitude of a cone.

The SLANT HIGHT of a pyramid or cone is the shortest straight line that can be drawn from the vertex to the perimeter or circumference of the base. Thus,

In the diagram the line A C represents the slant hight of the cone.

426. The **Frustum** of a pyramid or cone is the part which remains after cutting off the top by a plane parallel to the base. Thus,

The diagram C D E F represents the frustum of a cone.



427. By Geometry, there may be established the following

What is the Vertex of a pyramid? How is the Convex Surface formed? What is a Cone? The Altitude of a pyramid or cone? The Slant Hight? The *Frustum of a* pyramid or cone?

PRINCIPLES.

1. *The* CONVEX SURFACE *of a pyramid or cone is equal to the product of the perimeter or circumference of the base by half the slant hight.*

2. *The* ENTIRE SURFACE *of a pyramid or cone is equal to the convex surface plus the area of the base.*

3. *The* CONVEX SURFACE *of the frustum of a pyramid or cone is equal to half the product of the sum of the perimeters or circumferences of the two bases by the slant hight.*

4. *The* ENTIRE SURFACE *of a frustum of a pyramid or cone is equal to the convex surface plus the areas of the two bases.*

5. *The* CONTENTS *of a pyramid or cone are equal to the product of the area of the base by one third of the altitude..*

6. *The* CONTENTS *of a frustum of a pyramid or cone are equal to the sum of the areas of the two bases plus the square root of their product, multiplied by one third of the altitude.*

Exercises.

1. What is the surface of a square pyramid, each side of whose base is 3 feet, and the slant hight 24.05 feet?

Ans. 153.3 sq. ft.

2. Required the number of yards of canvas that will cover a conical tent the slant hight of which is 20 feet and circumference of the base 60 feet. *Ans.* $66\frac{2}{3}$ sq. yd.

3. If the slant hight of a frustum of a triangular pyramid is 12 decimeters, each side of the one base 15 decimeters, and of the other base 9 decimeters, how many square meters is its entire surface? *Ans.* 5.6449 sq. meters.

4. If one of the largest of the Egyptian pyramids is 477 feet in slant hight, and each side of its base, which is square, is 720 feet, what are the contents in solid yards? *Ans.* 2003200 cu. yd.

To what is the convex surface of a pyramid or cone equal? The entire surface? The convex surface of the frustum of a pyramid or cone? The entire surface? The contents of a pyramid or cone? Of a frustum of a pyramid or cone?

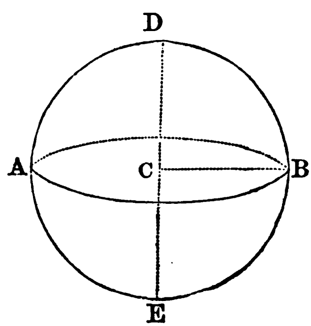
5. Required the number of cubic feet in a conical stack of hay whose hight is 21 feet and the diameter of whose base is 9.5 feet. *Ans.* 496.176 cu. ft.

6. The diameter of the larger end of a round spar is 30 inches, that of the smaller end 18 inches, and the length 45 feet; required its contents. *Ans.* 144.31+ cu. ft.

7. If the length of a stick of timber, in the form of the frustum of a pyramid, be 18 feet 8 inches, the side of its larger end 27 inches, and that of its smaller 16 inches, how many cubic feet are there in it? *Ans.* 61.228+ cu. ft.

SPHERES.

428. A **Sphere** is a volume bounded by a curved surface, all points of which are equally distant from a point within called the center.



429. The RADIUS of a sphere is a straight line drawn from the center to any point in the surface.

430. The DIAMETER of a sphere is a straight line drawn through its center, and terminated both ways by the surface. Thus,

In the diagram the line C B denotes the radius and D E the diameter of a sphere.

By Geometry, there may be proved the following

PRINCIPLES.

1. *The* SURFACE *of a sphere is equal to the product of the*

REVIEW QUESTIONS. What is Mensuration? (409) A Triangle? (410) A Quadrilateral? (412) A Circle? (418) A Prism? (421) A Pyramid? (424) A Cone? (425) — What is a Sphere? The Radius of a sphere? The Diameter?

circumference by the diameter, or to the product of 3.1416 *by the square of the diameter.*

2. *The* CONTENTS *of a sphere are equal to the product of the surface by one third of its radius, or to the product of one sixth of* 3.1416 *by the cube of the diameter.*

Exercises.

1. What is the surface of a cannon ball whose diameter is 9 inches? *Ans.* 254.46+ sq. in.

2. How many cubic meters in a sphere whose diameter is 12 centimeters? *Ans.* .000904+ cu. me.

3. Required the contents of a globe 15 inches in diameter. *Ans.* 1767.15 cu. in.

4. What is the surface of the earth, allowing it to be a sphere 7912 miles in diameter? *Ans.* 196663355.75+ sq. m.

SIMILAR FIGURES AND VOLUMES.

431. Two **Figures**, or two **Volumes**, are similar, when they exactly correspond in form, without regard to size.

From the relations of similar figures, or of similar volumes, to each other, which may be proved by Geometry, we have the following

PRINCIPLES.

1. *The areas of similar figures and volumes are to each other as the squares of their corresponding dimensions.* Hence,

2. *The corresponding dimensions of similar figures and volumes are to each other as the square roots of their areas.*

3. *The contents of similar volumes are to each other as the cubes of their corresponding dimensions.* Hence,

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To what is the surface of a sphere equal? The contents of a sphere? When are two Figures or two Volumes similar? What is the first Principle? The second? The third?

4. *The corresponding dimensions of similar volumes are to each other as the cube roots of their contents.*

Exercises.

1. If a triangle whose base is 20 feet has an area of 200 feet, what is the area of a similar triangle whose base is 10 feet?

SOLUTION. $20^2 : 10^2 :: 200 : 50$, *Ans.*

2. If a circle whose diameter is 12 feet has an area of 113.09 square feet, what is the area of a circle whose diameter is 15 feet? *Ans.* 176.70 sq. ft.

3. If it costs $125 to pave a rectangular court whose width is 40 feet, what will it cost to pave a similar court whose width is 30 feet? *Ans.* $70.31¼.

4. If a triangle whose altitude is 40 feet has an area of 1000 square feet, what is the altitude of a similar triangle whose area is 900 feet? *Ans.* 37.947 ft.

5. If the weight of a cannon ball 8 inches in diameter is 36 kilos, what is the weight of a similar ball 9 inches in diameter? *Ans.* 51.25+ kilos.

6. If a sphere of silver 1 inch in diameter be worth $6, what must be the diameter of another sphere to be worth $10368? *Ans.* 12 inches.

7. A bushel measure is 18½ inches in diameter; what must be the diameter of a half bushel measure of similar form? *Ans.* 14.68+ inches.

8. If the side of a cubical box is 2 feet, what must be the side of a similar box which shall contain 3 times as much? *Ans.* 2.88+ feet.

9. If a cylindrical pipe 20 centimeters in diameter will fill a cistern in 11¼ minutes, how long will it take a similar pipe 30 centimeters in diameter to fill it? *Ans.* 5 minutes.

10. If two men own together a conical stack of hay, which

REVIEW QUESTIONS. What is a Sphere? (428) The Radius of a sphere? (429) The Diameter of a sphere? (430)

is 16 feet in hight, how far down from the top must one of them take off for his part, if it is $\frac{1}{8}$ of the whole? *Ans.* 8 feet?

BOARD MEASURE.

432. Lumber, or sawed timber, as boards, planks, joists, and beams, are usually measured by board measure.

In Board Measure 1 foot is reckoned 1 foot long, 1 foot broad, and 1 *inch* thick. Hence,

433. To find the contents of boards, planks, joists, etc.

Multiply the product of the length and breadth, each taken in feet, by the number denoting the thickness in inches.

When the boards, planks, etc., are tapering, take half the sum of the breadth of the two ends for the breadth.

Since 1 foot board measure is 1 foot or 12 inches long, 1 foot or 12 inches broad, and 1 inch thick, it must be equal $12 \times 12 \times 1 = 144$ cubic inches. 144 cubic inches are contained in 1728 cubic inches, or in 1 cubic foot, 12 times. Hence,

12 *board feet* = 1 *cubic foot.*

Exercises.

1. What are the contents of a board 20 feet long and 16 inches broad? *Ans.* $26\frac{2}{3}$ bd. ft.

2. How many square feet in 2 planks, each 16 feet long, 18 inches wide, and 3 inches thick? *Ans.* 144 bd. ft.

3. What are the contents of 6 joists, 14 feet long, and 4 inches square? *Ans.* 112 bd. ft.

4. What is the cost of a stick of timber 24 feet long, 10 inches wide, and 6 inches thick, at 3 cents a square foot? *Ans.* $3.60.

5. What are the contents of a plank 22 feet long, and $3\frac{1}{2}$ inches thick, the width of the ends being 16 and 20 inches respectively? *Ans.* $115\frac{1}{2}$ bd. ft.

How is lumber usually measured? How do we find the contents of *boards, planks,* etc.?

GAUGING.

434. Gauging is the process of finding the capacity of casks.

The **Mean Diameter** of a cask is very nearly equal to the head diameter increased by two thirds of the difference between the bung and head diameters, or by three fifths when the staves are but slightly curved.

The capacity of a cask is that of a cylinder of the same length and mean diameter. Hence,

435. To find the capacity of casks,

Multiply the product of the square of the mean diameter and the length, expressed in inches, by .0034 *for liquid gallons, or by* .0129 *for liters.*

Multiplying by .0034 is the same as multiplying by .7854 and dividing by 231, and by .0129 the same as by .7854 and dividing by 61.022.

Since a liter is equal to one cubic decimeter, there will be in a cask 1000 times as many liters as cubic meters.

Exercises.

1. How many gallons in a cask whose mean diameter is 18 inches, and whose length is 30 inches? *Ans.* 33+ gallons.

2. How many gallons in a cask 36 inches long, 22 inches bung diameter, and 16 inches head diameter?
Ans. 48.96 gallons.

3. How many gallons in a cask whose length is 60 inches, bung diameter 36 inches, and head diameter 32 inches?

4. How many liters in a cask 1 meter long, and whose mean diameter is 60 centimeters? *Ans.* 282.744 liters.

What is Gauging? The Mean Diameter of a cask? What is the capacity of a cask? How can we find the capacity of casks? How may the capacity of a cask in liters be found, when its contents in cubic meters are known?

MEDIAL PROPORTION.

436. Medial Proportion, or Average, treats of mixing different articles.

This subject has sometimes been called *Alligation*, from the mechanical method formerly adopted of linking or tying together figures by a line, in the process of solving its questions.

CASE I.

437. To find the average value of given quantities of different values.

1. Let it be required to find the average value of a mixture of 8 lb. of sugar, worth 10 cents a pound, with 12 lb., worth 15 cents a pound.

OPERATION.

$$\begin{array}{rcr} \$.10 \times 8 & = & \$.80 \\ .15 \times \underline{12} & = & \underline{1.80} \\ 20 &) & \underline{2.60} \\ & & \$.13 \end{array}$$

8 pounds of sugar, at 10 cents a pound, are worth \$.80, and 12 pounds of sugar, at 15 cents a pound, are worth \$1.80; hence, the whole 20 pounds are worth \$.80 + \$1.80, or \$2.60.

If 20 pounds of the mixture are worth \$2.60, one pound of it is worth $\frac{1}{20}$ of \$2.60, which is \$.13.

RULE. *Divide the entire value of the mixture by the entire quantity, and the result will be the average value.*

Examples.

2. A farmer mixed 8 bushels of oats, worth 50 cents a bushel, 12 bushels of corn, worth 65 cents a bushel, and 10 bushels of barley, worth 60 cents a bushel; what was the average value of the mixture a bushel? *Ans.* \$.59$\frac{1}{3}$.

3. A grocer mixed together 18 lb. of oolong tea, at \$1 a pound, 6 lb. of souchong, at \$.60 a pound, and 6 lb. of hyson, at \$1.20 a pound. How much a pound is the mixture worth? *Ans.* \$.96.

What is Medial Proportion? Explain the operation. What is the Rule?

CASE II.

438. To find what quantities of different kinds must be taken to form a mixture of a given value.

1. Let it be required to find what quantities of coffee, at 14 cents, 16 cents, 19 cents, and 22 cents a pound, must be taken to form a mixture worth 18 cents a pound.

OPERATION.

$$18\text{ c.}\left\{\begin{array}{llll} 14\text{ c., to gain 1 c.} & \text{take} & \frac{1}{4}\text{ lb.} \\ 16\text{ c.,} \quad \text{“} \quad \text{“} & \text{“} & \frac{1}{2}\text{ lb.} \\ 19\text{ c., to lose 1 c.} & \text{“} & 1\text{ lb.} \\ 22\text{ c.,} \quad \text{“} \quad \text{“} & \text{“} & \frac{1}{4}\text{ lb.} \end{array}\right\} \times 4 = \left\{\begin{array}{l} 1\text{ lb.} \\ 2\text{ lb.} \\ 4\text{ lb.} \\ 1\text{ lb.} \end{array}\right.$$

If we take 1 lb. at 14 cents to form a mixture worth 18 cents, we gain 4 cents, and to gain 1 cent we must take $\frac{1}{4}$ of a lb.

If we take 1 lb. at 22 cents, we shall lose 4 cents, and to lose 1 cent, so as to balance exactly what we have just gained, we must take $\frac{1}{4}$ of a lb.

In like manner, we find we must take $\frac{1}{2}$ of a lb. of the 16 cent kind to gain 1 cent, and to lose 1 cent, to exactly balance that gain, we must take of the 19 cent kind 1 lb.

Therefore, we may take $\frac{1}{4}$ lb. of the coffee at 14 cents, $\frac{1}{2}$ lb. at 16 cents, 1 lb. at 19 cents, and $\frac{1}{4}$ lb. at 22 cents, or by multiplying these proportionals by 4, the least common multiple of the denominators of the fractions, which will make all the quantities integral, we may take 1 lb. at 14 cents, 2 lb. at 16 cents, 4 lb. at 19 cents, and 1 lb. at 22 cents, to form a mixture worth 18 cents a pound.

Also, since proportional quantities will remain proportional, when multiplied or divided by any quantity, an indefinite number of results, all answering the conditions of the question, may be obtained.

RULE. *Find how much must be taken of a kind whose value is less than the given average, to gain* 1 *of that average; also, how much must be taken of a kind whose value is greater than the given average, to lose* 1.

In like manner, compare the value of each of the other kinds with the average value.

If there are fractions in the results, multiply each of the

Explain the operation. What is the Rule?

numbers by the least common multiple of their denominators, and the several products will be the required parts, in integers.

We may, also, clear of fractions simply by multiplying each pair of results corresponding to couplets of quantities compared, by the least common multiple of the denominators of their own fractions.

Examples.

2. How much sugar, at 10, 14, 17, and 18 cents a pound, may be mixed together, so that the mixture shall be worth 16 cents a pound? *Ans.* 1, 3, 6, and 3.

3. How much rice, at 4, 6, and 11 cents a pound, may be mixed, so that the compound shall be worth 7 cents a pound?

4. A grocer wishes to mix brandy at 3, 5, and 7 dollars a gallon, with water, so that the mixture may be worth $4 a gallon; how many gallons of each kind may be taken?

5. A farmer bought pigs at $6 each, sheep at $9 each, and colts at $10 each; how many may he have bought if he paid for all on an average of $8 apiece? *Ans.* 2, 2, and 1.

CASE III.

439. To find the quantities of other kinds, when the quantity of one kind in the mixture is limited.

1. Let it be required to find how much coffee, at 16, 20, and 24 cents a pound, must be mixed with 10 lb., at 18 cents, so that the mixture may be worth 22 cents.

OPERATION.

$$22\text{ c.}\begin{cases}16\text{ c., to gain} & 1\text{ c. take} & \frac{1}{6}\text{ lb.}\\ 18\text{ c., ``} & 1\text{ c. ``} & \frac{1}{4}\text{ lb.}\\ 20\text{ c., ``} & 1\text{ c. ``} & \frac{1}{2}\text{ lb.}\\ 24\text{ c., to lose } 1+1+1\text{ c.} & & \text{``}\ \frac{1}{2}+\frac{1}{2}+\frac{1}{2}\text{ lb.}\end{cases}\times 40=\begin{cases}6\frac{2}{3}\text{ lb.}\\ 10\text{ lb.}\\ 20\text{ lb.}\\ 60\text{ lb.}\end{cases}$$

$$10\text{ lb.} \div \tfrac{1}{4}\text{ lb.} = 40\text{ times.}$$

By comparing the price of the articles with the average price (Art. 438), we find, by taking $\frac{1}{6}$ lb. of the 16 cent article, $\frac{1}{4}$ lb. of the 18 cent, and $\frac{1}{2}$ lb. of the 20 cent, there is a total gain of 3 cents, which may be

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Explain the operation.

balanced by taking $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=1\frac{1}{2}$ lb. of the 24 cent, at a corresponding loss of 3 cents.

But, of the 18 cent kind it is required to take 10 lb., or 40 times $\frac{1}{4}$ lb.; and taking the other proportional parts also 40 times as large, we have, as the required mixture, $6\frac{2}{3}$ lb. of the 16 cent article, 10 lb. of the 18 cent, 20 lb. of the 20 cent, and 60 lb. of the 24 cent.

RULE. *Find the proportional quantities as in the preceding case, and take these proportional quantities as many times as large as the proportional quantity found of the proposed rate is contained times in the given quantity.*

Examples.

2. A merchant has sugar at 9, 10, 13, and 14 cents a pound; how much may be mixed with 30 pounds of the 14 cent kind, to make a mixture worth 12 cents per pound?

3. How much water of no value may be mixed with 50 liters of wine, at \$.90 per liter, to reduce the price to \$.75 per liter? *Ans.* 10 liters.

4. Sold some cows at 80, 60, and 40 dollars each; also, 30 at 100 dollars each, and found that the whole averaged 70 dollars each; how many may there have been sold of the first three kinds? *Ans.* 1, 1, and 30.

CASE IV.

440. To find the quantity of each kind, when the entire quantity is limited.

1. How much sugar, of kinds at 10, 12, 18, and 20 cents a pound, must be taken to fill a box with a mixture of 200 pounds at 15 cents a pound?

OPERATION.

$$15\text{ c.}\begin{cases}10\text{ c., to gain } 1\text{ c. take } \frac{1}{5}\text{ lb.}\\ 12\text{ c., \quad `` \quad } 1\text{ c. \quad `` \quad } \frac{1}{3}\text{ lb.}\\ 18\text{ c., to lose } 1\text{ c. \quad `` \quad } \frac{1}{3}\text{ lb.}\\ 20\text{ c., \quad `` \quad } 1\text{ c. \quad `` \quad } \frac{1}{5}\text{ lb.}\end{cases}\Bigg\} \times \frac{375}{2} = \begin{cases}37\frac{1}{2}\text{ lb.}\\ 62\frac{1}{2}\text{ lb.}\\ 62\frac{1}{2}\text{ lb.}\\ 37\frac{1}{2}\text{ lb.}\end{cases}$$

$$200\text{ lb.} \div \frac{16}{15}\text{ lb.} = \frac{375}{2}\text{ times.}$$

Repeat the Rule. Explain the operation.

We find the proportional quantities (Art. 438) to be $\frac{1}{5}$ lb. of the 10 cent kind, $\frac{1}{3}$ lb. of the 12 cent, $\frac{1}{3}$ lb. of the 18 cent, and $\frac{1}{5}$ lb. of the 20 cent.

The sum of these quantities is only $\frac{16}{15}$ lb.; but the entire quantity of the proposed mixture must be 200 lb.; hence, each of the proportional parts must be taken as many times as large, as $\frac{16}{15}$ lb. is contained times in 200 lb., or $\frac{375}{2}$ times.

Taking $\frac{1}{5}$ lb., $\frac{1}{3}$ lb., $\frac{1}{3}$ lb., and $\frac{1}{5}$ lb., respectively, $\frac{375}{2}$ times, we have $37\frac{1}{2}$ lb. of the 10 cent kind, $62\frac{1}{2}$ lb. of the 12 cent, $62\frac{1}{2}$ lb. of the 18 cent, and $37\frac{1}{2}$ lb. of the 20 cent.

RULE. *Find the proportional quantities as in preceding cases, and take each of these quantities as many times as large as their sum is contained times in the given entire quantity.*

Examples.

2. What quantities of tea, worth \$.96, \$.90, and \$.78 per pound, respectively, may be taken to form a mixture of 112 pounds, at \$.88 per pound? *Ans.* $16\frac{32}{33}$, $67\frac{29}{33}$, $27\frac{5}{33}$.

3. A farmer has oats worth 40, 60, and 70 cents a bushel; what quantity of each kind may he take to make 40 bushels worth 50 cents a bushel?

4. A jeweler has gold 18, 19, and 24 carats fine; what quantity of each may he take to make 1 pound 20 carats fine? *Ans.* $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{4}$.

5. A grocer has sugar worth 7 cents a pound, which he would mix with some at 8 cents a pound, some at 10 cents, and some at 11 cents a pound. How much of each kind may he take to make a mixture of 90 pounds worth 9 cents a pound?

6. How much wine, at \$2.40, \$2.60, \$2.80, and \$2.90 per gallon, may be taken to make a hogshead worth \$2.70 per gallon? *Ans.* 18, 9, 9, 27.

What is the Rule? What is Medial Proportion? (436) What is it sometimes called? (436) What is the Rule in Case I.? (437) In Case II.? (438) In Case III.? (439)

SERIES.

441. A **Series** is a succession of numbers that depend on one another according to some fixed law.

The TERMS of a Series are the numbers composing it;

The EXTREMES are the first and last terms; and

The MEANS are all the terms between the first and last.

442. A series is *increasing* when the terms increase from left to right, and *decreasing* when they decrease from left to right.

ARITHMETICAL SERIES.

443. An **Arithmetical Series, or Progression,** is a series in which the terms vary by a common difference. Thus,

2, 4, 6, 8, 10, 12,

is an increasing arithmetical series, in which 2 is the common difference.

444. The first term, the last term, the number of terms, the common difference, and the sum of the terms, in an arithmetical series, are so related that, any three of them being given, the other two may be found.

CASE I.

445. To find any term in an arithmetical series.

Let $3 =$ the first term, $2 =$ the common difference, and $5 =$ the number of terms. Then,

2d term $= 3 + 2$; 3d term $= 3 + 2 \times 2$; 4th term $= 3 + 2 \times 3$; 5th term $= 3 + 2 \times 4$.

That is, when the first term, the common difference, and the number of terms are given, to find any term of the series,

To the first term add the product of the common difference by the number of terms which precede the required term, if the

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What is a Series? The Terms of a series? The Extremes? The *Means?* *An Arithmetical* Series?

series is increasing; or from the first term subtract the same if the series is decreasing.

Exercises.

1. If the ages of 5 persons are in arithmetical progression, the youngest being 15 years old, and the common difference is 2, what is the age of the oldest? *Ans.* 23 years.

2. A merchant bought 34 yards of cloth, and agreed to give 12 cents for the first yard, $12\frac{1}{3}$ cents for the second, $12\frac{2}{3}$ cents for the third, and so on; what did the thirty-third yard cost him? *Ans.* $22\frac{2}{3}$ cents.

3. I have 40 books; the first is worth $1.80, the second $1.77, the third $1.74, and so on, each being worth 3 cents less than the preceding; what is the value of the last book? *Ans.* $.63.

CASE II.

446. To find the common difference and the number of terms.

Let 3, 5, 7, 9, 11 be an arithmetical series.

Then, by the preceding case, the last term $11 = 3 + 2 \times 4$, and subtracting the first term 3, we have 2×4, or the product of the common difference by the number of terms less one.

Hence, to find the common difference,

Divide the difference of the extremes by the number of terms less one.

Also, to find the number of terms,

Divide the difference of the extremes by the common difference, and add one to the quotient.

Exercises.

1. The extremes of an arithmetical series are 5 and $27\frac{1}{2}$, and the number of terms 11; what is the common difference?

Ans. $2\frac{1}{4}$.

2. The extremes of an arithmetical series are 5 and $27\frac{1}{2}$, and the common difference $2\frac{1}{4}$; what is the number of terms?

How can we find the common difference? The number of terms?

3. A person traveling went the first day 3 miles, and increased his speed every day by 5 miles, till at last he went 58 miles in one day; how many days did he travel? *Ans.* 12 days.

CASE III.

447. To find the sum of all the terms of an arithmetical series.

Let	2,	4,	6,	8,	10,	12, be an arithmetical series,
and	12,	10,	8,	6,	4,	2, be the same inverted.

Then $14 + 14 + 14 + 14 + 14 + 14 = 84$, the sum of both series.

But 84 is equal to 14, the sum of the extremes, multiplied by 6, the number of terms; and half of 84, or of 14×6, is the sum of one of the series. Hence, to find the sum of all the terms,

Multiply the sum of the extremes by the number of terms, and take half the product.

Exercises.

1. The clocks of Venice strike from 1 to 24; how many strokes do one of these clocks make in one day? *Ans.* 300.

2. If a person on a journey travel the first day 30 miles, and each succeeding day a quarter of a mile less than he did the day before, how far will he travel in 30 days? *Ans.* $791\frac{1}{4}$ miles.

3. If 100 eggs be laid one yard distant from one another in a straight line, and a basket be placed one yard from the first one, what distance must a person travel to gather them singly and place them in the basket? *Ans.* 5 m. 1300 yd.

GEOMETRICAL SERIES.

448. A **Geometrical Series,** or **Progression,** is a series in which the terms vary by a common multiplier. Thus,

3, 9, 27, 81, 243,

is an increasing geometrical series, in which 3 is the common multiplier.

How do we find the sum of all the terms? What is a Geometrical Series?

The RATE, or RATIO, of a geometrical series is the common multiplier.

CASE I.

449. To find any term in a geometrical series.

Let 4 = the first term, 2 = the rate, and 5 = the number of terms. Then,

2d term $= 4 \times 2$; 3d term $= 4 \times 2^2$; 4th term $= 4 \times 2^3$; 5th term $= 4 \times 2^4$.

That is, when the first term, the rate, and the number of terms are given, to find any term of the series,

Multiply the first term by the rate raised to a power whose exponent is equal to the number of terms which precede the required term.

Exercises.

1. Find the 8th term of a geometrical series whose first term is 6 and rate 2. *Ans.* 768.

2. Find the 6th term of a geometrical series whose first term is 4096 and rate $\frac{1}{4}$. *Ans.* 4.

3. A gentleman dying left 11 sons, to whom he bequeathed his property, as follows: to the youngest he gave $1024; to the next, as much and a half; to the next, $1\frac{1}{2}$ of the preceding son's share, and so on. What was the eldest son's portion? *Ans.* $59049.

CASE II.

450. To find the rate of a series.

Let 4, 8, 16, 32, 64, be a geometrical series.

Then, by the preceding case, the 5th term $64 = 4 \times 2^4$, and dividing by the first term, we have 2^4, or the fourth power of the rate.

Hence, to find the rate of a geometrical series,

Divide the last term by the first, and extract that root of the quotient whose index is denoted by the number of terms less one.

What is the Rate or Ratio of a geometrical series? How do you find any *term?* The rate?

Exercises.

1. Find the rate of a geometrical series whose first term is 6, last term 768, and number of terms 8. *Ans.* 2.

2. Find the rate of a geometrical series whose first term is 4096, last term 4, and number of terms 6. *Ans.* $\frac{1}{4}$.

3. A man paid a debt by making 12 payments in geometrical progression, the first payment being $3, and the last $6144; what was the rate? *Ans.* 2.

CASE III.

451. To find the sum of all the terms of a geometrical series.

Let 4, 12, 36, 108, be a geometrical series; and multiplying it by the rate 3, we have

a second series = 12, 36, 108, 324, whose sum is 3 times
the first series, 4, 12, 36, 108. Subtracting, we have
twice the sum of the first = — 4, 0, 0, 0, 324, or 324 — 4; that is, the sum of the first = $\frac{324-4}{2}$, which is the difference between the first term and the product of the last term by the rate, divided by the rate less one. Hence, to find the sum of all the terms,

Multiply the last term by the rate, subtract the first term from the product, and divide the result by the rate less one.

Exercises.

1. Find the sum of a geometrical series whose extremes are 2 and 128, and rate 4. *Ans.* 170.

2. If the descendants of the 101 persons who landed at Plymouth, in the year 1620, had increased so as to double their number in every 20 years, how great would have been the aggregate to the year 1860? *Ans.* 413595.

3. A jockey offered to sell some fine horses to a young man not well versed in numbers, and receive in payment $1 for the first, $3 for the second, $9 for the third, and so on. The young man, thinking it a great bargain, agreed accordingly; what did the horses cost him, provided there were 12 of them? *Ans.* $265720.

How do we find the sum of all the terms?

ANNUITIES.

452. An **Annuity** is a fixed sum of money payable at the end of equal periods of time.

An annuity is said to be *forborne*, or in *arrears*, when payments are not paid when due.

453. The Amount, or Final Value, of an annuity is the sum of the amounts of all its payments at interest from the time each becomes due.

454. The Present Value of an annuity is such a sum as, put at interest, will, for the given time and rate, exactly amount to the annuity.

Pensions, rents, reversions, life insurance, etc., involve the principle of annuities.

CASE I.

455. To find the amount of an annuity at simple interest.

1. Required the amount of an annuity of $100, forborne five years, at 6 % simple interest.

At the end of the 5th year there will be due: the 5th year's payment, or $100 ; the 4th year's payment, $100, plus 1 year's interest, or $106; the 3d year's payment, $100, plus 2 years' interest, or $112; the 2d year's payment, $100, plus 3 years' interest, or $118; and the 1st year's payment, $100, plus 4 years' interest, or $124.

Hence, the sums due are $100 + $106 + $112 + $118 + $124, or $560.

But the sums due at the end of the 5th year form an arithmetical series, of which the annuity, or $100, is the *first term*, its interest for 1 year, or $6, is the *common difference*, and the number of years, or 5, is the *number of terms*. Hence,

Find the amount of the first payment for the last term of an arithmetical series, and then the sum of the series for the amount of the annuity.

What is an Annuity? When is an annuity said to be forborne, or in *arrears?* What is the Amount of an annuity? The Present Value of an *annuity?* How do we find the amount of an annuity at simple interest?

Exercises.

2. An annuity of $200 has been in arrears 8 years; what is the amount due, at 6 % simple interest? *Ans.* $1936.

3. To what will a rent of $450 per annum, payable quarterly, amount, if forborne for 11 years, at 6 % simple interest? *Ans.* $6546.37½.

4. If a salary of $450 a year be in arrears 10 years, to how much will it amount at 7 % simple interest? *Ans.* $5917.50.

CASE II.

456. To find the amount of an annuity at compound interest.

1. Required the amount of an annuity of $100, forborne five years, at 6 % compound interest.

At the end of the 5th year there will be due: the 5th year's payment, or $100; the 4th year's payment, $100, plus 1 year's interest, or $106; the 3d year's payment, plus 2 years' compound interest, or $112.36; the 2d year's payment, plus 3 years' compound interest, or $119.1016, and the 1st year's payment plus 4 years' compound interest, or $126.2476+.

Hence, the sums due are $100 + $106 + $112.36 + $119.1016 + $126.2476, or $563.709+.

But the sums due at the end of the 5th year form a geometrical series, of which the annuity, $100, is the first term, the amount of $1 for 1 year, or $1.06, is the rate, and the number of years the number of terms. Hence,

Find the amount of the first payment at compound interest for the last term of a geometrical series, and then the sum of the series for the amount of the annuity.

Exercises.

2. What is the amount of an annuity of $200 a year, forborne 5 years, at 7 % compound interest? *Ans.* $1150.146+.

3. If a person expends for 30 years $40 per annum for cigars, how much will they cost him at 7 % compound interest? *Ans.* $3778.39+.

How do we find the amount of an annuity at compound interest?

4. If you should deposit \$50 every 6 months in a savings bank, to how much would it amount in 25 years, at 3 % semi-annual compound interest? *Ans.* \$5639.84+.

CASE III.

457. To find the present value of an annuity at compound interest.

1. What is the present value of an annuity of \$100, to continue for 5 years, at 6 % compound interest?

The amount of the given annuity for 5 years, by the preceding case, is \$563.709+, and the present value of the annuity must be the present value of the amount (Art. 302). The amount of \$1 at compound interest for the given time and rate, from the table, Art. 319, is \$1.338225.

Hence, the present value is \$563.709 ÷ \$1.338225, or \$421.236+. Hence, to find the present worth of an annuity at compound interest,

Find the amount of the annuity, and divide it by the amount of \$1 at compound interest for the given time and rate.

Exercises.

2. What is the present value of a pension of \$1000 for 4 years, at 7 %? *Ans.* \$3387.207+.

3. What is the present value of an annual rent of \$154 for 19 years, at 5 %? *Ans.* \$1861.13.

4. Bought an estate for \$30000, payable in equal yearly installments of \$5000; how much ready money, at 6 %, should discharge the debt at the time of purchase? *Ans.* \$24586.62.

REVIEW EXERCISES.

1. What is the third power of $11\frac{2}{5}$? *Ans.* $1481\frac{68}{125}$.

2. Find the second power of the third power of 5. *Ans.* 15625.

3. Sold a field for \$484, receiving as many dollars per acre as there were acres; how many acres were there, and what was the price per acre? *Ans.* 22 acres, and \$22 per acre.

How is found the present worth of an annuity at compound interest?

4. There is a certain room, of a cubical form, which contains 1953.125 cubic feet; what is the length of each of its equal sides? *Ans.* 12.5.

5. I have 841 trees, which I wish to set out in a square grove; how many of the trees must be planted in each row?

6. What is the difference between $\frac{1}{2}$ of a solid foot and a solid $\frac{1}{2}$ foot, or a cube whose sides are each $\frac{1}{2}$ of a foot square? *Ans.* 3 solid $\frac{1}{8}$ feet.

7. If a lead pipe 1 inch in diameter will fill a cistern in 4 hours, in what time will 2 pipes, each $\frac{1}{2}$ of an inch in diameter, fill the same? *Ans.* 8 hours.

8. A grocer has two kinds of tea, one at 75 cents a pound, and the other $1.10; how must he mix them in order to afford the mixture at $1 a pound?

9. A man had 10 children whose several ages differed alike, the youngest being 6 years old, and the oldest 51; what was the difference between the ages of the ninth and tenth?

10. What will be the cost of painting a conical spire at $\frac{1}{8}$ of a dollar a square yard, if the slant hight of the spire be 50 feet, and the circumference at the base 26.7 feet? *Ans.* $14.83+.

11. If a horse be tethered equidistant from the four corners of a square lot containing exactly 10 acres, what must be the length of the rope to allow him to graze over every part of the lot? *Ans.* 28.27+ rd.

12. Construct a geometrical series, of which 12 is the first term, and 3072 the 5th term. *Ans.* 12, 48, 192, 768, 3072.

13. I can purchase a farm for $700 cash down, or for $924 to be paid in the course of 7 years, $\frac{1}{7}$ part of the price at the end of each year. Allowing compound interest at 6 %, which terms will be the most advantageous to me?

Ans. Cash down, by $36.86.

REVIEW QUESTIONS. What is a Series? (441) Terms of a Series? (441) Extremes? (441) The Means? (441) Arithmetical Series? (443) Geometrical Series? (448) Rate or Ratio of a Geometrical Series? (448)

Exercises in Analysis.

1. Required the greatest common divisor of $\frac{6}{5}$, $\frac{3}{4}$, and $\frac{9}{10}$.

SOLUTION. *$\frac{6}{5}$, $\frac{3}{4}$, and $\frac{9}{10}$, changed to equivalent fractions having the least common denominator, become $\frac{24}{20}$, $\frac{15}{20}$, and $\frac{18}{20}$.*

The greatest common divisor of 24, 15, *and* 18 *twentieths, is* 3 *twentieths, or* $\frac{3}{20}$.

Therefore, etc.

2. What is the greatest common divisor of $\frac{8}{9}$, $3\frac{3}{7}$, and $6\frac{2}{5}$?

Ans. $\frac{8}{315}$.

3. What is the greatest number that is contained an exact whole number of times in $\frac{2}{3}$, $\frac{3}{8}$, $\frac{6}{7}$, and $2\frac{1}{5}$?

4. What is the least common multiple of $2\frac{1}{4}$, $4\frac{1}{2}$, and $3\frac{3}{8}$?

SOLUTION. $2\frac{1}{4}$, $4\frac{1}{2}$, *and* $3\frac{3}{8}$, *changed to equivalent fractions having the least common denominator, become* $\frac{18}{8}$, $\frac{36}{8}$, *and* $\frac{27}{8}$.

The least common multiple of 18, 36, *and* 27 *eighths, is* 108 *eighths, or* $\frac{108}{8} = 13\frac{1}{2}$.

Therefore, etc.

5. What is the least common multiple of $\frac{3}{4}$, $\frac{6}{7}$, and $\frac{5}{8}$?

Ans. 30.

6. What is the sum of money with which can be purchased a number of hens at \$.75 each, a number of ducks at \$.37½ each, and a number of turkeys at \$2.06¼ each? *Ans.* \$8.25.

7. Find the square root of 225 by factoring.

SOLUTION. 225 *factored is equal to* $5 \times 5 \times 3 \times 3$.

Since the square root of a number is the factor which must be taken twice to form the number (Art. 391), 5 *and* 3, *or one of every two equal prime factors of the number, must be taken to compose its square root.*

Hence, 5×3, *or* 15, *is the square root of* 225.

8. Find the sixth root of 46656 by factoring. *Ans.* 6.

9. Find the cube root of $\frac{250}{686}$ by factoring. *Ans.* $\frac{5}{7}$.

10. A person being asked the hour of the day, said that the time past noon was equal to $\frac{4}{5}$ of the time till midnight. What was the time?

REVIEW QUESTIONS. What is a Unit? (1) A Quantity? (2) A Number? (3) Figures? (18) Notation? (16) Numeration? (17) Addition? (37) Subtraction? (42) Multiplication? (47) Division? (51)

SOLUTION. *The time to midnight is $\frac{5}{5}$ of itself; then $\frac{5}{5}+\frac{4}{5}$, or $\frac{9}{5}$, of the time to midnight, is equal to the time from noon to midnight, which is* 12 *hours.*

If $\frac{9}{5}$ of the time to midnight is equal to 12 *hours, $\frac{1}{5}$ is equal to $\frac{1}{9}$ of* 12 *hours, or* 1 *hour* 20 *minutes, and $\frac{4}{5}$ is equal to* 4 *times* 1 *hour* 20 *minutes, or* 5 *hours* 20 *minutes.*

Hence, the time was 20 *minutes past* 5 *o'clock, P. M.*

11. If the time of day is such that $\frac{2}{3}$ of the time past noon is $\frac{1}{6}$ of the time past midnight, what is the hour?

Ans. 4 o'clock, P. M.

12. What is the hour, if $\frac{1}{2}$ of the time past 10 o'clock, A. M. is the time till 10 o'clock, P. M.? *Ans.* 6 o'clock, P. M.

13. A, B, and C start at the same time from a given point, to travel in the same direction round an island 73 miles in circumference, A at the rate of 6, B of 10, and C of 16 miles per day; in what time will they be next together?

SOLUTION. *Since B travels* 4 *miles a day faster than A, he will gain an entire round of the island, or* 73 *miles, in $\frac{1}{4}$ of* 73 *days, or $18\frac{1}{4}$ days.*

Since C travels 10 *miles a day faster than A, he will gain an entire round in $\frac{1}{10}$ of* 73 *days, or $7\frac{3}{10}$ days.*

Hence, B cannot be with A except at the end of $18\frac{1}{4}$ days, or of some multiple of $18\frac{1}{4}$ days; and C cannot be with A except at the end of $7\frac{3}{10}$ days, or of some multiple of $7\frac{3}{10}$ days.

Therefore, C and B can both be with A for the first time, only after the lapse of a number of days expressed by the least common multiple of $18\frac{1}{4}$ and $7\frac{3}{10}$; and the least common multiple of $18\frac{1}{4}$ and $7\frac{3}{10}$ is $36\frac{1}{2}$.

Therefore, etc.

14. There is an island 73 miles in circumference, and 3 men all start together to travel round it in the same direction; A goes 5 miles a day, B 8, and C 10; when will they all come together again? *Ans.* In 73 days.

15. A and B, at the opposite extremities of a wood, 135 rods in a circuit, begin to go round it in the same direction, at

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REVIEW QUESTIONS. What is a Rule? (11) A Formula? (69) An Operation? (8) Which are the Fundamental Operations? (66) What is a Sign? (38) Symbols of Operation? (67) A Solution? (10) Analysis? (75)

the same time; A at the rate of 11 rods in 2 minutes, and B of 17 rods in 3 minutes. How many rounds will each make before the one will overtake the other?

Ans. A $16\frac{1}{2}$ rounds, and B 17 rounds.

16. At what time after 12 o'clock are the hour and minute hands of a watch next exactly together?

SOLUTION. *The minute hand of a watch passes over* 60 *minute spaces in an hour, and the hour hand over* 5 *such spaces; hence, the minute hand gains* 55 *minute spaces in* 60 *minutes, or* 1 *minute space in* $\frac{1}{55}$ *of* 60 *minutes.*

If the minute hand gains 1 *minute space in* $\frac{1}{55}$ *of* 60 *minutes, it will gain* 5 *minute spaces, or the distance the hands are apart at* 1 *o'clock, in* 5 *times* $\frac{1}{55}$ *of* 60 *minutes, or* 5 *minutes* $27\frac{3}{11}$ *seconds.*

Therefore, the hour and minute hands will be next together after 12 *o'clock at* 5 *minutes* $27\frac{3}{11}$ *seconds after* 1 *o'clock.*

17. A person looking at his watch, was asked the time of day. He replied that it was between 4 and 5, and the hour and minute hands were together; what was the exact time?

Ans. 21 minutes $49\frac{1}{11}$ seconds past 4 o'clock.

18. If a watch which is 10 minutes too fast on Tuesday noon, gains 3 minutes 10 seconds a day, what time will it indicate at $10\frac{1}{4}$ o'clock, A. M., of true time, on the following Sunday? *Ans.* 40 minutes $36\frac{7}{48}$ seconds past 10 o'clock, A. M.

19. A laborer agreed to work 20 days upon the condition that for every day he worked he should receive \$2, but for every day he was idle he should forfeit 50 cents; he received \$25. How many days did he work?

SOLUTION. *Had he worked all the time, he would have received* 20 *times* \$2, *or* \$40; *he therefore lost, by being idle,* \$40 — \$25, *or* \$15.

Since for each day that he was idle he lost \$2 + \$.50, *or* \$2.50, *he must have been idle as many days as* \$2.50 *are contained times in* \$15, *which are* 6.

Hence, he worked 20 *days* — 6 *days, or* 14 *days.*

REVIEW QUESTIONS. What is an Integer? (96) A Fraction? (131) A Common Fraction? (134) A Decimal Fraction? (176) A Denominate Number? (243) A Compound Denominate Number? (243) Reduction? (80)

20. A boy agreed to work 80 days on condition that he should receive 72 cents for every day he worked, and forfeit 48 cents for every day he was idle; at the expiration of the time he was in debt \$12; how many days had he been idle?

Ans. 58 days.

21. A laborer was hired 25 days; for every day he worked he was to receive \$1.25 and board, and for every day he was idle he was to pay board. At the end of the time he received \$23.75. The price of the board was \$.25. How many days did he work? *Ans.* 20 days.

22. Two brothers, A and B, have the same income. A saves $\frac{1}{4}$ of his, but B, by spending \$150 yearly more than A, at the end of 8 years finds himself \$400 in debt. What was the income of each?

SOLUTION. *If B, by spending* \$150 *yearly more than A, is in debt at the end of* 8 *years* \$400, *he spends each year* $\frac{1}{8}$ *of* \$400, *or* \$50, *more than his income.*

Therefore, \$150 — \$50, *or* \$100, *must be what A saves, which is* $\frac{1}{4}$ *of the income of each. If* $\frac{1}{4}$ *of the income of each is* \$100, *the whole income of each must be* 4 *times* \$100, *or* \$400.

Therefore, etc.

23. A and B have the same income. A saves $\frac{1}{8}$ of his, but B, by spending \$30 each year more than A, at the end of 8 years finds himself \$40 in debt. How much does each spend yearly? *Ans.* A \$175, and B \$205.

24. A and B have each an income of \$400 a year; A spends each year \$40 more than B; at the end of 4 years they both together save a sum equal to the income of either. What do they spend annually? *Ans.* A \$370, B \$330.

25. A, B, C, and D bought a grindstone, the diameter of which was 4 feet, each contributing an equal amount, and they wish to grind off their several shares successively. How much may each grind off, no allowance being made for the axle?

REVIEW QUESTIONS. What is Per Cent.? (262) Percentage? (263) Commission? (274) Brokerage? (275) Insurance? (278) Profit and Loss? (283)

SOLUTION. *Since each is evidently entitled to $\frac{1}{4}$ of the flat surface, when the first has ground off his portion, there will remain $\frac{3}{4}$ of that surface.*

Then, since the areas of circles are to each other as the squares of their diameters (Art. 431),

The whole stone : part remaining :: square of diameter of the whole stone : square of diameter of part remaining;

Whence, denoting the whole stone by unity, we have

$1 : \frac{3}{4} :: 4^2$: *square of diameter of part remaining; hence, the diameter of part remaining* $= \sqrt{\frac{3}{4} \times 4^2}$ *feet* $= 3.464$ *feet.*

Therefore, A grinds off 4 *ft.* — 3.464 *ft.* = .536 *feet* = 6.432 *inches.*

After the second has ground off his portion, there will remain $\frac{1}{2}$ of the stone. Whence,

$1 : \frac{1}{2} :: 4^2$: *square of diameter of part remaining; hence, the diameter of part remaining* $= \sqrt{\frac{1}{2} \times 4^2}$ *feet* $= 2.828$ *feet.*

Therefore, B grinds off 3.464 *feet* — 2.828 *feet* = .636 *feet* = 7.632 *inches.*

After the third has ground off his portion, there will remain $\frac{1}{4}$ of the stone. Whence,

$1 : \frac{1}{4} :: 4^2$: *square of diameter of part remaining; hence, the diameter of part remaining* $= \sqrt{\frac{1}{4} \times 4^2}$ *feet* $= 2$ *feet* $= 24$ *inches.*

Therefore, C grinds off 2.828 *feet* — 2 *feet* = .828 *feet* = 9.936 *inches, and there remains as D's share a part* 24 *inches in diameter.*

26. A, B, and C bought a grindstone 60 inches in diameter, which cost them $12, B and C contributing $3 each, and A the remainder. In order for each to get his share according to the sum contributed, how much must each grind off from the diameter, no allowance being made for the axle, A taking his portion first, and then B his?

Ans. A, 17.573+ inches; B, 12.426+ inches; and C, 30 inches.

27. Four ladies bought a ball of silk, 5 inches in diameter; how much of the diameter must each wind off so as to share the silk equally?

Ans. 1st, .45+ inches; 2d, .57+ inches; 3d, .82+; and 4th, 3.14+ inches.

REVIEW QUESTIONS. What is Interest? (288) Simple Interest? (291) *Annual Interest?* (310) Compound Interest? (317) Present Worth? (302) *Discount?* (302) Bank Discount? (307) Partial Payments? (312)

MISCELLANEOUS EXERCISES.

1. How many times may 95 be subtracted from 22515?

2. What part of 3 cents is $\frac{1}{3}$ of 2 cents?

3. On counting my sheep I found that $\frac{1}{2} + \frac{2}{7} + \frac{1}{6}$ of them numbered 80; how many had I? *Ans.* 84.

4. What number, when divided by $\frac{2}{3}$ of $\frac{4}{5}$ of $1\frac{1}{2}$, will produce 1? *Ans.* $\frac{4}{5}$.

5. What is the square of .1?

6. A farmer wishes to separate his farm of 113 A. 145 P. into lots of 12 A. 10 P. each; how many such lots can he have? *Ans.* $9\frac{171}{386}$.

7. What is the difference in time of two places 7′ of longitude apart?

8. William Hardy having engaged to travel 5 miles, became disabled, and was obliged to stop at the end of 3 m. 5 fur. $18\frac{1}{2}$ rd.; what part of his engagement had he completed?

9. How many hills of corn may be planted on an acre of ground, provided they are planted, in the square order, 4 feet apart each way, and none nearer the edge than 2 feet? *Ans.* 2722.

10. A thief having 20 miles the start of an officer in pursuit, goes 6 miles an hour, and the officer follows at the rate of 8 miles an hour; how long before he will overtake the thief? *Ans.* 10 hours.

11. A straight plank is $3\frac{1}{2}$ inches thick and $6\frac{1}{4}$ inches broad; what length must be cut off so as to contain $6\frac{1}{4}$ cubic feet of timber? *Ans.* $41\frac{1}{7}$ feet.

12. A man, who labored under an impediment in his speech, in calling for an article, said, "Give me a half of a—half of a—half of a—half of a gallon of vinegar." Taken at his word how much did he ask for? *Ans.* 2 gills.

Review Questions. What is Ratio? (320) Proportion? (326) Compound Proportion? (332) Partnership? (334) Equation of Payments? (338) Averaging of an Account? (345) Settlement of an Account? (350)

13. When gold sells at a premium of 45 % in currency, how much can be bought for $1 of currency? *Ans.* $.68 $\frac{28}{29}$.

14. If 30 % is lost by selling shoes at 84 cents a pair, at what price should they be sold to gain 20 %? *Ans.* $1.44.

15. How much stock, at 93¼ %, can be purchased for $540, a commission of ⅛ % being charged on the stock purchased? *Ans.* $578 $\frac{26}{83}$.

16. A gentleman promised his son a new arithmetic, on condition that he would go to a certain orchard through three gates, and get a sufficient number of apples, that on his return he could leave half what he had and half an apple more at the first gate, and half the remainder and half an apple more at the second gate, and half of what he had left and half an apple more at the third gate, without cutting any, and then have one remaining. How many must he get? *Ans.* 15.

17. A can do a piece of work in 3 days, B can do three times as much in 8 days, and C five times as much in 12 days; in what time would they all do it together? *Ans.* 21⅓ hours.

18. A merchant sold a lot of coffee at 15 cents a pound, and lost 10 %; soon after he sold another lot, to the amount of $525, and gained 40 %. How many pounds were there in the last lot, and what was the price per pound at which it was sold? *Ans.* 2250 lb., at 23⅓ cents.

19. A man bought a house for $1575, and repaired it for a tenant, who agreed to pay him a rent of $220 per annum, which was 12 % of the money paid for the house and its repairs; what was the cost of repairing it? *Ans.* $258.33⅓.

20. What is the difference between the annual and compound interest of $300 for 4 years at 6 %? *Ans.* $.26+.

21. A note, on 3 months, dated January 6, was discounted March 4; for what time was the discount? *Ans.* 36 days.

REVIEW QUESTIONS. What are Taxes? (353) Duties? (358) *Internal Revenue*? (361) Customs? (363) Exchange? (369) Inland *Exchange*? (376) Foreign Exchange? (379)

22. If you should, on June 20th, buy a note dated January 20th, drawn for $40, on 8 months, what should you pay for it, reckoning bank discount at 2 % a month? *Ans.* $37.52.

23. A, B, and C join their capitals, which are in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$; at the end of 4 months A withdraws $\frac{1}{2}$ of his capital, and at the end of 9 months more they divide their profits, $2840; what should each receive?

Ans. $1020, $1040, and $780.

24. If 100 liters of wine cost 250 francs, what must be the price per gallon, in United States money, to gain 20 per cent.?

25. If 3 lb. of tea be worth 4 lb. of coffee, and 6 lb. of coffee be worth 20 lb. of sugar, how many pounds of sugar can be had for 9 lb. of tea? *Ans.* 40 pounds.

26. An express train leaves Chicago for a point 120 miles west, at 2 o'clock, and goes at the rate of 25 miles per hour; at what time must a slow train, which goes 15 miles in 50 minutes have left, so as not to be overtaken by the express train?

Ans. 12 hours 8 minutes.

27. The merchandise balance of an account is $200 due by averaging the account from June 11; what will be its cash value, if payment be delayed till Sept. 11, or for 3 months, allowing interest at 6 %? *Ans.* $203.

28. A guardian paid his ward $6460 for $5000 which he had in his hands 4 years; what rate of interest did he allow him? *Ans.* $7\frac{3}{10}$ %.

29. How many square feet of 1 inch boards may be sawed from a beam 24 feet long, and whose rectangular ends are 16 inches by 12 inches, if no allowance is made for the thickness of the saw? *Ans.* 384 sq. ft.

30. If the weight of a hectoliter of wheat is 75 kilograms, what weight of wheat will fill a bin 2 meters long, 1.4 meters broad, and 1 meter deep? *Ans.* 2100 kilos.

REVIEW QUESTIONS. What is a Power? (385) A Root? (388) Involution? (386) Evolution? (390) Square Root? (391) Cube Root? (396)

31. In a mixture called coffee, $\frac{1}{2}$ of the whole plus 25 pounds is coffee, and $\frac{1}{3}$ part less 5 pounds is chiccory; what per cent. of the whole is chiccory? *Ans.* $29\frac{1}{6}$ per cent.

32. A has due him $144, payable in 7 months, but the debtor agrees to pay $\frac{1}{2}$ down, and $\frac{1}{3}$ in 4 months; in what time should he pay the balance?

33. What must be the face of a 60 days' note, when money is worth 6 %, to allow of taking $3958 from a bank?

34. If the specific gravity of iron in bars is 7.8 times that of water, and a liter of water weighs a kilogram, what is the weight of a bar of iron 4 meters long, 1 decimeter broad, and 3 centimeters thick? *Ans.* 93.6 kilos.

35. If a pipe 6 inches bore will discharge a certain quantity of water in 3 hours, in what time will 3 pipes, each 3 inches bore, discharge 3 times that quantity? *Ans.* 12 hours.

36. If the merchandise balance of an account is $600, due April 21, what was its cash value, interest at 7 %, on the preceding January 1st? *Ans.* $587.17.

37. A cat watches a mouse which is distant 24 feet, and stealing towards it, advances 5 feet every quarter of an hour, while the mouse goes away 3 feet in the same time; now, allowing the cat the last quarter of an hour to advance 7 feet, how many hours will she be in catching the mouse? *Ans.* $2\frac{3}{4}$ h.

38. A park in the form of a rectangle is 40 rods long and 36 rods wide; what is the length of a straight walk between its opposite corners? *Ans.* 53.81+ rods.

39. A circular garden, containing one acre and forty-one square rods, has a graveled walk of uniform width just within the circle, that takes up 12 square rods of the ground; what is the diameter of the garden, and the width of the walk?

Ans. Diameter of garden, 16 rd. nearly; width of walk, 4+feet.

REVIEW QUESTIONS. What is Mensuration? (409) A Point? (401) A Line? (402) A Plane Figure? (407) When are figures or volumes *similar*? (431) What is a Series? (441) An Arithmetical Series? (443) A *Geometrical* Series? (448) An Annuity? (452)

APPENDIX.

ROMAN NOTATION.

458. The **Roman Notation,** so called because it was used by the ancient Romans, employs, in expressing numbers, seven capital letters:

I,	V,	X,	L,	C,	D,	M,
one,	five,	ten,	fifty,	one hundred,	five hundred,	one thousand.

459. All other numbers may be expressed by combining these letters in accordance with the following

PRINCIPLES.

1. *Repeating a letter repeats the number it denotes.*

2. *By writing a letter denoting a less number* BEFORE *a letter denoting a greater, the number expressed is the* DIFFERENCE *of the numbers.*

3. *By writing a letter denoting a less number* AFTER *a letter denoting a greater, the number expressed is the* SUM *of the numbers.*

4. *A dash (—), makes the number expressed by it a thousand fold.*

Roman Table.

I.	denotes	One.	XVII.	denotes	Seventeen.
II.		Two.	XVIII.		Eighteen.
III.		Three.	XIX.		Nineteen.
IV.		Four.	XX.		Twenty.
V.		Five.	XXX.		Thirty.
VI.		Six.	XL.		Forty.
VII.		Seven.	L.		Fifty.
VIII.		Eight.	LX.		Sixty.
IX.		Nine.	LXX.		Seventy.
X.		Ten.	LXXX.		Eighty.
XI.		Eleven.	XC.		Ninety.
XII.		Twelve.	C.		One hundred.
XIII.		Thirteen.	D.		Five hundred.
XIV.		Fourteen.	M.		One thousand.
XV.		Fifteen.	$\overline{\text{X}}$.		Ten thousand.
XVI.		Sixteen.	$\overline{\text{M}}$.		One million.

METRIC SYSTEM.

460. This system, first adopted by the French in 1795, and by far the most simple and comprehensive that has yet been devised, has now been legalized or adopted by almost all civilized countries.

The metric system, by legislative action, in 1864, was recommended to be taught in the schools of Connecticut.

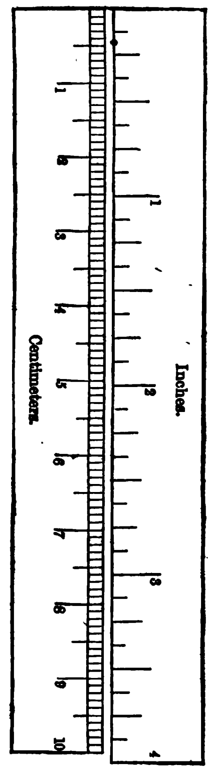
461. Its principal denominations are pronounced thus:

Mē'ter, Âre, Lī'ter, Stêre, Grăm, Mĭl'-li-me-ter, Cĕn'ti-me-ter, Dĕc'i-me-ter, Dĕk'a-me-ter, Hĕc'to-me-ter, Kĭl'o-me-ter, Mȳr'i-a-me-ter, Cĕn'ti-âre, Hĕc'târe, Mĭl'li-lī-ter, etc.

The scales in the margin exhibit a decimeter, or a tenth of a meter, divided into centimeters and millimeters, and four inches divided into eighths of an inch.

462. The equivalents expressed in the tables (Art. 232-238) are recognized, in the United States, in the construction of contracts, and in all legal proceedings.

A DECIMETER.



LIFE INSURANCE.

463. The **Expectation of Life** is the average number of years that persons at different ages live, as shown by life statistics.

The rates of life insurance are based upon the expectation of life of persons of the age of the applicant for a policy.

464. The Carlisle Table and the Wigglesworth Table, showing the expectation of life, as presented on the following page, are chiefly used in the United States.

The former is based on life statistics in Great Britain, and the *latter on life* statistics in this country.

TABLES.

Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.	Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.	Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.	Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.
0	38.72	28.15	24	38.59	32.70	48	22.80	22.27	72	8.16	9.14
1	44.68	36.78	25	37.86	32.33	49	21.81	21.72	73	7.72	8 69
2	47.55	38.74	26	37.14	31.93	50	21.11	21.17	74	7.33	8.25
3	49.82	40.01	27	36.41	31.50	51	20.39	20.61	75	7.01	7.83
4	50.76	40.73	28	35.69	31.08	52	19.68	20.05	76	6.69	7.40
5	51.25	40.88	29	35.00	30.66	53	18.97	19.49	77	6.40	6.99
6	51.17	40.69	30	34.34	30.25	54	18.28	18.92	78	6.12	6.59
7	50.80	40.47	31	33.68	29.83	55	17.58	18.35	79	5.80	6.21
8	50 24	40.14	32	33.03	29.43	56	16.89	17.78	80	5.51	5.85
9	49.57	39.72	33	32.36	29.02	57	16.21	17.20	81	5.21	5.50
10	48.82	39.23	34	31.68	28.62	58	15.55	16.63	82	4.93	5.16
11	48.04	38.64	35	31.00	28.22	59	14.92	16.04	83	4.65	4.87
12	47.27	38.02	36	30.32	27.78	60	14.34	15.45	84	4.39	4.66
13	46.51	37.41	37	29.64	27.34	61	13.82	14.86	85	4.12	4.57
14	45.75	36.79	38	28.96	26.91	62	13.31	14.26	86	3.90	4.21
15	45.00	36.17	39	28.28	26.47	63	12.81	13.66	87	3.71	3.90
16	44.27	35.76	40	27.61	26.04	64	12.30	13.05	88	3.59	3.67
17	43.57	35.37	41	26.97	25.61	65	11.79	12.43	89	3.47	3.56
18	42.87	34.98	42	26.34	25.19	66	11.27	11.96	90	3.28	3.73
19	42.17	34.59	43	25.71	24.77	67	10.75	11.48	91	3.26	3.32
20	41.46	34.22	44	25.09	24.35	68	10.23	11.01	92	3.37	3.12
21	40.75	33.84	45	24.46	23.92	69	9.70	10.50	93	3.48	2.40
22	40.04	33.46	46	23.82	23.37	70	9.18	10.06	94	3.53	1.98
23	39.31	33.08	47	23.17	22.83	71	8.65	9.60	95	3.53	1.62

The premiums of life insurance are generally reckoned at a certain sum, per $1000 of insurance, payable annually, semi-annually, or quarterly.

The payments may be limited to one or more, by the policy; and the insurance may be for life or for a certain number of years.

Exercises.

1. A man 52 years of age gets his life insured for $4000, at the rate of $31.89, to be paid semi-annually. To how much will the premiums amount should he pay them for ten years? $2551.20.

2. If a man at the age of 35 insures his life for $2000, on the plan of 10 annual payments of $52.40 on $1000, and should die at the age of 49, how much more will his family receive than the premiums he paid? $952.

TABLE SHOWING THE NUMBER OF DAYS.

FROM ANY DAY OF	TO THE SAME DAY OF NEXT.											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	234
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	152	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

When February is included between the points of time, a day must be added in leap year.

465. The following exercises will furnish additional examples of the application of the Vermont rule for partial payments (Art. 316).

1. A note for $1000 was given July 1, 1864.

INDORSEMENTS. January 1, 1865, $100; September 1, 1866, $223.99; December 25, 1866, $12.

How much was due, by simple interest, January 1, 1867?

Ans. $803.12.

2. A note for $700 was given February 4, 1864.

INDORSEMENTS. December 18, 1864, $164; June 24, 1865, $200; September 11, 1865, $120; July 5, 1866, $60.

What was due on this note, by annual interest, Nov. 28, 1866?

Ans. $233.50.

3. A note for $625.50 given October 1, 1864.

INDORSEMENTS. January 1, 1865, $200; November 1, 1865, $20; January 1, 1866, $300.

How much was due, by simple interest, May 1, 1866?

Ans. $143.79.

4. A note for $1000 was given January 1, 1866.

INDORSEMENTS. April 1, 1866, $24; August 1, 1866, $4; December 1, 1866, $6; February 1, 1867, $60; July 1, 1867, $40.

What will be due, by annual interest, June 1, 1870?

Ans. $1130.62.

THE END.